

**Comparative Modal Analysis of Rifle Barrels  
with Different Cross Sections using Finite Elements**

by

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## **KEYWORDS**

Barrel Cross Section

Barrel Rigidity

Cantilever

COMSOL 4.3b

Finite Element Analysis

Modal Analysis

Natural Frequencies

Rifle Barrel

## NOMENCLATURE

$\rho$	Density (kg/m <sup>3</sup> , lb/in <sup>3</sup> )
$E$	Modulus of Elasticity (Pa, psi)
$\nu$	Poisson's Ratio (-)
$M$	Mass Matrix (kg, lbm)
$C$	Damping Matrix (kg/s, lbm/s)
$K$	Stiffness Matrix (N/m, lb/in)
$U$	Displacement Vector (m, in)
$\bar{U}$	Nodal amplitude from vibration (m, in)
$R^{\text{ext}}$	External time dependent forces (N, lb)
$f_n$	Modal frequency (Hz)
$K_n$	Constant dependent on modal shape $n$
$I$	Area moment of inertia (m <sup>4</sup> , in <sup>4</sup> )
$\omega$	Weight per unit length (kg/m, lb/in)
$g$	Gravitational acceleration constant (m/s <sup>2</sup> , in/s <sup>2</sup> )
$l$	Length of the beam (m, in)

## GLOSSARY

- Eigenmode** A normal mode of vibration of an oscillating system, also known as a modal shape or characteristic shape
- Eigenfrequency** A natural resonant frequency of an oscillating system, also known as a natural frequency or modal frequency
- Modal Analysis** Measuring the dynamic response of structures and or fluids under vibrational excitation

## **ACKNOWLEDGMENT**

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## **ABSTRACT**

This project investigated the natural frequencies of vibration and corresponding mode shapes of rifle barrels with different geometric cross sections. Different cross sections are being used by manufacturers today to increase the accuracy of rifles at long distances. The modal shapes of the barrels are the points of interest in determining which barrel cross section is more rigid for the weight. In this study, barrel geometries with different cross sections were created using SolidWorks and then modal analysis was carried out using COMSOL Multi physics 4.3b. The finite element models were verified by comparison with theoretical solutions when available. The rigidity of the barrels were compared to their weights and recommendations are given for optimal design.

# 1. Introduction

## 1.1 Background

A rifle is a firearm that is used for a variety of applications (e.g. military [1], competition, and hunting) for a target that is at extended ranges. There are a multitude of rifle configurations, but the majority of rifle will have the following parts as shown in Figure 1.1. The rifle bolt is a sliding metal component that houses the firing pin and is used to position the cartridge into the breach, close the breach, and eject the spent casing. The ejection/loading port is a hole in the receiver that allows rounds to be loaded and spent casing to be ejected. A receiver is part of the firearm that houses the operating parts of a firearm, such as the bolt, trigger group, magazine port and barrel connection. A sight, whether it's traditional "iron" sights or an optic allows the operator to aim the rifle. A barrel is a metal tube, where the expansion of gases propels the bullet or projectile at a high velocity out of the end of a barrel. The rifle stock is used by the operator to hold and position the rifle into a firing position. The trigger guard protects the trigger from accidentally being actuated. A trigger is the device that allows the operator to fire the rifle.

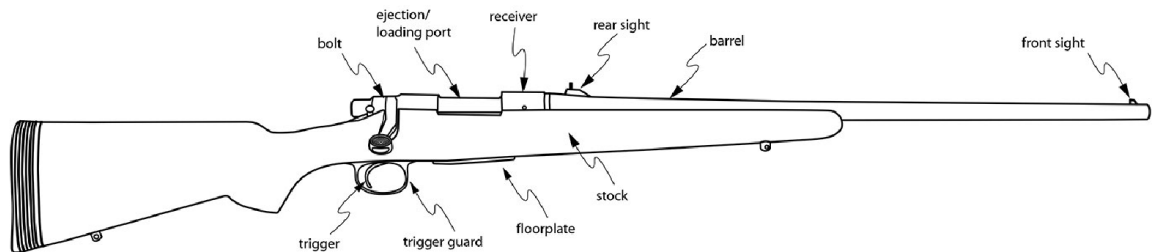


Figure 1.1: Major parts of a rifle [2]

Accuracy and weight are two important characteristics to consider when designing a long range rifle. A long range rifle needs to be inherently accurate, meaning that with a lack of external forces, the rifle is capable of having the bullets' point of impact be repeatable each time the rifle is shot. Accuracy of the rifle, as the distance of the target increases, becomes increasingly important. For example, if the end of the gun barrel is moving up to one minute of arc off the center, the bullet at 100 yards will be off by 1.047 inches. This would not affect the marksman if the goal is hunting an animal which is larger than a circle with a diameter of 2.094 inches. However, the same rifle used at

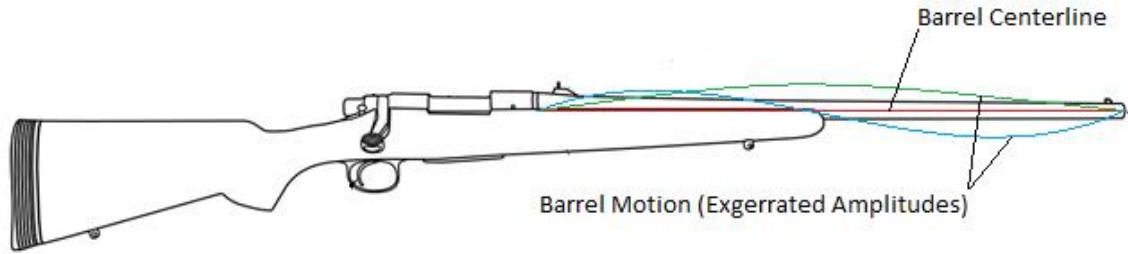
1,000 yards could have the bullet fire in a circle with a radius of 10.47 inches. With the larger circle for point of impact, this effectively eliminates the use of a rifle for all applications, except military applications, where the marksman is firing at something as large as a vehicle to disable it.

The accuracy of the rifle barrel can be affected by many things such as condition, temperature, ammunition compatibility, and rigidity. Barrel condition is how smooth the inside of the barrel is. For example a “worn out” barrel could be the result of corrosion and pitting on the inside of the barrel, where the rifling is no longer continuous. This affects the rifle’s accuracy by changing the gas flow which results in nonsymmetrical loading acting on the bullet or projectile.

Barrel temperature is important because as the rifle is fired the barrel will heat up and depending on the consistency of the firing, the barrel can cool in between shots. The barrel goes through thermal expansion and contraction changing its shape. Depending on the material the stock is made out of and how the barrel is attached to the rest of the rifle, as the barrel expands it can begin to interact with the stock. This applies force on the barrel and can move it out of position.

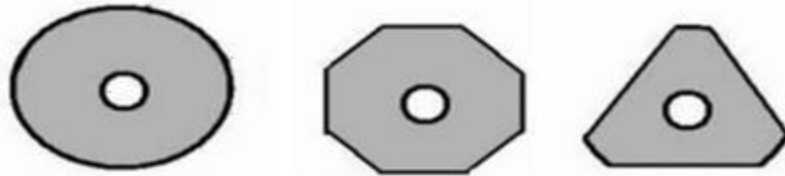
Barrel/ammunition compatibility is how well the barrel rifling and ammunition work with each other. The rifling of a barrel is often optimized for a weight of bullet. For example a heavy bullet will require a higher twist weight in the rifling to allow the bullet to spin. If a lighter bullet (of the same caliber) is fired from the same barrel, it could damage the bullet and rifle barrel. If a heavier bullet is used in a rifle barrel that has a lower twist rate, used for lighter bullets and a heavier bullet is fired from the rifle, it could not put an effective spin on the bullet, making it equivalent to being fired from a smooth bore rifle, making the bullet more likely to destabilize and tumble during flight.

Barrel rigidity is the overall stiffness of the barrel/rifle system. When the rifle is fired, the pressure from the expanding gases acts on the rifle, forcing the rifle cartridge against the back of the breach [3]. The expanding gas from the igniting powder is what creates the forces that cause the recoil in the rifle and vibrate the barrel. If the barrel is not sufficiently rigid the end of the barrel will flex causing the flight path of the bullet to be affected, see Figure 1.2.



**Figure 1.2: Exaggerated examples of barrel motion [2]**

The rifle, if used either for military or sporting (e.g. hunting, competition) purposes, is operated by a person with limited strength and endurance. For the person to perform well over an extended period of time, the rifle should be as light as possible. The barrel is one of the largest components of a rifle. It is usually constructed out of steel making it the heaviest component. This makes the barrel a targeted component for reducing weight. Weight reduction is desired to improve the marksman ability to hold and use the rifle. Manufacturers also want to reduce the weight of the rifle, to use less material and keep costs lower. The barrel weight can be reduced by making non-circular cross sections as shown in Figure 1.3.



**Figure 1.3: Rifle Barrel Cross Sections, Circular (left), Octagonal (middle), and Triangular (right)**

A barrel can be attached to the rifle with varying methods (e.g. threaded, welded, pinned, bolted, or latched connection). The barrel is connected to the receiver and this is the primary connection. When the barrel is only held into the rifle stock by the attachment to the receiver this is called a “free floated” barrel. The benefit of the free floated barrel is that the barrel will not be in contact with the rifle; depending on the stock materials, the stock can fluctuate due to environmental factors and distort the rifle barrel; this affects the distance accuracy of the rifle. Older military bolt action rifles were not free floated and capable of great accuracy. They employed barrel bands to hold the rifle to the stock, while it is still attached to the receiver. The issue with the use of barrel

bands is that they will greater constrain the barrel and cause a change to the barrel's vibration modes in an unpredictable manner [4].

## 1.2 Problem Description

Accuracy of the rifle and weight reduction of the barrel are directly correlated to each other in the design of a rifle. When the rifle barrel is thinned to minimize weight, the barrel becomes less rigid and the rifle's inherent accuracy is affected. To optimize the accuracy and weight of a rifle designers have utilized multiple methods to increase the rigidity of a barrel by minimizing the weight of the barrel. One method is for manufacturers to design a non-circular barrel cross section, see Figure 1.3. The Remington VTR, which uses a triangular shaped cross section would be an example of this method. A second method is to connect the barrel to the stock using a metal band, known as a barrel band. An alternate method for increasing barrel accuracy is to add an adjustable mass to the end of the barrel, known as a tuner [1]. The most basic barrel shape's natural frequencies cannot be hand calculated. The complexity of this problem and its variables demonstrates that a finite element model should be used to calculate the barrel's natural frequencies and modal shapes.

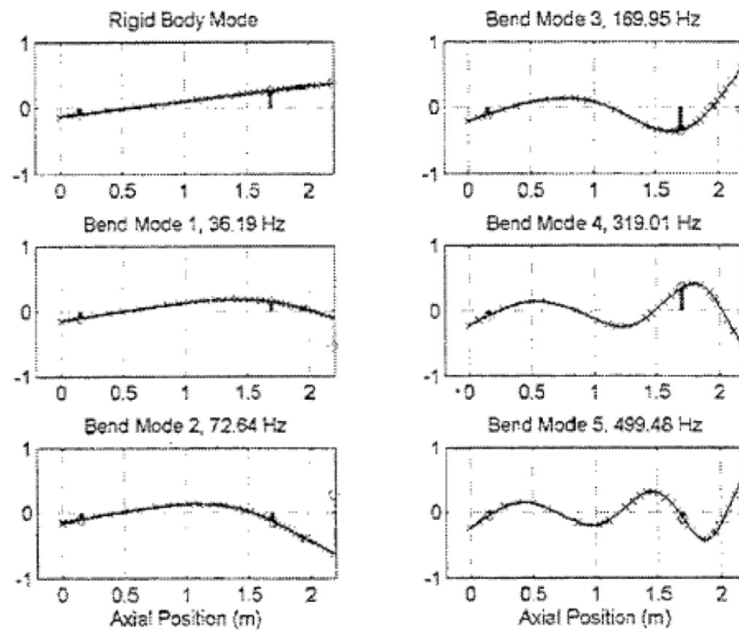
The Slovak republic study [1] used two calibers, 5.56 mm and 7.62 mm. This study will focus on a single caliber. The .308 in (7.62 mm) will be used for this study, as this caliber is the one that would be used for extended ranges. The Slovak Republic study also focused on three barrel type for each caliber, a circular tube, an assault rifle barrel, and an assault rifle barrel with a muzzle compensator on its end. This study will focus on three barrel cross sections; see Figure 1.3, with the straight constant cross section and also tapered barrels with constant changing cross sections.

The barrels will be made out of 4140 steel, a common material for barrels. The properties are listed below in Table 1.1.

**Table 1.1: Material Properties used for 4140 Steel [5]**

Material Property	Value [Units]
Young's Modulus, E	210 [GPa]
Poisson's Ratio, $\nu$	.3
Density, $\rho$	7850 [kg/m <sup>3</sup> ]

The objective of this study is an exploration of natural frequencies of different cross sectional shapes of barrels to determine how much the cross sectional shape of the barrel affects the barrel's modal shape. In a study to design a vibration absorber for a gun barrel the mode shapes were found using an FEA code in MATLAB [6]. The results are shown below in Figure 1.4. The modal shapes only include bending in the vertical plane and rigid body motion. In the MATLAB analysis the barrel was a free barrel design, which would allow for the rigid body motion seen in the upper left column in Figure 1.4.



**Figure 1.4: Damped Mode Shapes and Natural Frequencies for Eight Rod [Vibration] Absorber [6]**

The Slovak Republic's study was three dimensional and found mode shapes that were in torsion and axial extensions, as shown in Figure 1.5. The Slovak study fixed the left end of the barrel, which would prevent the rigid body motion seen in Littlefield, et al's study. The barrel being fixed would also make the torsional Eigenmodes possible. Hak In

Gimm, et al [7] study found when they performed modal analysis on a gun barrel system that each natural frequency in the vertical direction had a nearly identical natural frequency, where the modal shape was in the horizontal direction, as shown in Table 1.2.

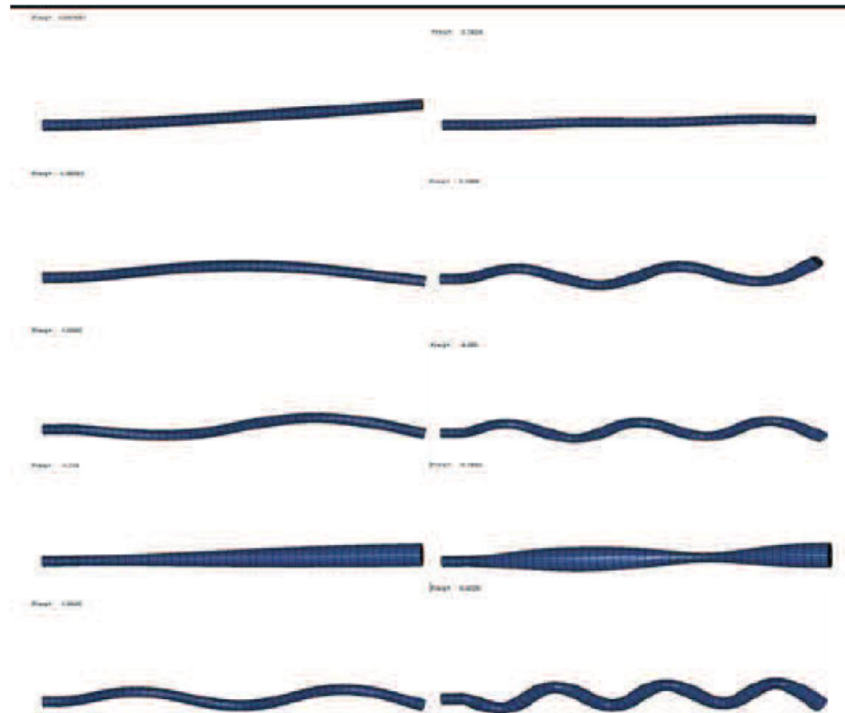


Figure 1.5: Eigenmodes for Cylindrical Barrel (From Left Side to the Right Side is from 1 to 10 Respectively) [1]

Table 1.2: Numerical results from the static and modal analysis [7]

Mode No.	CASE 1	CASE 2	CASE 3
	Y/Z [Hz]	Y/Z [Hz]	Y/Z [Hz]
1	22.2/ 22.4	21.6/ 22.0	31.8/ 35.4
2	103.2/ 103.4	94.8/ 96.5	78.6/ 79.1
3	262.5/ 263.3	234.7/ 237.1	113.0/ 114.7
4	495.4/496.0	440.4/442.9	117.8/117.8
5	535.2/623.4	592.0/612.2	199.2/199.6
Deflection, mm	0.87	0.86	0.34
Stress, MPa	10.7	49.9	10.5

The second objective of the study is to determine which cross sectional shape of barrel is best for accuracy and weight constraints by comparing the frequency achieved (higher means a more rigid barrel) and the weight of the barrel. It will also be determined which barrel cross section, if any, is more influenced by outside boundary conditions, such as a barrel band.

## 2. Methodology

### 2.1 Modal Analysis

Modal analysis in structural mechanics is used to obtain the natural frequencies and modal shapes of structures during free vibrations. The mode shapes and natural frequencies are important to understand to better design a structural system for noise, and vibration applications. When applying an oscillating force to a structure the structure's response is a set deformation. As the rate of oscillation nears the natural frequencies of a structure the response of the structure is a sharp increase in deformation. The deformation that is present at the natural frequencies of the structure are considered the mode shapes of the structure.

The basic form of Newton's law is  $F=ma$ . This can be represented in a multiple dimensional form as shown below [8] in equation (1). Where:  $M$  is the mass matrix;  $C$  is the damping matrix;  $K$  is the stiffness matrix;  $U$  is the displacement vector;  $R^{ext}$  is the externally applied time dependent forces.

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{R^{ext}\} \quad (1)$$

When considering a modal analysis is being performed, the damping matrix is assumed to be negligible and is omitted. It is also free vibration; therefore the external force vector is also omitted. The vibrational motion is considered to be sinusoidal as shown in equation (2), where:  $\bar{U}$  is the nodal amplitudes from the vibration.

$$\{U\} = \{\bar{U}\} \sin \omega t \quad \{\ddot{U}\} = -\omega^2\{\bar{U}\} \sin \omega t \quad (2)$$

The corresponding equation for free vibrations is shown equation (3). Equation (3) can then be solved to get  $n$  number of modes. Where  $\omega^2$  is an eigenvalue, and  $\omega$  is a natural frequency. The matrix  $[K] - \omega^2[M]$  is called the dynamic stiffness matrix.

$$([K] - \omega^2[M])\{\bar{U}\} = \{0\} \quad (3)$$

## 2.2 Exact Solution of a Cantilever Beam

A constant cross section beam with the left end fixed and the right end free has been solved and a solution provided in Roark's formula for stress and strain [9].

$$f_n = \frac{K_n}{2\pi} \sqrt{\frac{EIg}{wl^4}} \quad (4)$$

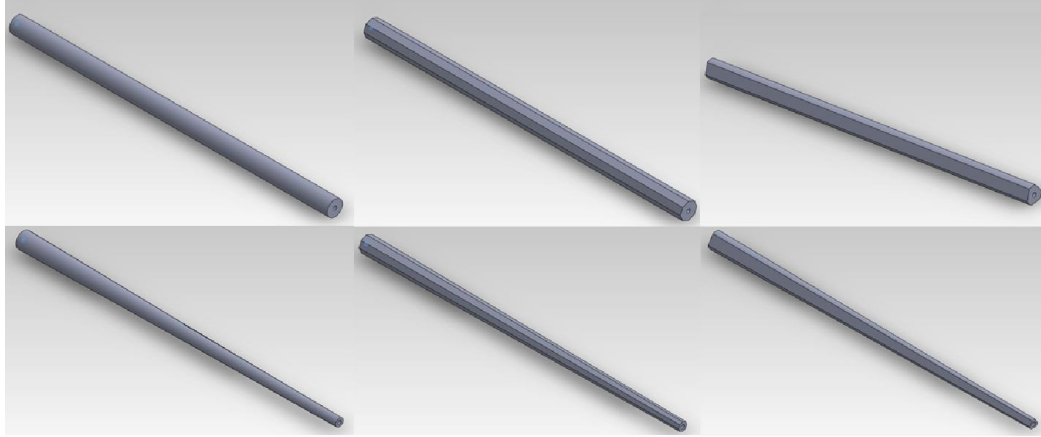
Where  $f_n$  is the modal frequency in hertz;  $K_n$  is the constant dependent on the desired mode;  $E$  is the young's modulus;  $I$  is the area moment of inertia;  $g$  is the gravitational constant;  $w$  is the load per unit length,  $l$  is the length of the beam.

It is necessary to validate the finite element model. In order to do this the first five modes for a square cantilever beam will be calculated to be used as a baseline for the finite element models' results.

## 2.3 Finite Element Modeling

### 2.3.1 3-D Modeling

To create the three dimensional shapes required for the square cross section test beams and the different barrel cross sections, CAD software was used. The CAD software used was Solidworks 2011. The constant cross section barrels were created by sketching the cross section on the YZ plane, and extruded in the X direction to the length of 22, 24, and 26 inches. The tapered barrels were created by sketching the fixed ends cross section on the YZ plane. A secondary parallel plane was created at lengths of 22, 24, and 26 inches. To sketch the cross section of the barrel at the free end of the barrel, Solidworks' lofting feature was used to create a solid model of the tapered barrel. Each barrel has a .30 inch diameter hole cut through the solid model along the X axis in order to simulate the barrel. The rifling of the barrel was ignored in order to be able to better mesh the solid model similar to other studies [1] and [10]. Table 2.1 lists the barrel geometries that were created and examined in this study. Figure 2.1 shows the three cross section shapes of the barrel for both the constant and tapered barrels.



**Figure 2.1: 3-D Solid Models of Rifle Barrels with the Different Cross Section**

**Table 2.1: Barrel Configurations Examined**

1	22 inch Circular Cross Section Full Barrel
2	24 inch Circular Cross Section Full Barrel
3	26 inch Circular Cross Section Full Barrel
4	22 inch Circular Cross Section Tapered Barrel
5	24 inch Circular Cross Section Tapered Barrel
6	26 inch Circular Cross Section Tapered Barrel
7	22 inch Octagonal Cross Section Full Barrel
8	24 inch Octagonal Cross Section Full Barrel
9	26 inch Octagonal Cross Section Full Barrel
10	22 inch Octagonal Cross Section Tapered Barrel
11	24 inch Octagonal Cross Section Tapered Barrel
12	26 inch Octagonal Cross Section Tapered Barrel
13	22 inch Triangular Cross Section Full Barrel
14	24 inch Triangular Cross Section Full Barrel
15	26 inch Triangular Cross Section Full Barrel
16	22 inch Triangular Cross Section Tapered Barrel
17	24 inch Triangular Cross Section Tapered Barrel
18	26 inch Triangular Cross Section Tapered Barrel

### **2.3.2 Importing Geometry**

The rifle barrel models were analyzed using the eigenfrequency and frequency analysis modules in COMSOL 4.3b. As the geometry was created in Solidworks, the file geometries were saved as a STEP 214 file. The STEP files were then converted to COMSOL geometry using COMSOL import tool. Using COMSOL, the 3D model had material properties (Table 1.1) added to the geometry. The boundary conditions were applied following the sub-domain section in COMSOL. The larger end of the tapered barrel is fixed with a zero displacement in all directions. For the constant cross section barrels the ends are the same size, so only one of the ends is fixed. The rest of the barrel is able to move freely. This boundary condition simulates a bolt action rifle that has a free floated barrel, which is when the barrel is only attached to the receiver and is not touching the rifle's stock.

To simulate a barrel band no more geometry models were needed in COMSOL. A section of the geometry was partitioned on the barrel geometry. The partitioned section had an additional boundary condition placed on it. The area was set to a zero displacement in the Y and Z directions, but unrestricted in the X direction (along the axis of the barrel).

### 3. Results and Discussion

#### 3.1 Square Cross Section Beam Test Case

##### 3.1.1 Exact Solution of a Square Cross Section Cantilever Beam

The solution for the modal frequencies of a cantilevered beam of constant cross section was provided solved in Table 16.7, 3b in Roark's formulas for Stress and Strain. The constant given for  $K_n$  are given in Table 3.1 [9].

**Table 3.1: Modal Frequencies of Square Cross Section Beam**

Bending Modes	Mode Constant ( $K_n$ )	Modal Frequencies
n=1	3.52	60.87592
n=2	22.0	380.4745
n=3	61.7	1067.058
n=4	121	2092.61
n=5	200	3458.859

Using a square cross section beam with the material properties of 4140 steel (Table 1.1), and a length of 26 inches the frequency was solved for each of the five modes. The values are given in Table 3.1. These values were compared to the finite element analysis of the baseline model using COMSOL.

#### 3.2 COMSOL Boundary Conditions and Mesh Determination

The COMSOL module for three dimensional Eigenfrequency was opened with the geometries imported from SolidWorks. Once the geometry was imported, the material was defined (see Table 1.1). This was completed by opening the material subdomain and the default values for density, young's modulus, and Poisson's ratio were entered. The boundary conditions (2.3.2) applied on the geometry are a fixed constraint on the ends of the barrels that were located on the YZ plane. When the barrel bands were added, no additional geometry was needed. A partition .25 inches long was created midway down the length of the barrels. The barrel partition had a constraint to prevent motion in the Y and Z direction but can freely move in the X direction, simulating the effects of a barrel band.

The mesh elements chosen were tetrahedral with free mesh parameters. Each model was originally meshed using COMSOL's predefined "extremely coarse" mesh size. This allowed for the fastest computing times. The mesh size was then iteratively decreased to observe changes in results, see Figure 3.1. It was observed that the mesh size quickly approaches a steady solution. This approach was done for each model. In order to verify that the modal frequencies provided by the finite element solution were accurate results, the square cross section test beam was conducted first. The results were then compared to the exact solution provided in Table 3.1.

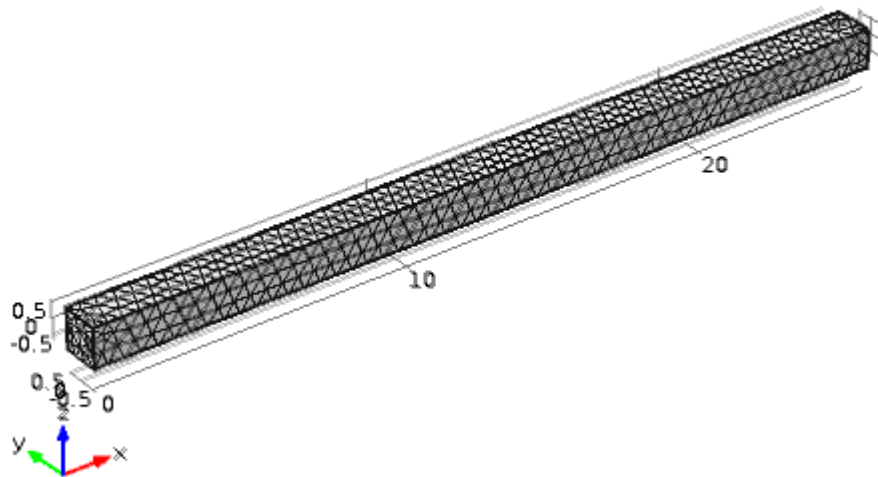


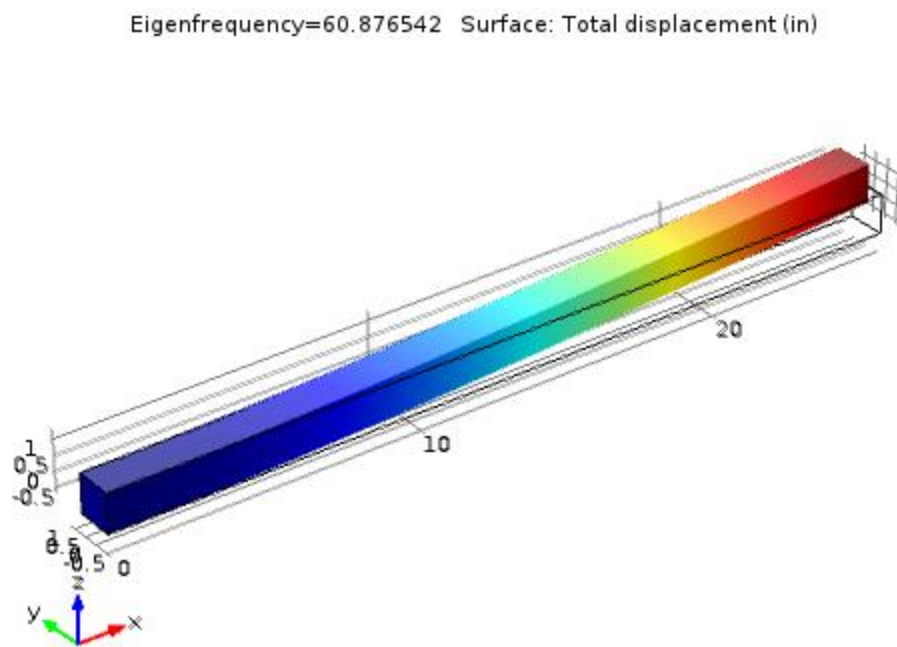
Figure 3.1: Mesh for the Square Test Beam

### 3.3 Finite Element Solution of a Square Cross Section Cantilever Beam

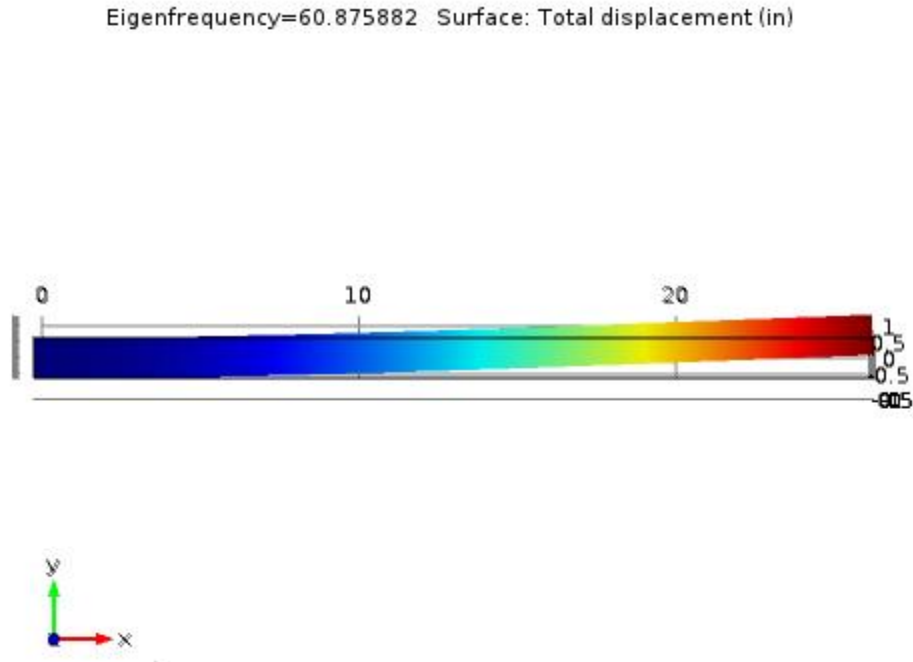
#### 3.3.1 3-D Finite Element Solution

The square cross section test case was first modeled to ensure that the ensuing analysis would give accurate results given the mesh and boundary conditions that were applied. The geometry was meshed using a coarse setting. With the coarse mesh, the results were not aligning with the exact solution. The mesh density was increased to have a total of 4,120 tetrahedral elements. Five modes were chosen to match the exact solution. These

modes are shown in Appendix A. The modes selected were because the majority of their displacement is in the XZ plane. Torsion modes and axial extension modes are not shown. The exact solution only considered mode shapes in a vertical direction, as a result the torsion and axial extensions are not considered. This is also seen in other barrel modal analysis, [10] and [1]. Figure 3.2 shows the square cross section test beam's first modal shape and that it is primarily in the vertical, XZ plane. The frequency occurs at 60.876542 hertz. Then at a frequency of 60.875882 Hertz the square test beam has a modal shape that deforms in a similar manner, except rotated 90 degree to the XY plane, as shown in Figure 3.3.



**Figure 3.2: Square Cross Section Test Beam's First Mode**



**Figure 3.3: Square Test Beam's mode in the XY plane.**

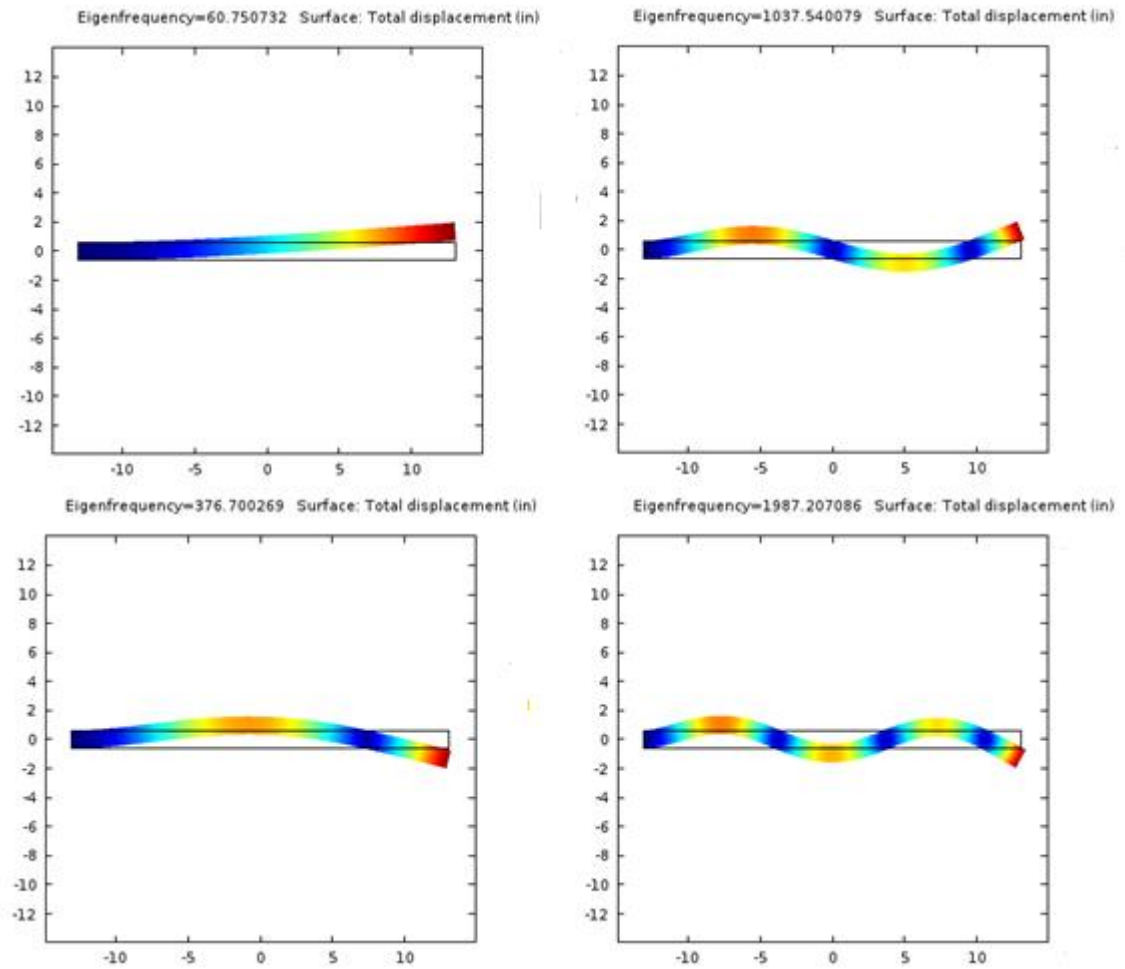
The results of the modal frequencies were in agreement for the first mode of the exact solution. The modal frequencies error increases as the mode number increases as shown in Table 3.2. The results are similar to what was found in Varmint AI's study [10].

### 3.3.2 2-D Finite Element Solution

A two dimensional modal analysis of the square cross section test beam was performed to compare the first five modes to the exact solution. Due to the restriction of two dimensions it was assumed that the first five modes would be the same as the modes provided by the exact solution.

The model was constructed to be a 26 inch long rectangle with a height of 1.25 inches. The same material properties were used for the 2D rectangle as the 3D models. Under the solid mechanics subdomain, a plane strain assumption was used and this resulted in agreement with the exact solution. A free triangle mesh was used in order and the maximum element size was decreased to .26 inches in order to provide an accurate result.

The results agree with the exact solution with low error for the first mode. The error increases as the mode number increases. The frequencies also compared well with the 3D results, see Table 3.2. The modal frequencies are shown in Appendix A. Figure 3.4 shows the first four modes of the 2D test beam. The shapes of the test beam agree with Figure 1.4 and Figure 1.5.



**Figure 3.4: 2D Test Beam's First Four Modes**

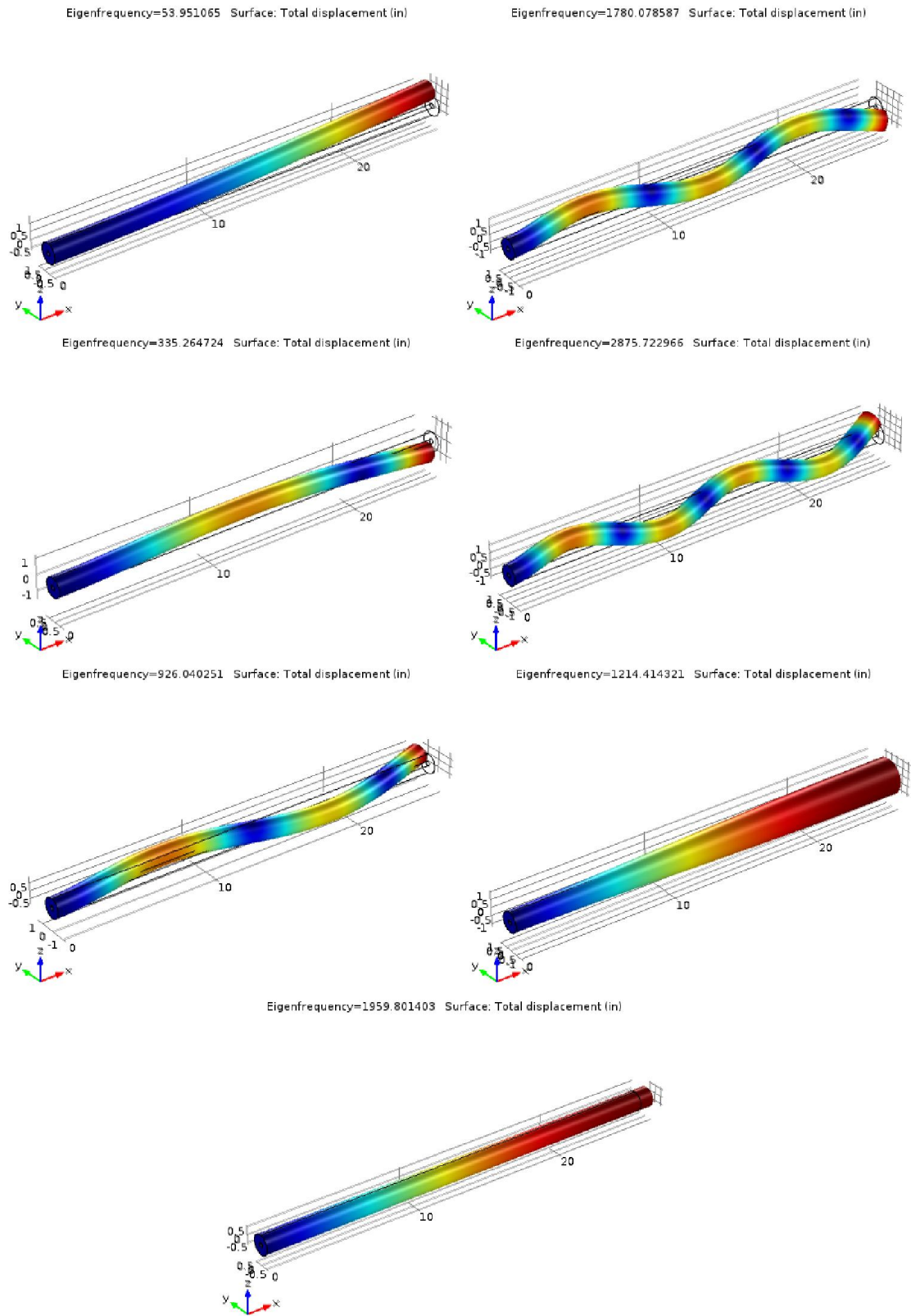
**Table 3.2: Modal Frequency Comparison of the exact solution to the 2D and 3D finite element analysis**

<b>Mode</b>	<b>Exact Solution</b>	<b>2D FEM Results</b>	<b>2D % Error</b>	<b>3D FEM Results</b>	<b>3D % Error</b>
1	60.87592442	60.750732	-0.205651779	60.876542	0.001014477
2	380.4745276	376.700269	-0.991987205	377.472039	-0.79542014
3	1067.058107	1037.540079	-2.766299965	1039.687954	-2.63253536
4	2092.609902	1987.207086	-5.036907064	1991.36286	-5.08430904
5	3458.859342	3195.016445	-7.628031991	3201.792678	-8.0288354

### **3.4 Circular Cross Section Barrel**

#### **3.4.1 Constant Cross Section Circular Barrel**

The next case that was examined was the circular barrel cross section. The circular cross section is the most common cross section used for rifle barrels. The geometry of the barrel is constant cross section and only varies from the first geometry by its circular cross section. The barrel has a diameter of 1.25 inches. The length of the barrel was varied from 26 inches in length to 24 inches and 22 inches. The 22 inch barrel's model was solved using a mesh of 9,447 elements. The 24 inch barrel's model was solved using a mesh of 10,611 elements. The 26 inch barrel's model was solved using a mesh of 11,068 elements. Each length of barrel had similar modal shapes as the square test beam, as shown in Figure 3.5.

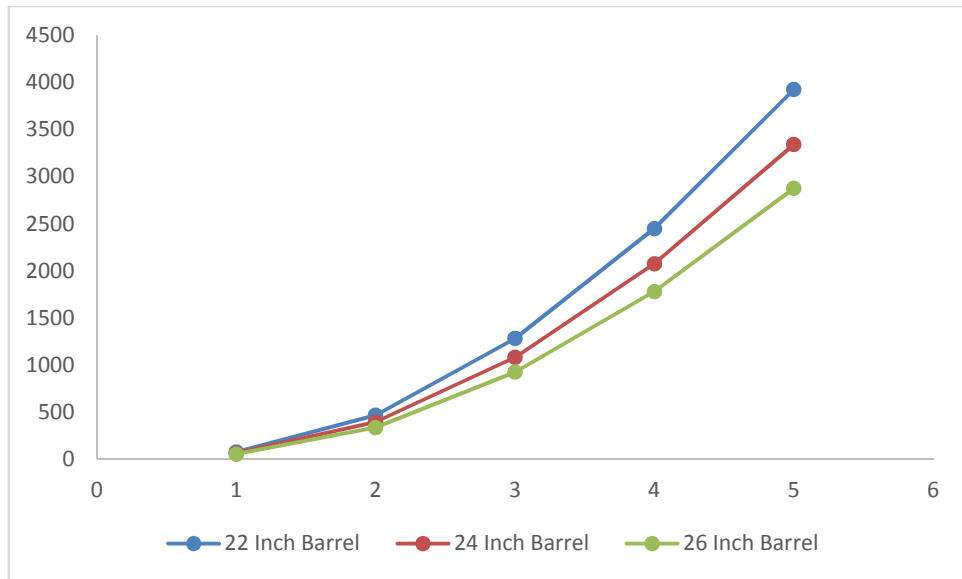


**Figure 3.5: Circular Cross Section Full Barrel Modal Shapes**

Each barrel length exhibited the same pattern of having two natural frequencies close to one another in a pair. Table 3.3 lists the natural frequencies of the three sets of barrel lengths. The full list of frequencies is listed in Appendix B.

**Table 3.3: Circular Cross Section Full Barrel Modal Frequencies**

Mode		Frequency [Hz]		
		22 in Barrel	24 in Barrel	26 in Barrel
<b>Bending</b>	1	75.297021	63.300788	53.951065
	2	466.624149	392.743609	335.264724
	3	1282.16025	1082.1296	926.040251
	4	2448.81175	2074.84502	1780.07859
	5	3925.64685	3341.05611	2875.72297
<b>Torsion</b>	1	1435.16764	1315.61387	1214.41432
<b>Extension</b>	1	2315.82907	2123.4248	1959.8014



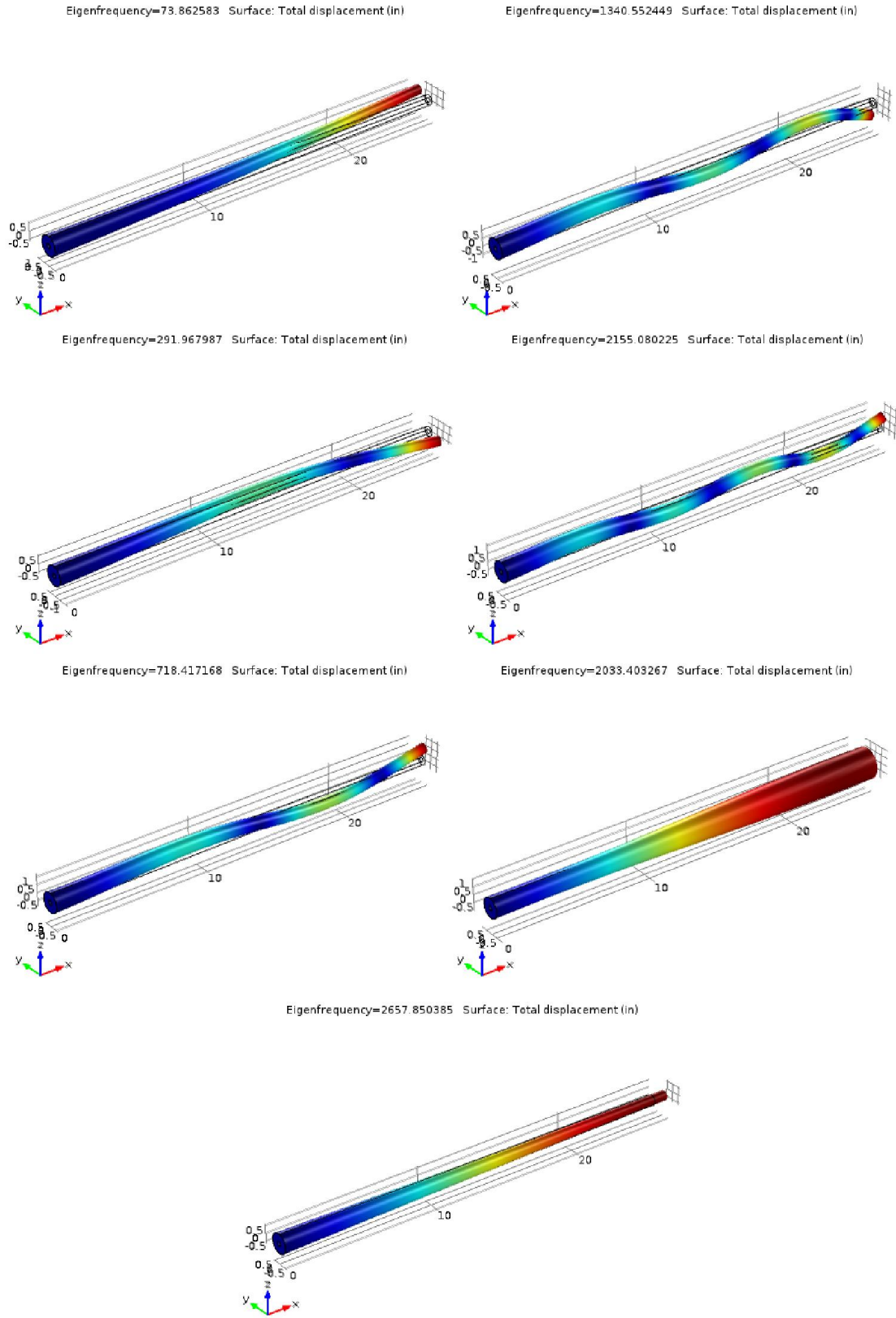
**Figure 3.6: Natural Bending Frequencies for Circular Barrels**

As the barrel lengths decrease the natural frequencies for each of the bending shapes increases, as shown in Figure 3.6. This corresponds well with Equation (4), where the

natural frequency of a cantilevered beam is inversely proportional to the square of the beam length.

### **3.4.2 Tapered Circular Cross Section Barrel**

The next case that was examined was the circular barrel cross section tapered down from a 1.25 in diameter to a diameter of .625 inches at its free end. The circular cross section with a taper makes the geometry closer to actual rifle barrel designs. The length of the barrel was varied from 26 inches in length to 24 inches and 22 inches. The 22 inch barrel's model was solved using a mesh of 7,140 elements. The 24 inch barrel's model was solved using a mesh of 7,975 elements. The 26 inch barrel's model was solved using a mesh of 8,276 elements. Each length of barrel had similar modal shapes as the square test beam, as shown in Figure 3.7.

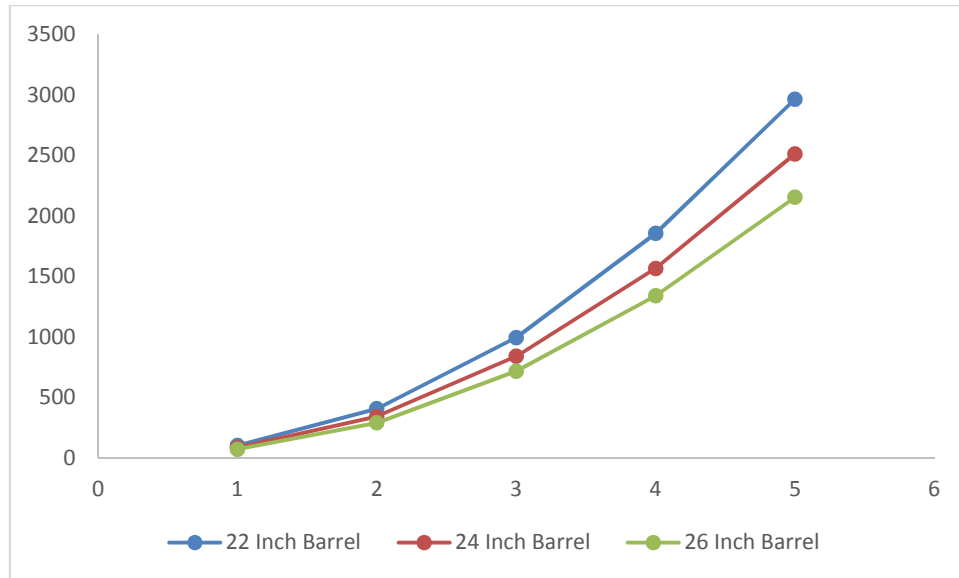


**Figure 3.7: Circular Cross Section Tapered Barrel Modal Shapes**

The tapered barrel exhibited the same behavior as the first two geometries, with every mode shape with a majority of vertical displacement is paired with a mode shape that is the same, except with horizontal displacement. Table 3.4 shows the modal frequencies of the tapered barrels, which have a modal shape with the majority of the displacement in the vertical plane. The full list of frequencies is in Appendix B. The straight tapered barrels consistently have a higher natural frequency than the constant cross section barrels. The inverse proportionality to the beam length still remains for the tapered barrel as shown in Figure 3.8. Another difference between the full barrel circular cross section and the tapered barrel circular cross section is the order that the modal shapes take place. The torsional mode occurs between the third and fourth bending mode in the straight circular barrel, see Table 3.3. The torsional mode occurs between the fourth and fifth bending modes in the tapered circular barrel, see Table 3.4.

**Table 3.4: Circular Cross Section Tapered Barrel Modal Frequencies**

<b>Mode</b>		<b>Frequency [Hz]</b>		
		<b>22 in Barrel</b>	<b>24 in Barrel</b>	<b>26 in Barrel</b>
<b>Bending</b>	<b>1</b>	103.218268	86.761819	73.862583
	<b>2</b>	407.243098	342.696952	291.967987
	<b>3</b>	996.321773	841.238162	718.417168
	<b>4</b>	1855.68525	1567.00179	1340.55245
	<b>5</b>	2964.62229	2512.98371	2155.08023
<b>Torsion</b>	<b>1</b>	2403.30423	2203.05487	2033.40327
<b>Extension</b>	<b>1</b>	3144.98921	2880.37362	2657.85039



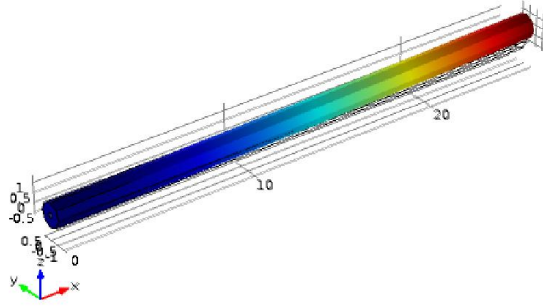
**Figure 3.8: Natural Bending Frequencies for Tapered Circular Barrels**

### 3.5 Octagonal Cross Section Barrel

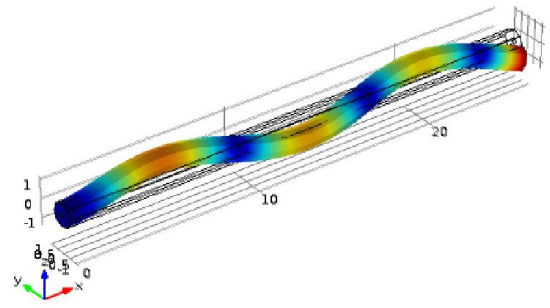
#### 3.5.1 Constant Cross Section Octagonal Barrel

The next case that was examined was the barrel of octagonal cross section. The octagonal cross section is the second most common cross section used for rifle barrels. The barrels in this case have constant octagonal cross section and are of various lengths. The barrel has an octagonal cross section that is circumscribed by a circle with a diameter of 1.25 inches. The length of the barrel was varied from 26 inches in length to 24 inches and 22 inches. The 22 inch barrel's model was solved using a mesh of 4,148 elements. The 24 inch barrel's model was solved using a mesh of 4,732 elements. The 26 inch barrel's model was solved using a mesh of 8,830 elements. Each length of barrel had similar modal shapes as the square cross section test beam, as shown in Figure 3.9.

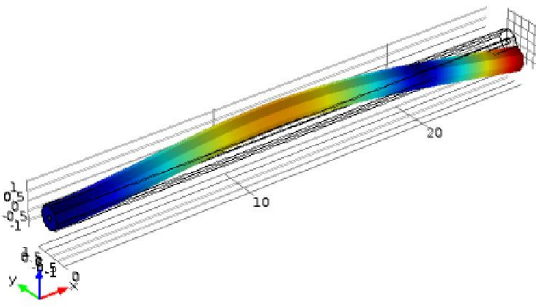
Eigenfrequency=51.388016 Surface: Total displacement (in)



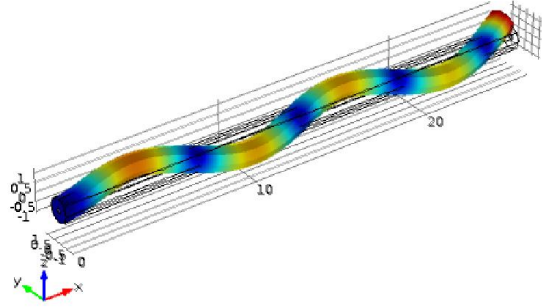
Eigenfrequency=1701.238133 Surface: Total displacement (in)



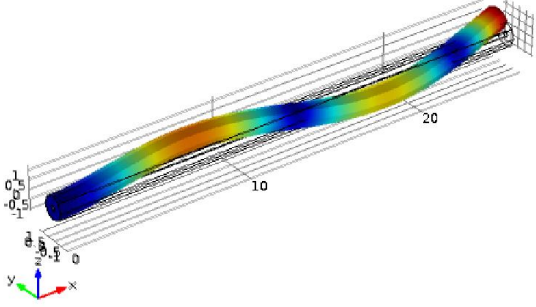
Eigenfrequency=319.340097 Surface: Total displacement (in)



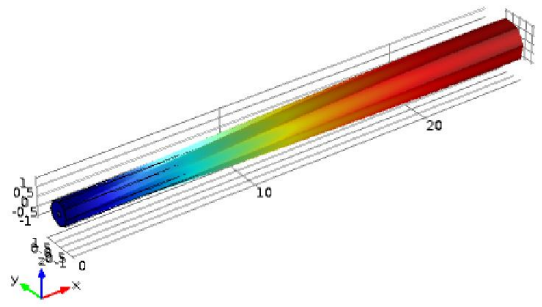
Eigenfrequency=2752.415918 Surface: Total displacement (in)



Eigenfrequency=883.114005 Surface: Total displacement (in)



Eigenfrequency=1207.357164 Surface: Total displacement (in)



Eigenfrequency=1960.121095 Surface: Total displacement (in)

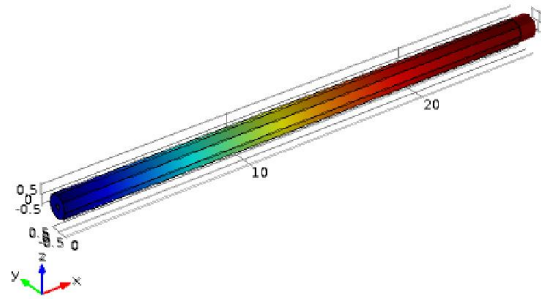
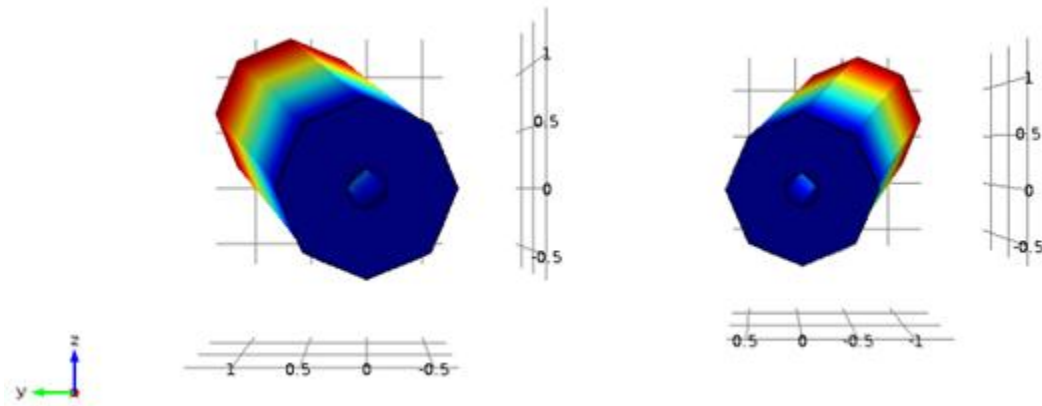


Figure 3.9: Octagonal Cross Section Full Barrel Modal Shapes

Each barrel length exhibited the same pattern of having two natural frequencies close to one another in a pair. The octagonal barrels' modal shapes that are paired together appear to be orthogonal the same as the circular cross section, except it is no longer in the vertical and horizontal planes. Figure 3.10 shows the first and second modal shape for the 26 inch barrel. The view on the left shows the first mode where the barrel moves in a plane that is at a 45 degree angle from the XZ plane. This also corresponds with one of the ridges of the octagon.



**Figure 3.10: First Bending Modal Shape, view from the YZ plane**

Table 3.5 lists the first five bending modes and the first torsional and axial extension modes. The full list of frequencies is listed in Appendix C.

**Table 3.5: Octagonal Cross Section Full Barrel Modal Frequencies**

		Frequency [Hz]		
Mode		22 in Barrel	24 in Barrel	26 in Barrel
<b>Bending</b>	1	71.932733	60.460071	51.388016
	2	445.750366	375.262678	319.340097
	3	1225.92961	1035.059506	883.114005
	4	2352.44604	1992.153269	1701.238133
	5	3775.104485	3210.496724	2752.415918
<b>Torsion</b>	1	1434.002825	1314.352929	1207.357164
<b>Extension</b>	1	2316.347504	2123.872342	1960.121095

The octagonal barrel when compared to the circular barrel has lower bending modal frequencies. Due to the octagonal cross section being circumscribed in 1.25 diameter circle the cross sectional area of the octagonal barrel is less. The area moment of inertia

would be less than the circular barrel. Based on equation (4), the modal frequency is proportional to the square root of the area moment of inertia, this implies the octagonal barrel should have a lower first modal frequency, as it does. The octagonal cross section torsional mode occurs between the third and fourth bending mode, the same as the circular cross section barrel. The octagonal torsional modal frequency is consistently lower than the circular torsion modal frequency.

### **3.5.2 Tapered Octagonal Barrel**

As it was done with the circular cross section, the next case that was examined was the octagonal barrel cross section tapered down from circumscribed circle with a 1.25 in diameter circle to a circle with a diameter of .625 inches. The octagonal cross section with a taper makes the geometry closer to actual rifle barrel designs. The length of the barrel was varied from 26 inches in length to 24 inches and 22 inches. The 22 inch barrel's model was solved using a mesh of 3,219 elements. The 24 inch barrel's model was solved using a mesh of 3,472 elements. The 26 inch barrel's model was solved using a mesh of 4,463 elements. Each length of barrel had similar modal shapes as the square test beam, as shown in Figure 3.11.

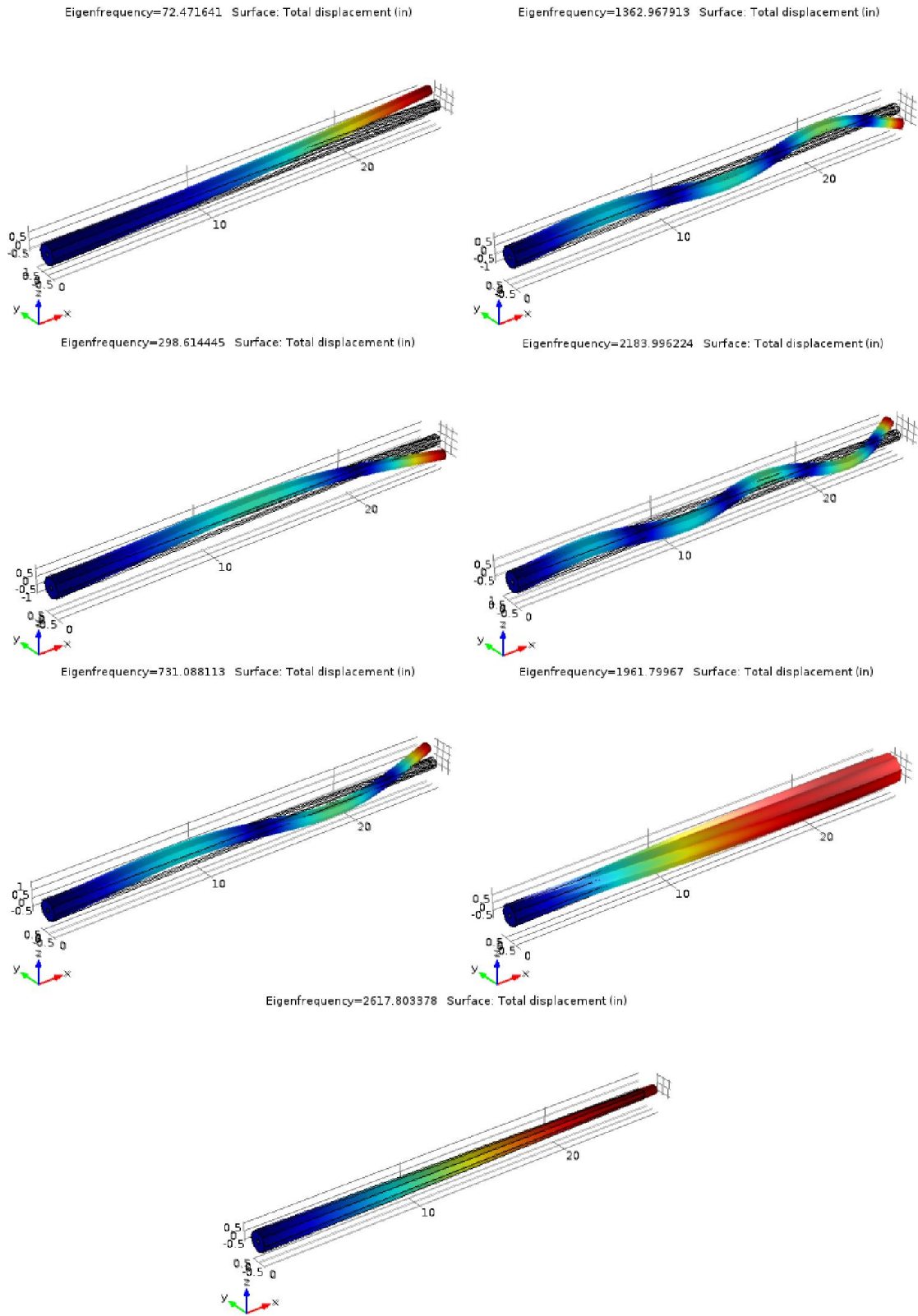


Figure 3.11: Octagonal Cross Section Tapered Barrel Modal Shapes

**Table 3.6: Octagonal Cross Section Tapered Barrel Modal Frequencies**

		<b>Frequency [Hz]</b>		
<b>Mode</b>		<b>22 in Barrel</b>	<b>24 in Barrel</b>	<b>26 in Barrel</b>
<b>Bending</b>	1	101.230076	85.070636	72.471641
	2	416.304435	350.18399	298.614445
	3	1016.21242	856.320725	731.088113
	4	1886.542294	1593.412953	1362.967913
	5	3008.358869	2547.755751	2183.996224
<b>Torsion</b>	1	2322.882485	2128.226957	1961.79967
<b>Extension</b>	1	3095.19018	2838.101837	2617.803378

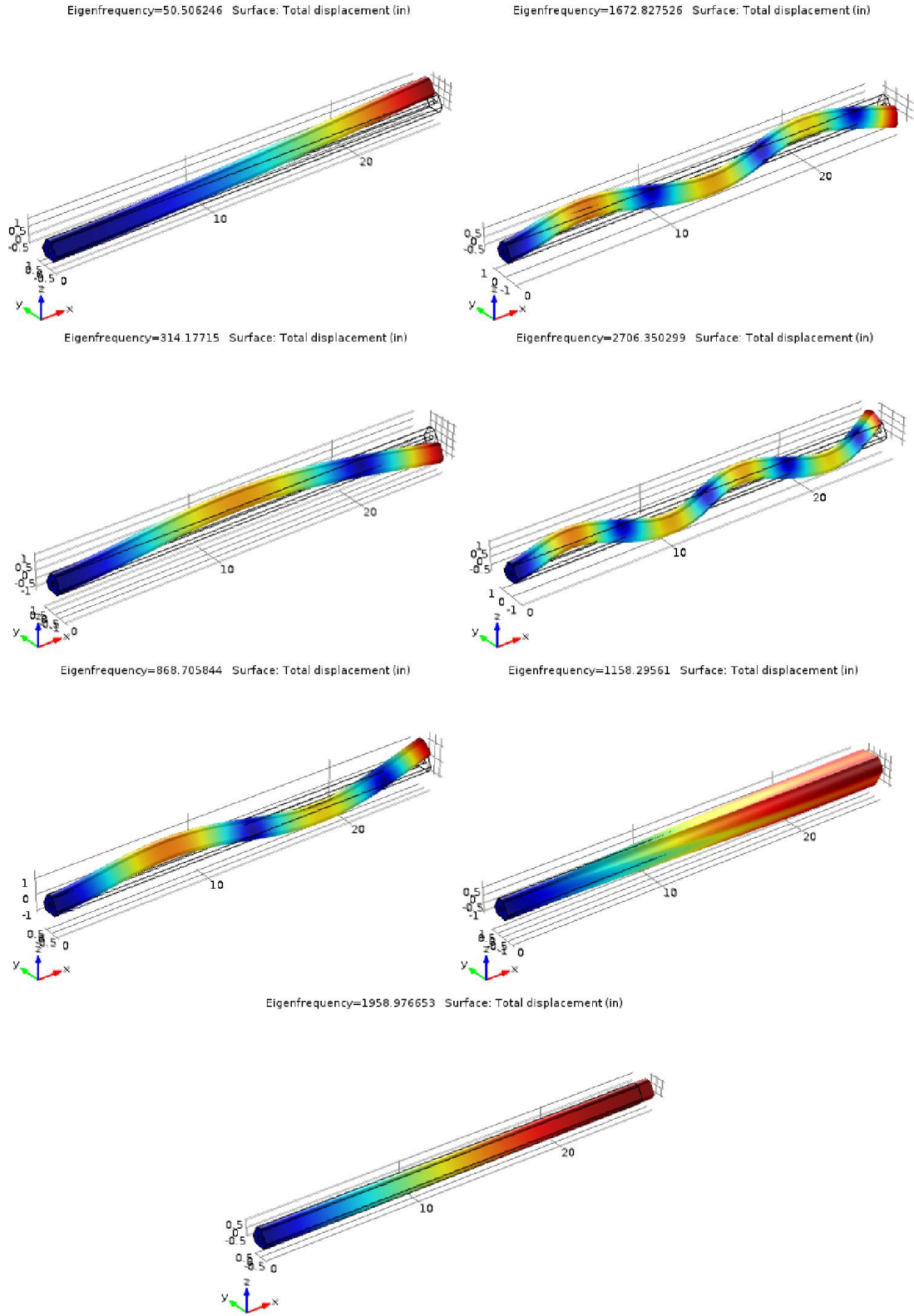
The five bending, torsional, and axial modes are listed in Table 3.6 for the three barrel lengths. The full list of frequencies is in Appendix C. The tapered octagonal barrels' first bending mode had lower frequencies than the circular tapered barrel, however the remaining bending modes had higher frequencies than the tapered circular barrel. The tapered octagon barrel and the full octagon barrel both have a lower frequency for the first torsional mode than the circular barrel. The tapered octagonal barrel had lower frequency for the axial extension mode when compared to the tapered circular barrel, opposite of the relationship the full barrels exhibited. The tapered octagonal barrel also exhibited the same pattern of having higher modal frequencies than the full octagonal barrel, the same as the circular cross section barrels.

### **3.6 Triangular Cross Section Barrel**

#### **3.6.1 Constant Cross Section Triangular Barrel**

The next case that was examined was the triangular barrel cross section. The triangular cross section is a new cross section used for rifle barrels that is an option for Remington 700 bolt action rifles. The barrels in this case are a constant cross section. The barrel has a triangular cross section that is an isosceles triangle with the distance from the centroid to the peak of 1 inch. The peaks of the triangle are then rounded off by a 1.25 inch diameter circle to achieve the shape shown in Figure 1.3. The length of the barrel was varied from 26 inches in length to 24 inches and 22 inches. The 22 inch barrel's model was solved using a mesh of 9,140 elements. The 24 inch barrel's model was

solved using a mesh of 9,764 elements. The 26 inch barrel's model was solved using a mesh of 10,656 elements. Each length of barrel had similar modal shapes as the square test beam, as shown in Figure 3.12.



**Figure 3.12: Triangular Cross Section Full Barrel Modal Shapes**

**Table 3.7: Triangular Cross Section Full Barrel Modal Frequencies**

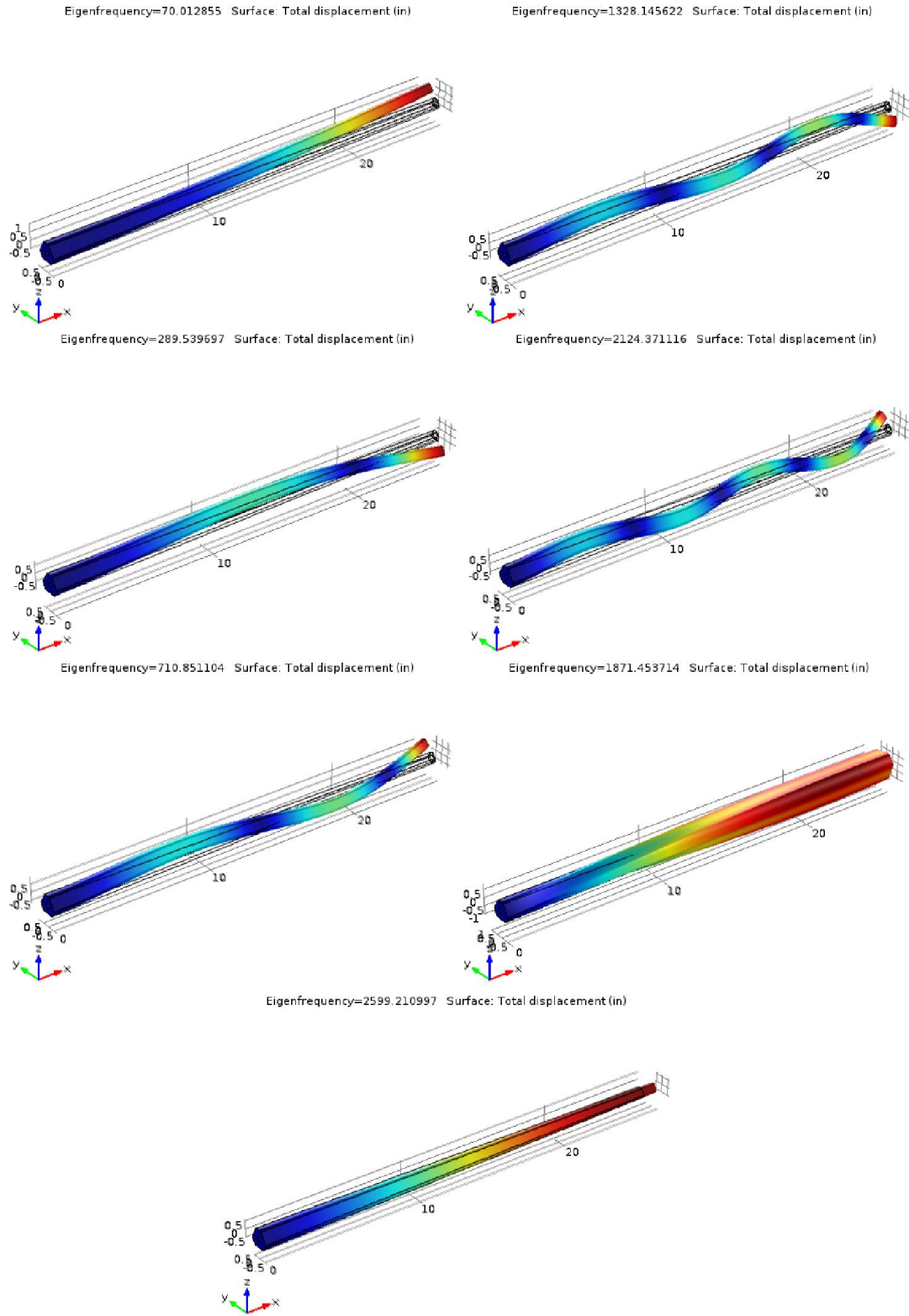
		<b>Frequency [Hz]</b>		
<b>Mode</b>		<b>22 in Barrel</b>	<b>24 in Barrel</b>	<b>26 in Barrel</b>
<b>Bending</b>	1	70.574772	59.261798	50.506246
	2	437.382987	368.022336	314.17715
	3	1203.752135	1015.814662	868.705844
	4	2303.32246	1950.439719	1672.827526
	5	3701.290208	3145.618597	2706.350299
<b>Torsion</b>	1	1369.076168	1254.895245	1158.29561
<b>Extension</b>	1	2316.066358	2122.199802	1958.976653

The five bending, torsional, and axial modes are listed in Table 3.7 for the three barrel lengths. The full list of the triangular cross section barrel is located in Appendix D. The triangular cross section barrel when compared to the circular cross section barrel has lower bending modal frequencies. Due to the triangular cross section, being circumscribed by a 1.25 diameter circle the cross sectional area of the triangular barrel is less. Due to this the area moment of inertia would be less than the circular barrel. Based on equation (4), the modal frequency is proportional to the square root of the area moment of inertia, implying the triangular barrel should have a lower first modal frequency. The torsional mode occurs at the same point between the third and fourth bending mode for the Triangular cross section, the same as the circular barrel. The triangular cross section has lower torsional modal frequencies than the corresponding circular cross section barrels. However, the axial extension modes were relatively the same as the circular cross section modal frequencies.

### **3.6.2 Tapered Triangular Barrel**

The last case that was examined was the triangular barrel cross section tapered barrel. The triangular cross section is tapered down from the cross section used in the full barrel triangular cross section down to a cross section that is an isosceles triangle with the distance from the centroid to the peak of .5 inches. The peaks of the triangle are then rounded off by a .625 inch diameter circle. The length of the barrel was varied from 26 inches in length to 24 inches and 22 inches. The 22 inch barrel's model was solved using a mesh of 7,707 elements. The 24 inch barrel's model was solved using a mesh of

8,290 elements. The 26 inch barrel's model was solved using a mesh of 9,043 elements. Each length of barrel had similar modal shapes as the square test beam, as shown in Figure 3.13.



**Figure 3.13: Triangular Cross Section Tapered Barrel Modal Shapes**

**Table 3.8: Triangular Cross Section Tapered Barrels Modal Frequencies**

		<b>Frequency [Hz]</b>		
<b>Mode</b>		<b>22 in Barrel</b>	<b>24 in Barrel</b>	<b>26 in Barrel</b>
<b>Bending</b>	1	98.039357	82.379183	70.012855
	2	403.563427	338.936412	289.539697
	3	987.955502	832.267511	710.851104
	4	1837.643262	1550.195767	1328.145622
	5	2928.025129	2478.275043	2124.371116
<b>Torsion</b>	1	2213.012729	2028.366045	1871.453714
<b>Extension</b>	1	3020.554515	2821.416619	2599.210997

The five bending, torsional, and axial modes are listed in Table 3.8 for the three barrel lengths. The full list of frequencies is in Appendix D. The tapered triangular cross section barrels' first bending mode had higher frequencies than the triangular cross section full barrel. However, as with the other tapered barrels, the following bending modes were lower than their corresponding full barrels. As with the triangular and circular cross section full barrels the tapered triangular barrel has lower modal frequencies than the circular cross section tapered barrel.

### **3.7 Barrel Bands**

When a free floated barrel is not used, it is common to have the barrel banded to the stock of the rifle. The full barrel with the three different cross sections at a length of 26 inches were run to compare to the corresponding free floated barrels. The barrel band was located at the mid length of the barrels at a distance of 13 inches, see Figure 3.14. The cross sections of the barrel were the same size as the free floated barrels that were run. The circular cross section barrel used 11,088 tetrahedral elements. The octagonal cross section barrel used a mesh of 5,575 tetrahedral mesh elements. The triangular cross sectional barrel used a mesh of 11,270 tetrahedral elements.

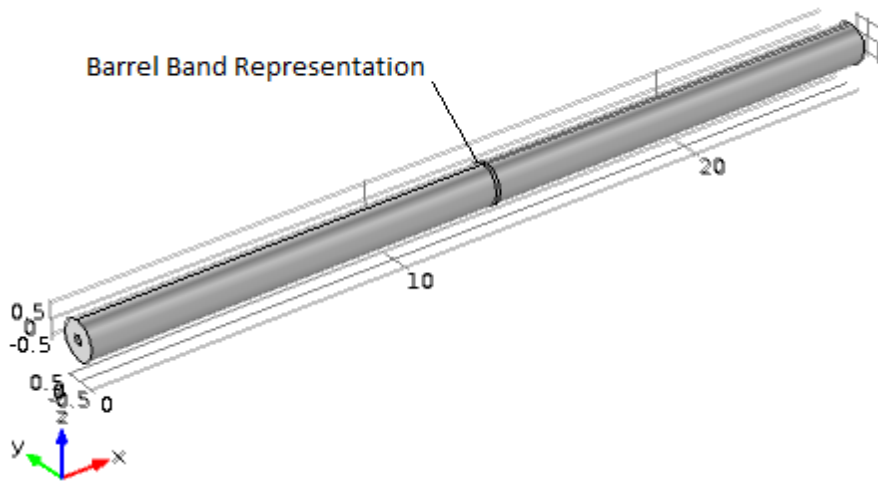
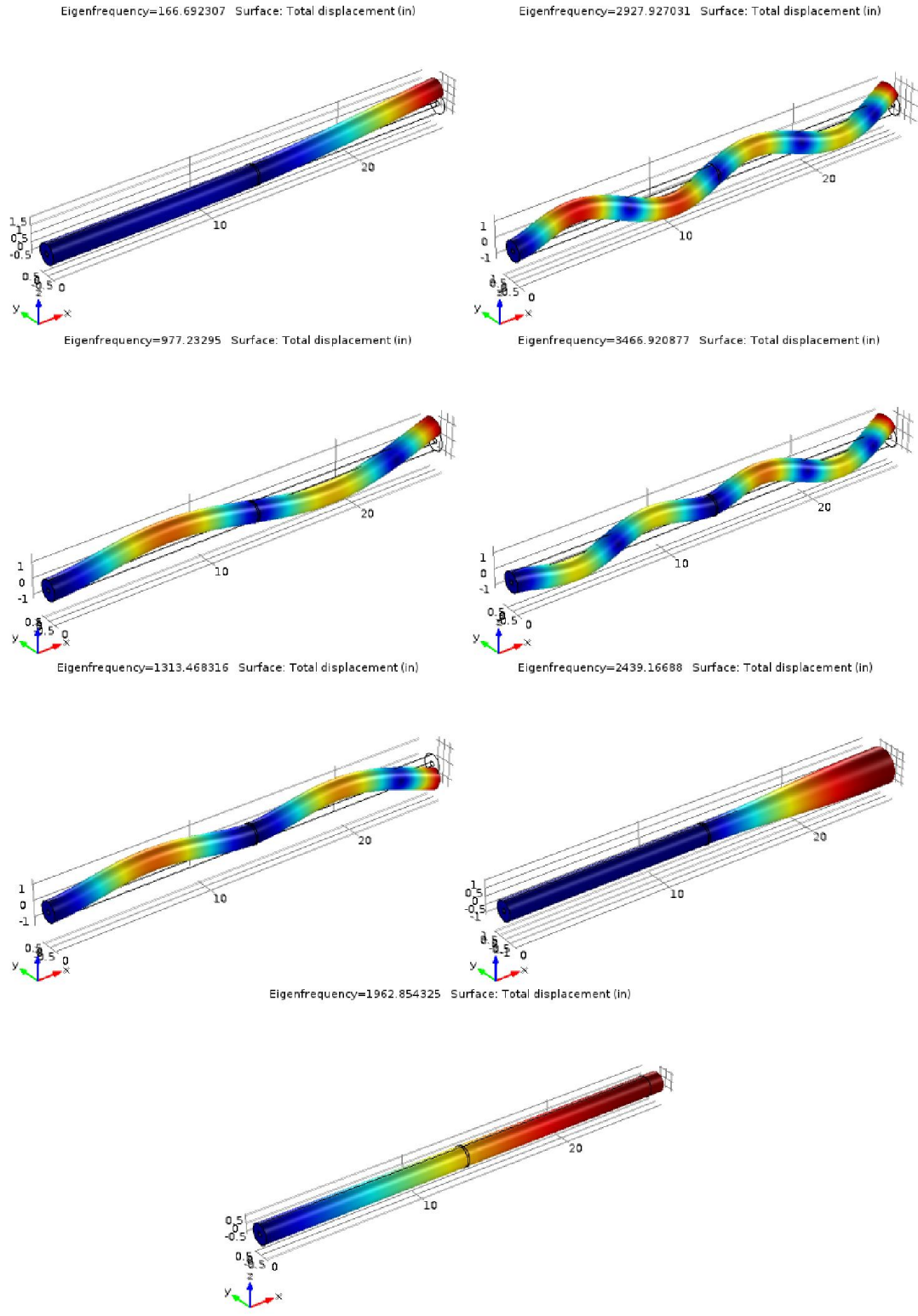


Figure 3.14: Banded Circular Cross Section Barrel's COMSOL Model

Table 3.9: Modal Frequencies for Banded Barrels

		Frequency [Hz]		
Mode		Circular	Octagonal	Triangular
<b>Bending</b>	1	166.692307	161.25536	157.974391
	2	977.23295	944.088668	923.858493
	3	1313.468316	1260.597927	1233.069992
	4	2927.927031	2820.627448	2763.516411
	5	3466.920877	3337.710917	3267.489808
<b>Torsion</b>	1	1962.854325	1963.861645	1962.77501
<b>Extension</b>	1	2439.16688	2438.881508	2328.187848

Table 3.9 shows the bending, torsional and axial extension modal frequencies for all three cross sections for 26 in barrels. The banded barrels had pairs of modal frequencies like the free floated barrels. For every bending modal shape in the vertical direction an orthogonal modal shape and frequency exists. The full list of modal frequencies is shown in Appendix E. The modal shapes of the barrel can be seen in Figure 3.15.



**Figure 3.15: Modal Shapes of Banded Circular Cross Section Barrel**

All three cross sections have the same mode shapes at different frequencies, but their relation to each other frequencies was the same as the free floated barrels. The circular cross section barrel has the highest modal frequencies, with the triangular barrel the lowest modal frequencies. The modal shapes changed from the free floated barrels. The modal shapes have only odd number of nodes. There are two different three node modal shapes and two types of five node modal shapes. The first type appears the same as the third bending shapes as the free floated barrels. The middle node, located at the barrel band has zero displacement, but the barrel is midway through the sinusoidal peak. The next three node modal shape is a higher frequency and the middle node again has zero displacement, but the node is located at the peak of the sinusoidal shape.

### **3.8 Final Summary: Rigidity versus Weight**

The three different cross sections have performed as expected given the variables in equation (4). The area moment of inertia is reduced when the octagonal and triangular cross section are circumscribed inside of the baseline diameter, indicating that the modal frequencies of the barrels will be smaller. Equation (4) also shows the barrel's weight is inversely proportional to the modal frequency. With the two variables having opposing effects on the modal frequency a barrel shape that allows for a large area moment of inertia and reduce weight would be the optimum design.

Table 3.10 compares the weights and frequencies of the different barrels. The weight of each barrel derived by multiplying the density of 4140 steel times the volume calculated using Solidworks. The first frequency calculated from the finite element model is listed next to the weight of each barrel. The full barrels were compared to the full barrels and the tapered barrels are compared to the tapered barrels. The barrels of the same length were compared to each other. For example the weight for the 22 inch barrel of octagonal cross section was compared to the weight of the 22 inch barrel of circular cross section. The octagonal cross section full barrels are 10.58% lighter than the circular barrel. The same process was done for the 1<sup>st</sup> frequency, giving a frequency reduction of 4.5%. Once the percent reduction in weight and frequency is known the

weight reduction was divided by the frequency reduction to obtain the ratio of percent weight reduction to percent frequency reduction of 2.36.

The triangular cross section full barrels reduce the weight of the barrels by 16.56%. The reduction in frequency is more significant at approximately 6.35%. However, the weight to frequency reduction is an average of 2.61 for the triangular cross sections compared to 2.32 for the octagonal cross section. Making the triangular barrel the more efficient for weight reduction.

**Table 3.10: Weight vs. Frequency Reduction Comparison**

<b>Barrel</b>	<b>Weight [lb]</b>	<b>1st Frequency</b>	<b>Weight Reduction</b>	<b>Frequency Reduction</b>	<b>Weight / Frequency Reduction Ratio</b>
22 Inch Circular Cross Section Full Barrel	7.22496	75.297021	N/A	N/A	N/A
24 Inch Circular Cross Section Full Barrel	7.88384	63.300788	N/A	N/A	N/A
26 Inch Circular Cross Section Full Barrel	8.53988	53.951065	N/A	N/A	N/A
22 Inch Circular Cross Section Tapered Barrel	3.63236	103.268716	N/A	N/A	N/A
24 Inch Circular Cross Section Tapered Barrel	3.9618	86.861819	N/A	N/A	N/A
26 Inch Circular Cross Section Tapered Barrel	4.29124	73.862583	N/A	N/A	N/A
22 Inch Octagonal Cross Section Full Barrel	6.461	71.929794	10.57%	4.47%	2.364506827
24 Inch Octagonal Cross Section Full Barrel	7.04888	60.460071	10.59%	4.49%	2.359983762
26 Inch Octagonal Cross Section Full Barrel	7.63676	51.388016	10.58%	4.75%	2.226059688
22 Inch Octagonal Cross Section Tapered Barrel	3.58408	101.228899	1.33%	1.98%	0.672908396
24 Inch Octagonal Cross Section Tapered Barrel	3.91068	85.070636	1.29%	2.06%	0.625730405
26 Inch Octagonal Cross Section Tapered Barrel	4.23728	72.471641	1.26%	1.88%	0.667735716
22 Inch Triangular Cross Section Full Barrel	6.02932	70.574772	16.55%	6.27%	2.638723592
24 Inch Triangular Cross Section Full Barrel	6.57744	59.261798	16.57%	6.38%	2.597016497
26 Inch Triangular Cross Section Full Barrel	7.12556	50.506246	16.56%	6.39%	2.59375845
22 Inch Triangular Cross Section Tapered Barrel	3.33132	98.036108	8.29%	5.07%	1.635633124
24 Inch Triangular Cross Section Tapered Barrel	3.6352	82.37342	8.24%	5.17%	1.595368804
26 Inch Triangular Cross Section Tapered Barrel	3.93908	70.001243	8.21%	5.23%	1.569797626

The tapered cross section did not have the same result as the full barrels. The tapered octagonal cross section saw a larger reduction in frequency than its reduction in weight. The triangular cross section tapered barrel when compared to the circular barrel did see a weight reduction greater than the frequency reduction.

## 4. Conclusions

The expected outcomes of the study were met. It is evident that rifle barrels, regardless of their cross section, will have the same modal shapes. The variation in natural frequency was a result of the changes in area moment of inertia and the weight of the barrels. The Roark's formula for calculating the barrel modes, equation (4), shows that the barrel modal frequencies will be proportional to the square of the area moment of inertia and inversely proportional to the square of the barrel length. The solutions obtained from the finite element models are in agreement with the Roark's equation.

The study did show that the weight of the barrel can be reduced by a greater amount than the frequency allowing for a lighter barrel without sacrificing the rigidity of the barrel by the same amount. The triangular cross section barrels have the highest weight percent reduction over frequency percent reduction ratios.

Regardless of the cross section all barrels the same modal shapes when the additional barrel band boundary condition was used. This indicated that the modal shape and vibrational modes were consistent and could be predicted through FEA. The barrel band effected the barrel's modal frequencies greater than the barrel cross sections. This indicates that further modeling of a complete rifle would offer greater insight in the modes of vibration for the barrel in a rifle.

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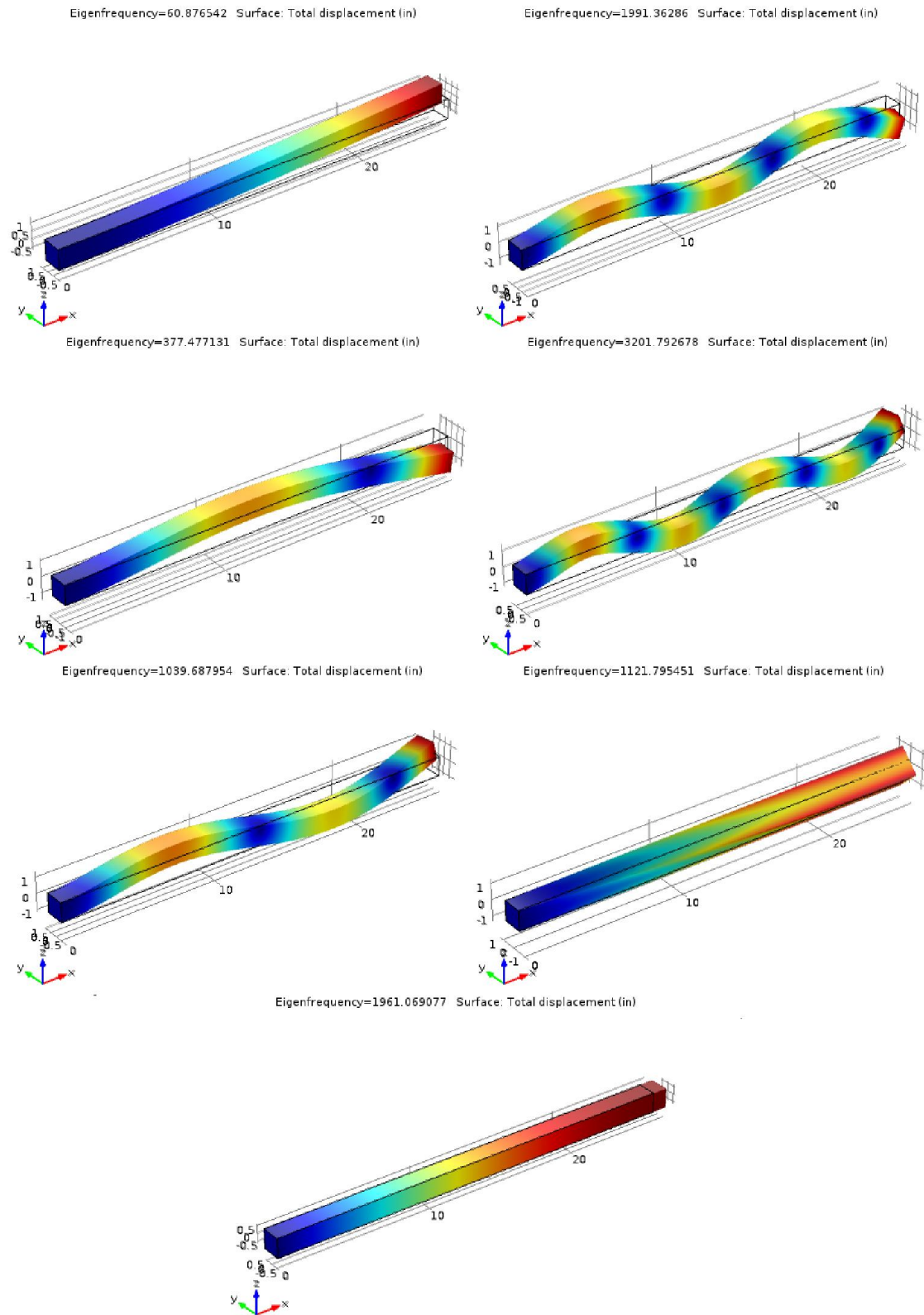
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## Appendix A: Square Test Beam Modal Shapes and Frequencies

The three dimensional and two dimensional modal shapes of the square test beam are shown below.

**Table A.1: Three Dimensional Square Test Beam Frequencies**

<b>Mode</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>
1	60.875882	Bending
2	60.876542	Bending
3	377.472039	Bending
4	377.477131	Bending
5	1039.672679	Bending
6	1039.687954	Bending
7	1121.795451	Torsional
8	1961.069077	Axial Extension
9	1991.324517	Bending
10	1991.36286	Bending
11	3201.727815	Bending
12	3201.792678	Bending



**Figure A.1: Basic Mode Shapes of the 3D Square Test Beam**

## Appendix B: Circular Cross Section Barrels' Modal Frequencies

The three dimensional modal shapes of the circular cross section barrels are shown below.

**Table B.1: Modal Frequencies for Circular Constant Cross Section Barrels**

	22 Inch Barrel		24 Inch Barrel		26 Inch Barrel	
Mode	Frequency [Hz]	Mode Type	Frequency [Hz]	Mode Type	Frequency [Hz]	Mode Type
<b>1</b>	75.297021	Bending	63.300788	Bending	53.951065	Bending
<b>2</b>	75.301337	Bending	63.304664	Bending	53.952334	Bending
<b>3</b>	466.624149	Bending	392.743609	Bending	335.264724	Bending
<b>4</b>	466.647551	Bending	392.756439	Bending	335.269891	Bending
<b>5</b>	1282.113914	Bending	1082.019773	Bending	926.030407	Bending
<b>6</b>	1282.160265	Bending	1082.129604	Bending	926.040251	Bending
<b>7</b>	1435.167639	Torsional	1315.613865	Torsional	1214.414321	Torsional
<b>8</b>	2315.829073	Axial	2074.845017	Bending	1780.078587	Bending
<b>9</b>	2448.61172	Bending	2074.885391	Bending	1780.089229	Bending
<b>10</b>	2448.811752	Bending	2123.424796	Axial	1959.801403	Axial
<b>11</b>	3925.375998	Bending	3340.807417	Bending	2875.607968	Bending
<b>12</b>	3925.646848	Bending	3341.056107	Bending	2875.722966	Bending

**Table B.2: Modal Frequencies for Circular Cross Section Tapered Barrels**

	<b>22 Inch Barrel</b>		<b>24 Inch Barrel</b>		<b>26 Inch Barrel</b>	
<b>Mode</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>
<b>1</b>	103.218268	Bending	86.761819	Bending	73.862583	Bending
<b>2</b>	103.268716	Bending	86.781409	Bending	73.884833	Bending
<b>3</b>	406.933501	Bending	342.696952	Bending	291.967987	Bending
<b>4</b>	407.243098	Bending	342.854202	Bending	292.085529	Bending
<b>5</b>	996.321773	Bending	840.901969	Bending	718.10485	Bending
<b>6</b>	997.029405	Bending	841.238162	Bending	718.417168	Bending
<b>7</b>	1854.419427	Bending	1566.369668	Bending	1340.004597	Bending
<b>8</b>	1855.685252	Bending	1567.001794	Bending	1340.552449	Bending
<b>9</b>	2403.30423	Torsional	2203.054868	Torsional	2033.403267	Torsional
<b>10</b>	2963.322434	Bending	2511.606761	Bending	2154.029565	Bending
<b>11</b>	2964.622285	Bending	2510.983711	Bending	2155.080225	Bending
<b>12</b>	3144.989211	Axial	2880.373615	Axial	2657.850385	Axial

## Appendix C: Octagonal Cross Section Barrels' Modal Frequencies

The three dimensional modal shapes of the octagonal cross section barrels are shown below.

**Table C.1: Modal Frequencies for Octagonal Cross Section Full Barrels**

	<b>22 Inch Barrel</b>		<b>24 Inch Barrel</b>		<b>26 Inch Barrel</b>	
<b>Mode</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>
<b>1</b>	71.929794	Bending	60.460071	Bending	51.388016	Bending
<b>2</b>	71.932733	Bending	60.461603	Bending	51.388614	Bending
<b>3</b>	445.750366	Bending	375.262678	Bending	319.340097	Bending
<b>4</b>	445.775879	Bending	375.275205	Bending	319.353645	Bending
<b>5</b>	1225.929691	Bending	1035.059506	Bending	883.044357	Bending
<b>6</b>	1226.010391	Bending	1035.125338	Bending	883.114005	Bending
<b>7</b>	1434.002825	Torsional	1314.352929	Torsional	1207.357164	Torsional
<b>8</b>	2316.347504	Axial	1992.153269	Bending	1701.174281	Bending
<b>9</b>	2352.44604	Bending	1992.261947	Bending	1701.238133	Bending
<b>10</b>	2352.604761	Bending	2123.872342	Axial	1960.121095	Axial
<b>11</b>	3775.104485	Bending	3210.496724	Bending	2752.363698	Bending
<b>12</b>	3775.402481	Bending	3210.670495	Bending	2752.415918	Bending

**Table C.2: Modal Frequencies for Octagonal Cross Section Tapered Barrels**

	<b>22 Inch Barrel</b>		<b>24 Inch Barrel</b>		<b>26 Inch Barrel</b>	
<b>Mode</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>
<b>1</b>	101.228899	Bending	85.070636	Bending	72.471641	Bending
<b>2</b>	101.230076	Bending	85.074757	Bending	72.475635	Bending
<b>3</b>	416.300917	Bending	350.18399	Bending	298.614445	Bending
<b>4</b>	416.304435	Bending	350.200177	Bending	298.618678	Bending
<b>5</b>	1016.208438	Bending	856.320725	Bending	731.088113	Bending
<b>6</b>	1016.21242	Bending	856.35885	Bending	731.095537	Bending
<b>7</b>	1886.542294	Bending	1593.336724	Bending	1362.967913	Bending
<b>8</b>	1886.552397	Bending	1593.412953	Bending	1362.985868	Bending
<b>9</b>	2322.882485	Torsional	2128.226957	Torsional	1961.79967	Torsional
<b>10</b>	3008.358869	Bending	2547.492627	Bending	2183.845834	Bending
<b>11</b>	3008.383212	Bending	2547.755751	Bending	2183.996224	Bending
<b>12</b>	3095.19018	Axial	2838.101837	Axial	2617.803378	Axial

## Appendix D: Triangular Cross Section Barrels' Modal Frequencies

The three dimensional modal shapes of the triangular cross section barrels are shown below.

**Table D.1: Modal Frequencies for Triangular Cross Section Barrels**

	22 Inch Barrel		24 Inch Barrel		26 Inch Barrel	
Mode	Frequency [Hz]	Mode Type	Frequency [Hz]	Mode Type	Frequency [Hz]	Mode Type
<b>1</b>	70.574772	Bending	59.261798	Bending	50.506246	Bending
<b>2</b>	70.578766	Bending	59.264379	Bending	50.510813	Bending
<b>3</b>	437.382987	Bending	368.016446	Bending	314.17715	Bending
<b>4</b>	437.417553	Bending	368.022336	Bending	314.200586	Bending
<b>5</b>	1203.752135	Bending	1015.77805	Bending	868.705844	Bending
<b>6</b>	1203.798554	Bending	1015.814662	Bending	868.74026	Bending
<b>7</b>	13690.076168	Torsional	1254.895245	Torsional	1158.29561	Torsional
<b>8</b>	2303.32246	Bending	1950.439719	Bending	1672.657819	Bending
<b>9</b>	2303.590307	Bending	1950.581098	Bending	1672.827526	Bending
<b>10</b>	2316.066358	Axial	2122.199802	Axial	1958.976653	Axial
<b>11</b>	3700.825098	Bending	3145.618597	Bending	2705.68083	Bending
<b>12</b>	3701.290208	Bending	3145.997671	Bending	2706.350299	Bending

**Table D.2: Triangular Cross Section Tapered Barrel Modal Frequencies**

	<b>22 Inch Barrel</b>		<b>24 Inch Barrel</b>		<b>26 Inch Barrel</b>	
<b>Mode</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>
<b>1</b>	98.036108	Bending	82.37342	Bending	70.001243	Bending
<b>2</b>	98.39357	Bending	82.379183	Bending	70.012855	Bending
<b>3</b>	403.394638	Bending	338.936412	Bending	289.502297	Bending
<b>4</b>	403.563427	Bending	338.977974	Bending	289.539697	Bending
<b>5</b>	987.126466	Bending	831.373045	Bending	710.104816	Bending
<b>6</b>	987.955502	Bending	832.267511	Bending	710.851104	Bending
<b>7</b>	1835.783426	Bending	1548.644164	Bending	1326.474094	Bending
<b>8</b>	1387.643262	Bending	1550.195767	Bending	1328.145622	Bending
<b>9</b>	2213.012729	Torsional	2028.366045	Torsional	1871.453714	Torsional
<b>10</b>	2927.573569	Bending	2475.784368	Bending	2122.24302	Bending
<b>11</b>	2928.025129	Bending	2478.275043	Bending	2124.371116	Bending
<b>12</b>	3080.554515	Axial	2821.416619	Axial	2599.210997	Axial

## Appendix E. Barrel Band Modal Frequencies for all Cross Sections

The three dimensional modal frequencies of the banded barrels for all cross sections are shown below.

**Table E.1: Barrel Band Modal Frequencies**

	<b>Circular Cross Section</b>		<b>Octagonal Cross Section</b>		<b>Triangular Cross Section</b>	
<b>Mode</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>	<b>Frequency [Hz]</b>	<b>Mode Type</b>
<b>1</b>	166.692307	Bending	161.25536	Bending	157.974391	Bending
<b>2</b>	166.774157	Bending	161.305311	Bending	158.018202	Bending
<b>3</b>	977.23295	Bending	944.088668	Bending	923.858493	Bending
<b>4</b>	977.646502	Bending	944.182436	Bending	924.037119	Bending
<b>5</b>	1313.468316	Bending	1260.597927	Bending	1233.069992	Bending
<b>6</b>	1313.5584	Bending	1260.659214	Bending	1233.209812	Bending
<b>7</b>	1962.854325	Axial	1963.861645	Axial	1962.77501	Axial
<b>8</b>	2439.16688	Torsional	2438.881508	Torsional	2328.187848	Torsional
<b>9</b>	2927.927031	Bending	2820.627448	Bending	2763.516411	Bending
<b>10</b>	2928.64215	Bending	2821.224915	Bending	2764.218681	Bending
<b>11</b>	3466.920877	Bending	3337.710917	Bending	3267.489808	Bending
<b>12</b>	3467.344654	Bending	3338.00489	Bending	3268.711968	Bending