Study of Interior Ballistics

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Abstract

The knowledge and comprehension of the processes that occur inside a firearm's tube, since the propeller charge ignition, until the projectile's exit, is important in developing armament, in an economic and safe way. With these processes, mathematical modelling, which is currently a very used approach, it is possible to optimize the explosive charge and the used materials. This study was performed in the scope of the project FIREND, as a result of the cooperation between the Academia Militar and the Instituto Superior Tcnico. The developed model was applied to a system, weapon, propeller charge and projectile, already existent, and the obtained results were validated with the software PRODAS and charts related to the present system. To better understand the importance of the variables involved, it is made a sensitivity analysis to several ballistic parameters, maintaining invariable the dimensions of the studied weapon. In the results analysis, it is verified that the rotating band is a very important element, significantly altering the pressure and speed values. It was also verified that with propeller charges of higher deflagration speed it is possible to obtain higher values of maximum pressure and speed at exiting.
Keywords: Internal ballistic; Artillery Projectile; Pressure; Velocity; Mathematic modelling; Sensibility analysis.

Nomenclature

A	-	Cross Area
b	-	Covolume
c	-	Propellant charge
d	-	Diameter
D	-	Web thickness
f	-	Web fraction
l	-	Initial length of the combustion chamber
L	-	Tube length after l
m_2	-	Effective mass 2
$\overline{p}$	-	Mean pressure
p_s	-	Base pressure
p_B	-	Breech pressure
p_r	-	Resistive pressure
R_L	-	Lagrange Ratio
u_g	-	Gas velocity
u_s	-	Projectile speed
V_i	-	Free volume behind the projectile
x	-	Projectil position

Greek symbols

β	-	Burn rate coefficient
γ	-	Specific Heat Ratio
θ	-	Form factor
λ	-	Propellant force
ρ	-	Density
ϕ	-	Fraction of propellant burnt

1. Introduction

This work falls within a broader project called FRIEND, in which it is intended to develop a high calibre projectile to be propelled by an Artillery howitzer in order to support forest fires fighting. The main data that is intended to supply for the FIREND project are the pressure and temperatures that occur during the whole internal ballistic process. Thus, to better understand the entire process inside a howitzer tube, since the propeller agent is ignited up until the projectile exits, it is made a sensitivity analysis to several ballistic parameters and it is also analysed the way these parameters affect the shoot.

To recover the results it is developed a mathematical modelling that is based on the Carlucci model discribed in the book BALLISTIS Theory and Design of Guns and Ammunition [1], considered as the book of reference in the current work, performing some changes, in order to be closer to the reality. In the results analysis, due to the impossibility of measuring the pressures and speed in real shooting, reference values are considered.

The study area in which the current work is inserted may be considered as a branch of mechanic and chemistry called by Ballistic, which is concerned with the study of phenomena related with projectiles departure, flight and arrival, and also with the effects provoked by the projectile impact in the target. Supporting itself in the laws of physics and experimental rehearsals, throughout time, methods of study have been developed, both at a theoretical and computing modelling level and at an experimental level.

As to be able to study ballistic in more detail, this is normally divided in three distinct areas, referring to different moments of the shooting. The internal ballistic, where the present work is inserted, approaches the gun powders properties and all processes that occur since the propeller charge ignition up until the projectile exits, relying in several areas of mechanics, such as thermodynamics and the fluids mechanic. The external ballistic is responsible by the study of processes that occur after the projectile exits the firearm and during its course in the atmosphere (translation and rotation movements). Lastly, the terminal ballistic (or of effects) studies the effects caused by the impact of the projectile in the goal.

There are also those that consider the intermediate or transition ballistic, referred to the period of time that lasts since the projectile exits the howitzer up until this is no longer acted upon the gases resultant from the propeller charge combustion.

2. Goals

It is intended to develop a projectile, light and with as little environmental impact as possible, that carries chemical substances to fight fires, ensuring desirable exiting speeds and stability conditions during flight, as well as adaptability to the existing charts. In order to reach these goals, it is necessary to quantify the adverse conditions to which the ammunition is subjected to during the whole shooting process. This way, it will be possible to select new materials capable of withholding high pressures and temperatures. To determine these values it was developed a mathematical model based on the Carlucci's model in the book BALLISTIS [1], making some alterations considered relevant.

3. Internal Ballistic Mathematical Model

The current model is within the scope of the zero dimensional models, since it is not considered density variations of the gas resulting from the gunpowder combustion in any of the directions of the cylindrical coordination system.

Model Hypothesis:

To the model development it is assumed several simplification hypothesis, next presented: (i) the thermal losses through the howitzer tube walls are disregarded; (ii) the volumetric mass of the gas resultant of the combustion does not vary spatially (zero dimensional model); (iii) to simplify the equations, in the continuity equation (and only in this equation) it is considered that all explosive charge is instantly converted into gas; (iv) the howitzer tube has constant section and equal to the lower value measured; (v) the propellant gas is not considered ideal, being used the Noble-Abel state equation; (vi) both the friction resistance (contact between projectile and tube) and the air inside the tube, are taken into account substituting the projectile real mass for an effective mass; (vii) after all the propellant charge transformed itself into gas (instant called in the literature as burnout), the evolution of the gas is isentropic.

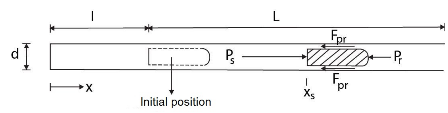

Figure 1: Simplified scheme of the howitzer's tube's inside

From the continuity equation:

$$u_g = \frac{x}{x_s} u_s, \tag{1}$$

with $u_g(0) = 0$.

Verifying that the gas speed varies linearly since the breech up until the projectile base.

Applying the equation of motion for the gas and making a balance of forces to the projectile, results can be obtained from pressure similar to that presented in [1], the average pressure in the combustion chamber being given by:

$$\overline{p} = p_s \left(1 + \frac{c}{3m_2}\right) \tag{2}$$

And defining *Lagrange Ratio*, R_L as:

$$R_L = \frac{1 + \frac{c}{2m_2}}{1 + \frac{c}{3m_2}} \tag{3}$$

The pressure in breech is:

$$p_B = \overline{p} R_L \Rightarrow p_B = \frac{c\phi\lambda}{A x_{s1}} R_L \tag{4}$$

With the definition of the central ballistic parameter M, defined by Corner in 1950 [2]:

$$M = \frac{A^2 D^2}{m_2 c \lambda \beta^2} \left[\frac{1 + \frac{c}{3m_2}}{\left(1 + \frac{c}{2m_2}\right)^2} \right] \tag{5}$$

Considering the Noble-Abel state equation, it is also defined:

$$l_1 = \frac{cb}{A} \; ; \; x_{s1} = x_s - l_1 \phi$$

Verifying that x_s varies according to f as following:

$$\frac{dx_s}{df} = -\frac{M x_{s1}}{1 + \theta f} \tag{6}$$

To $\theta = 0$:

From equation (6) considering the form factor as null, solving the differential equation and defining Q, to simplify the equations. The pressure in the breech can be determined, up until the burnout, through the following expression:

$$p_B = Q \frac{(1-f) M l}{l_1 + e^{M(1-f)} (Ml - l_1)} \tag{7}$$

with:

$$Q = \frac{\lambda c R_L}{V_i},$$

being V_i the initial volume, $V_i = Al$.

In order to determine the maximum pressure, f is determined, numerically such as:

$$l_1 - e^{M(1-f)} \left(Ml - l_1\right) \left[M\left(1-f\right) - 1\right] = 0 \tag{8}$$

With the value of f determined in (8), applying it in (7), it is determined the attained maximum pressure value.

After the burnout, the expression used to determine the pressure is:

$$p_B = Q \left(\frac{x_s - l_1}{l e^M - l_1} \right)^{-\gamma} e^{-M} \tag{9}$$

For $\theta \neq 0$:

If the form factor is different from zero, and following the same line of reasoning as in $\theta = 0$, one has, until the burnout:

$$p_B = Q \frac{l}{\frac{x_s}{\phi} - l_1} \tag{10}$$

with:

$$x_s = l \left(\frac{1+\theta}{1+\theta f} \right)^{M/\theta} + c_1 \left[(1+\theta f) c_2 - (1+\theta f)^2 (M+\theta) - (1+\theta f)^{-M/\theta} c_3 \right] \tag{11}$$

Being: $c_1 = \frac{M l_1}{\theta(M+\theta)(M+2\theta)}$; $c_2 = (1+\theta)(M+2\theta)$;

$$c_3 = \theta(1+\theta)^{M/\theta + 2}$$

Also with this form factor, in order to determine the maximum pressure, the following equation has to be solved numerically:

$$(1+\theta f)^{-\frac{M}{\theta}-1} - (M+\theta)\left(1-f\right)\left(1+\theta f\right)^{-\frac{M}{\theta}-2} = c_4 \tag{12}$$

with:

$$c_4 = \frac{c_1 \left(M+\theta\right)\left(1+\theta\right) - c_1 c_2}{l(1+\theta)^{\frac{M}{\theta}} - c_1 c_3}$$

From this equation it is obtained the value of f, which provides the maximum pressure. Knowing this, it is calculated the respective x_s with (13) and p_{Bmax} with (10).

After the burnout, to determine the pressure, the following expression is used:

$$p_B = p_{Bc} \left(\frac{x_s - l_1}{x_c - l_1} \right)^{-\gamma} \tag{13}$$

In the study of the projectile speed, the equations are the same independently of θ. With ϕ being a dimensionless parameter, with the goal to simplify the equations, already defined in [1]:

$$\Phi = \frac{2}{1-\gamma} \left[\left(\frac{x_s - l_1}{x_c - l_1} \right)^{1-\gamma} - 1 \right]$$

One has up until the burnout:

$$u_s^2 = \frac{\lambda c M (1-f)^2}{m_2 + \frac{c}{3}} \tag{14}$$

As from the burnout, the projectile speed is given by:

$$u_s^2 = \frac{\lambda c}{m_2 + \frac{c}{3}}(M + \Phi) \quad (15)$$

Considering the rotating band:

It is presented a complement to the developed model, in which through the introduction of an initial condition it is intended to simulate the effect of the rotating band.

The suggested initial condition are as following:

$$t = 0\ ;\ x_s = l\ ;\ u_s = 0\ ;\ f = f_0\ ;\ \phi = \phi_0$$

The differential equation is now:

$$\frac{dx_s}{df} = -\frac{Mx_{1s}\,(f_0 - f)}{(1-f)(1+f\theta)} \quad (16)$$

One has up until the burnout:

$$u_s^2 = \frac{\lambda c M\,(f_0 - f)^{\,2}}{m_2 + \frac{c}{3}} \quad (17)$$

and

$$u_c^2 = \frac{\lambda c M f_0^2}{m_2 + \frac{c}{3}} \quad (18)$$

As from the burnout, the projectile speed is given by:

$$u_s^2 = \frac{\lambda c}{m_2 + \frac{c}{3}}\left(M f_0^2 + \Phi\right) \quad (19)$$

For $\theta = 0$:

Determining x_s, using the following equation:

$$x_s = \frac{\Omega}{(1-f)^{M(1-f_0)} e^{Mf}} \quad (20)$$

with:

$$\Omega = (1-f)^{1+M(1-f_0)} l_1 [e^{fM} + e^M E_n(z)] + l(1-f_0)^{M(1-f_0)} e^{Mf_0} - (1-f_0)^{1+M(1-f_0)} l_1 [e^{f_0 M} + e^M E_n(-n)]\ ;$$

$$n = (f_0 - 1)M\ \ ;\ z = M(1-f)$$

For $\theta \neq 0$:
To determine x_s, the following expression is used:

$$x_s(1+\theta f)^{\frac{M(1+f_0\theta)}{\theta(1+\theta)}}(1-f)^{\frac{M(1-f_0)}{1+\theta}} = Ml_1 \int (f_0 - f)(1+\theta f)^{\frac{M(1+\theta f_0)}{\theta(1+\theta)}}(1-f)^{\frac{M(1-f_0)}{1+\theta}} df + const. \quad (21)$$

With the initial condition $x_s(f_0) = l$, we can obtain the value of the constant and using again the function exponencial integral it is possible to deduce an equation for $x_s = x_s(M, , f_0, f, l_1, l)$. As it turns out, the simple change of the initial condition for a value of $f_0 \neq 1$, greatly increases the complexity of mathematical problem.

To any value of θ, the equation that determined the evolution of pressure is:

$$p_B = Q\frac{\phi l}{x_s - l_1\phi} \quad (22)$$

Central ballistic parameter correction
It was considered of interest to verify which was the alteration in the results due to the approximation in equation of β:

$$D\frac{df}{dt} = -\beta\overline{p} \approx \beta p_B \quad (23)$$

The previously presented deductions were made using the approximation, it is now made its accounting, understanding to which point it affects or not the results, applying the exact result where it is considered the average pressure instead of the pressure in the . The alteration makes that in the developed equations M is substituted by MR_L^2, term which was designated by M_1.

4. Models limitations

As the main limitations of the model, we have the inability to exactly represent the rotating band, which is a key element in the pressure created in the combustion chamber and that, at the same time, is responsible by the projectiles flights stability through the rotation movement introduced as it moves inside of the striated tube. The burning rate coefficient is considered constant, by lack of data that specify which are the values of this parameter as it is developed the internal ballistic process and, lastly, the fact that the model only presents values near the real ones in case you consider the charge 2 (as shown below), since it is intended that it properly works for the different charges.

5. Results

Before the study of internal ballistics, it was determined that the **ideal charge**, defined as the charge of gunpowder that, making an increase in mass, minimizes errors obtained. It was found as shown in figures 2 and 3 that charge 2 minimizes the errors of speed and pressure.

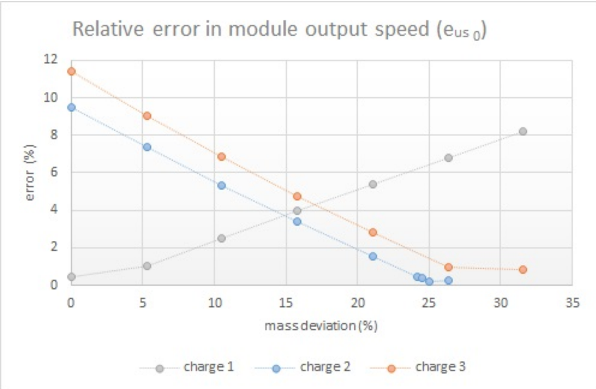

Figure 2: Speed error to the output as a function of deviation in mass

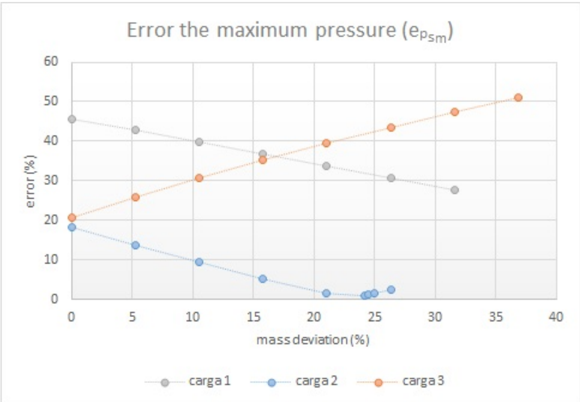

Figure 3: Pressure error as a function of deviation in mass

The table 1 shows that a slight increase in weight also contributes to a reduction of the speed error.

Table 1: Evolution of the errors of $u_{s_{saida}}$ and p_{Sm} to charge 2 as a function of deviation in mass

Deviation (%)	e u_{s_0} (%)	e p_{s_m} (%)
0	9.50	18.27
10.53	5.34	9.43
15.79	3.40	5.30
21.05	1.54	1.36
24.21	**0.47**	**0.92**
24.50	0.37	1.12
25.00	0.21	1.48

The model development being made, the recovery of results went through several phases, starting with a simpler solution ($f_0 = 1$, Without corrections), which does not consider neither corrections in mass nor in M and in friction. Applying these, it was obtained the situation in which it was minimized the error regarding the reference values ($f_0 = 1$, Best solution), lastly it was made the complementary analysis in which starting from an initial condition it is tried to simulate the rotating band ($f_0 = 0.9606$).

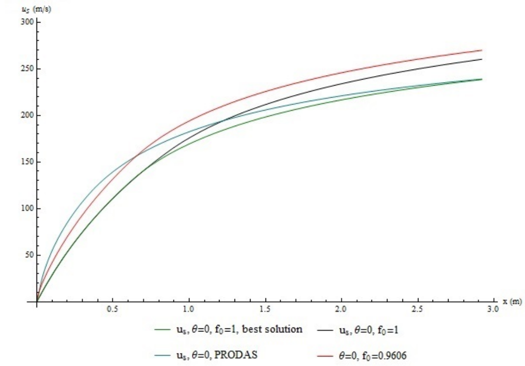

Figure 4: Simultaneous comparison of models Projectile speed.

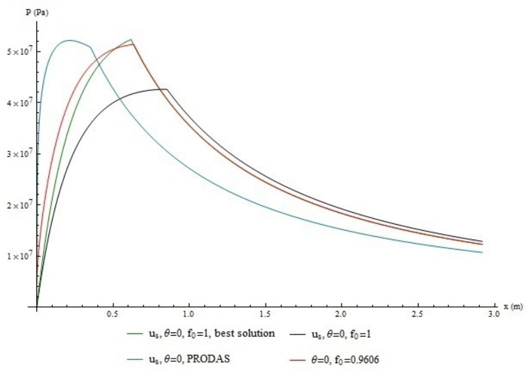

Figure 5: Simultaneous comparison of models Pressure in the base of projectile.

Table 2: Results from the several developed models and the reference values

	e u_{s_0} (%)	e p_{s_m} (%)
$f_0 = 1$	42.64	260.28
$f_0 = 1$, best solution	52.37	238.57
$f_0 = 0.9606$	52.09	269.88
PRODAS	52.17	239.04
Firing tables	-	237.7

Table 3: Errors from the several developed models and the reference values

	e_{psm} (%)	e_{us_0} (%)
$f_0 = 1$	18.27	9.50
$f_0 = 1$, best solution	0.38	0.37
$f_0 = 0.9606$	0.153	13.54
PRODAS	0	0.56
Firing tables	-	0

It is also shown the graphs that show how the pressure curves and speed influence one another being for such changed the value of burning rate β.

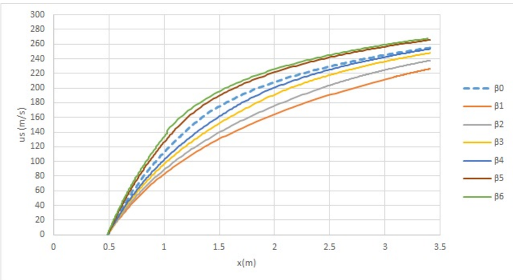

Figure 6: Projectile speed for several β.

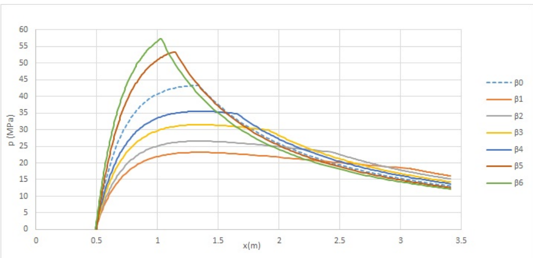

Figure 7: Breech pressure for several β.

6. Conclusions

For the armoury development, either the propeller system or the projectile, it is necessary a good understanding of the phenomena that occur as the projectile is moving in the inside of the firearm, to ensure the safety of the users, as well as the exit speed of the projectile at the barrel to attain the desired goal. The need to understand and account for the phenomena that occur during this process is not an easy task, being a field of study that is being developed for many years, seeking each time more the accuracy of the obtained results, many of which, due to the complexity of the equations that compose the developed models, are only possible to obtain due to a rising ability of computers calculation. It is noteworthy that the acquisition of experimental data has also been evolving accordingly with the development of technology.

In the present work it was performed an analytical model aiming to simulate the shot made by an artillery howitzer, going from an analysis mainly based on thermodynamics and fluids mechanic. It was used as a reference the book Ballistics [1] and, from this, it was considered other hypothesis, such as considering the covolume of the gas produced by the gunpowder combustion.

The projectile taken into account for the current work was the M107 155 mm, as to compare the results obtained by the developed model with values already tabulated.

There was the opportunity to see the system working together, i.e., watching the howitzer M114 A1 155 mm/23 shooting with the projectile M107 155 mm with the propeller charge M3A1 green bag.

The model being developed, the first concern was to verify if this was close to the reference results, either qualitatively, through the curves described by the pressure and speed graphs, as well as quantitatively, by comparing the models results with reference values.

In a first analysis of the models results, it was verified that without applying corrections to the mass the values were a little deviated of the intended, which was what was expected since the model does not concretely represent the rotating band, being its effect taken into account as an increase in mass in the projectile. Applying this alteration in mass, it was verified that the developed model presents good results for the charge 2 of the charge M3A1 green bag, when compared with the reference values, which was designated as ideal charge to the model. With the model validated to, at least, the charge 2, afterwards there was a sensibility analysis in order to verify how certain simplification hypothesis influence the results obtained, as well as some ballistic parameters affect or alter in a more or less significant way.

It was started the sensibility analysis by making a comparison between the model developed in the current work and the one developed in [1], this way being verified that the application of the perfect gases state equation in high pressure environments is not the best choice to be made. It was obtained an alteration in the maximum pressure, (approximately 9%) when considered the gas covolume, which makes sense since, with high pressures and temperature it is unavoidable the interaction of the gases resultant from the combustion.

It was verified that a high importance parameter in this study was the burning rate coefficient (β), which, with its values alteration, was possible to analyse how the pressure and speed curves alter and how these are related, concluding in this work that a bigger maximum pressure value, leads to a higher speed at the barrel. It was noteworthy that those with higher burning rate present higher maximum pressure and exit speed values. This results from these having the ability to release a bigger amount of gases in the first moments of the projectile movement. If the β value is reduced, the maximum pressure is lower, but the pressure at exit is higher, which is justified with the occurrence of a lower speed in transforming the solid charge into gas, leading to the burnout occurring with the projectile neared the barrel.

Aiming to be even neared to the results obtained with the reference values, the friction was represented through an increase in mass in the projectile and was considered exact solution of equation of β.

It was also considered a complement to the model developed in which, through the introduction of an initial condition that intended to simulate the existence of the rotating band, it was verified that the results obtained in the previous model with the mass correction, were more accurate; however, in this study complement, the results were considered satisfactory since the projectiles mass is not directly altered.

References

[1] D. E. Carlucci and S. S. Jacobson, BALLISTICS, Theory and Design of Guns and Ammunition, S. Edition, Ed. CRC Press, 2014.

[2] J. Corner, Theory of Interior Ballistics of Guns. Jhon Wiley and Sons, New York, 1950.