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THEORY AND APPLICATION OF MATHEMATICAL MODELING OF SHOULDER-FIRED WEAPONS

PART I: M16A1 RIFLE

BY

PAUL E. EHLE

ALBERT E. RAHE

1 NOVEMBER 1972

FINAL REPORT

U.S. ARMY ARMAMENT COMMAND

RESEARCH DIRECTORATE

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A mathematical model of the M16A1 Rifle was constructed by personnel of the Research Directorate to serve as (1) a vehicle to extend the state-of-the-art in weapon modeling, (2) a basis with which to compare performance characteristics of other weapons such as the XM19 Rifle, (3) an aid to troubleshoot the weapon, (4) a means to increase present knowledge of the M16A1, and (5) a way to explore and demonstrate the capabilities and limitations of mathematical models of weapons.

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20. For each component mass, equations of motion are written. These allow a total of eleven degrees of freedom plus a nearly arbitrary number of degrees of freedom for the drive spring. To provide accuracy data, one rotational degree of freedom is allowed for weapon pitch motion. Expressions are derived for the many diverse forces acting on the masses. The resulting equations are solved on an IBM 360/65 digital computer. A sensitivity analysis is conducted, and the results are compared with those from a similar model of the XM19 Rifle. The greatest differences between the M16A1 and the XM19 sensitivities were found to be the effect of the ignition delay and the drive spring on cycle time and the mounting conditions on accuracy.

The methods developed during model construction are applicable to the modeling of many other weapons. Part II of this two-report series is an application of these techniques to the XM19 Rifle.

FOREWORD

Acknowledgement is made to a number of persons in the Research Directorate, Weapons Laboratory, Rock Island Arsenal, who contributed toward the completion of this project. Mr. Albert J. Patsche and Mr. Paul A. Cacari participated in the early explorations of dynamic spring and buffer behavior. Mr. Lanny D. Wells and Mr. Robert B. Long applied to several test cases the method to couple complex models (developed in Appendix G).

Mr. Timothy L. Brosseau, BRL, contributed much valuable information through telephone conversations.

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1.0 INTRODUCTION

1.1 Objectives

The objectives of the study described in this report were:

a. to develop mathematical computer models for the M16A1 and XM19 Rifles.

b. to investigate alternative ways to model various events in the firing cycle.

c. to perform various sensitivity studies using the models.

d. to evaluate proposed design changes.

NOTE: The term "model" used in this report will indicate mathematical computer model.

This study was divided into two parts:

Volume I deals with the construction and the use of a model (M16 model) of the M16A1 Rifle. It is exploratory, philosophical, and it describes alternative ways to handle various forces and treats a variety of topics related to modeling in addition to providing a detailed description of the M16A1 model and results.

Volume II describes the development of a mathematical computer model of the XM19 Rifle (MXM model). In contrast, the MXM model is less exploratory and the report goes directly to the point of describing exactly what is in the model, what the results of exercising it are, and what conclusions can be drawn. The work was designed to be an application of present techniques that would directly result in a practical model.

1.2 Material Presented in This Volume

The material presented in Volume I of this report deals with the construction and the use of a mathematical model (M16 model) of the M16A1 Rifle. With the M16 model, the motions of the hammer, bolt, bolt-carrier, main gun, buffer body, buffer weights and an optional number of segments of the drive spring can be predicted. The M16 model has eleven degrees of freedom plus an extra number dependent upon how many element masses are chosen to represent the drive spring. For most purposes, other than an in-depth study of the spring itself, five element masses are adequate. One of the degrees of freedom accounts for the pitch motion of the weapon. The M16 model incorporates a simple model of the human operator (shooter) consisting of linear and torsional springs and dampers, and it also includes a simple exterior ballistics analysis so that major trends in weapon dispersion can be evaluated. Detailed interior ballistics and gas-transmission analyses are not included; experimental pressure-time curves are used.

The construction of the M16 model of the M16A1 Rifle serves a number of purposes. It is first a vehicle for the extension of the state of the art in weapon modeling. For this role, the M16A1 Rifle is ideally suited. Rifles are readily available, and much information is already known with which theory can be compared. Second, a model of this weapon provides a logical basis for comparison of performance of other weapons such as the XM19 Rifle. Statements about sensitivity to changes in parameters in one weapon are more meaningful if one knows how another weapon responds to similar changes in these parameters. Third, the model serves as a practical tool for any proposed modifications to this weapon. Fourth, the model can be used for troubleshooting should any difficulties arise. In the past, manufacturing problems have arisen that are ideally suited for exploration through a model.[1] Fifth, as is demonstrated in this report, such a model can add to the present store of information about the M16A1 Rifle. Effects can be explored that are virtually impossible to study experimentally. Finally, the model provides a means of exploring and demonstrating the capabilities and limitations of mathematical weapon models.

Unique features of the M16 model include not only detailed and efficient analyses of the buffer, drive spring, and magazine, but also an exploration of weapon dispersion through a simple model of the shooter. An extensive sensitivity analysis was also performed that provides information on trends previously unavailable. Also, in connection with the development of the model, many new approaches were explored and some of these are discussed in the appendices to this report. Topics discussed include impact between masses, the coupling of complex mathematical models without simultaneous solution of all equations, and a

[1] Grandy, A.J.; Duffy, J.A.; Horchler, M.H.; and Ehle, P.E.; "Investigation of Bolt/Bolt-Carrier Clearances in the M16A1 Rifle", Technical Note TN-1159, Frankford Arsenal, Philadelphia, Pa. (May 1971)

unique fluid flow analysis.

One of the most significant aspects of the report is the sensitivity analysis. This analysis provides quantitative information about the M16A1 previously unavailable from experiment and demonstrates one of the important uses of a model. According to the M16 model, burst accuracy of this weapon can be improved by a stiffening of the shoulder and the upper torso of the shooter, a decrease in breech pressure, and increase in moment of inertia of the rifle, a decrease in drive-spring constant, and the positioning of the center of gravity of the buffer weights midway between the possible extremes. An increase in cyclic rate can be achieved, according to the model, by a stiffening of the shooter's shoulder, an increase in gas pressure in the bolt-carrier cavity and at the breech, a decrease in mass of the bolt and bolt-carrier, and the positioning of the center of gravity of the buffer weights in the most forward position possible. Many of these trends, but not their degree, are obvious.

The remainder of Volume I deals with the development and exercise of the model, and the discussion of study areas important for future model development. First, modeling philosophy is discussed that is essential for the placement of the subject of this report in proper perspective. Then after a brief description of weapon operation, the general equations of motion used to describe this rifle are discussed. From the solution of these equations, typical predictive capabilities of the model are demonstrated and many predictions are compared with experiment. For insight into the response of weapon performance to changes in various parameters, the sensitivity analysis is discussed. (Note that some of the model predictions were made while the model was in an advanced but not final state of development. However, the results are substantially representative of the present model. Continual changes and refinements are a natural part of the modeling process). A number of comparisons are made between the predictions of the M16 model and those of the MXM model of the XM19 Rifle described in Volume II. On the basis of the M16 model predictions, conclusions are drawn and recommendations are made. In the appendices, detailed derivations of each force expression are given. Various topics for future emphasis are also discussed.

A conversion table is included so that the reader can quickly relate cycle time to shots per minute and make conversions between English and metric systems. Finally, computer programs are provided to present a clearer impression of the M16A1 and drive spring models. Both computer programs are written in FORTRAN IV and were run on IBM 360/65 computers. Run times for a 3 shot burst generally was less than 1 minute, but this varied with the number of finite elements used to model the drive springs.

2.0 MATHEMATICAL COMPUTER MODELS

2.1 Modeling Philosophy

Since the M16A1 Rifle has been in service for a number of years and has undergone extensive tests and design changes, one may wonder why a mathematical model of the M16A1 was not undertaken long ago. In the past, weapon development has been basically a trial-and-error procedure. The techniques of mathematical modeling for the comprehensive description of small-caliber automatic weapons simply were not developed until recently. Some early modeling work was done at Frankford Arsenal in 1967[2]. However, one of the earliest attempts at a comprehensive model was made at Rock Island in 1969[3]. Models of artillery weapons had been developed before that time, but the problems of modeling these two types of weapons are quite different, especially in the areas of automatic fire and man-weapon interaction. Modeling efforts at WECOM and elsewhere have recently increased, and people now have a better idea of what can be modeled effectively, what is essential to model in detail, what modeling can provide, and what the problems and limitations are. In this section of the report, an attempt is made to present a clearer idea of current modeling philosophy upon which the M16A1 model is based.

2.1.1 Four Major Areas

A number of general purposes exist for which a model of any automatic weapon can be used. Eventually, with little or no resort to experimentation, highly developed math models will be constructed with which quick, inexpensive, accurate, and nearly complete information can be provided in four major areas: (1) weapon concept evaluation (2) design (3) evaluation of existing weapons (4) troubleshooting.

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[2]Kucsan, S.; Shinaly, F.; Jaslow, H.; and Zaid, M.; "Small Arms Systems Simulation," Frankford Arsenal Report R-1848, Philadelphia, Pa. (Jun 1967)

[3]Ehle, P.E., "Mathematical Model of the Stoner 5.56mm Medium Machine Gun, XM207," WECOM Technical Report 70-114, AD 862081L, Research and Engineering Directorate, Rock Island, Ill (Oct 1969)

Today, this goal is partially realized. For concept work, one can use a model to predict gross operational characteristics, provide some reliability information, and partially explore compatibility with existing mounting systems. (A good example of this capability is contained in Ref [4].) For design, one can now use the models only as part of a trial-and-error procedure. At the very least, this procedure can be feasibly automated and many configurations can be analyzed by the computer until specified design criteria such as cyclic rate are met. To have the computer directly solve for the detailed specifications of a general configuration by which these criteria will be satisfied is also conceivable. Developments in optimization methodology will contribute to these design capabilities. However, the underlying design concept will still be the product of a human designer. Completely automated design is not in the foreseeable future. In the evaluation of existing weapons, the models can locate tendencies toward malfunctions, examine certain aspects of reliability and accuracy, and assess compatibility of the weapon with various mounting systems. For troubleshooting, one can use a model to locate the sources of many problems, to determine the ramifications, and to indicate alternative solutions. The full potential of mathematical modeling of weapons is far from being reached in the four major areas.

2.1.2 Simple Models

For many purposes, very simple models are the most effective. For example, in early concept studies, the inclusion of fine details is often unjustified. Also, one phenomenon may be very precisely modeled, but this advantage is somewhat negated by gross approximations in other parts of the model. In addition, when one divides a complicated physical process into a large number of smaller ones, often the result is that, instead of making one large guess, many smaller guesses are made; and the sum is no better than the accuracy of its parts. Also, computer time may soar unnecessarily. Thus, for a particular problem at hand, highly detailed approaches are sometimes neither necessary nor productive.

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[4] Interim ADPACS Report - DESVAL - USAWECOM Task Group Report, SWERR-R, Rock Island, Ill (Jul 1972)

However, if one is to more fully explore the potential of math models, to look beyond the problems of the present, and to increase accuracy, one must at some time dig deeply into the details of the various physical phenomena. Thus, by the inclusion of seemingly unnecessary detail, especially in exploratory models, one is in effect planning for the future, increasing the ability of engineers to model future weapons, and advancing the state of the art. If one is to understand complex physical processes, these problems must be divided into parts that can be individually analyzed. Inclusion of these parts in a model also enables one to assess their relative importance. On balance, highly detailed models often serve a purpose beyond the immediate problem at hand and should be encouraged if time and resources permit.

2.1.3 Extreme Attitudes

One can often find two extreme attitudes concerning the value of math models of weapons. The first attitude is that these models are worthless because certain phenomena such as the complete human response of the shooter are too complex to model, or because some model predictions are not in agreement with experiment, or because certain expressions used in the models were not rigouously derived. The second attitude is that a model can do virtually everything and can even set the manufacturing tolerance limits on every part of the mechanism. The truth, of course, lies somewhere between these extremes.

2.1.3.1 First Attitude

The first attitude is unjustified because of the reasons stated below:

(a) Although phenomena exist too complex to analyze precisely now, an approximate model often retains much of the essence of the phenomenon and can be used for trend predictions that provide important insight into weapon characteristics. Perfection is desirable, but is not necessary.

(b) Inaccuracy of certain model predictions does not necessarily imply inaccuracy of others. For example, an incorrect peak force may not have much effect on cycle time predictions if the impulse level is basically correct. Also, disagreement with an experimental measurement does not automatically mean that the prediction is in error. In making a presentation at a nonlinear mathematics conference held recently at Rock Island, Professor R. Rivlin of Lehigh University remarked that in his opinion, good experiments are more difficult to achieve than good theory. Even if the measurements are accurately made, considerable care must be exercised in their interpretation. For example, in the M16 model, extraction force is taken as the applied force acting on the cartridge case to resist motion; it does not include inertia forces because the case and bolt are considered to be a single mass during extraction. However, when measured by strain gages on the extractor, extraction force does include the force needed to overcome the inertia of the case. Many sources of experimental errors exist, just as many such sources in theory exist. Certainly, the final authority rests with accurate experimentation, but not all experimentation is accurate.

(c) Theoretical rigor should be sought where possible, but is not essential for useful weapon models. In many cases, it is unjustified. For example, one may naively assert that a model is seriously deficient because, in determination of the extraction force, no plastic effects were considered during the expansion of the cartridge case against the chamber wall. However, inclusion of these effects would not necessarily make the model more accurate because of the large uncertainties in coefficient of friction and gas-pressure distribution. Also, a power series used to obtain a reasonable approximation to a highly complex phenomenon may not converge in the mathematical sense. A model is not invalid because someone thinks that, for example, Coulomb damping rather than viscous damping should have been used in one of the force expressions; in reality, probably neither is correct. The ultimate criterion for an expression is whether it works satisfactorily for the purposes intended, not whether it was rigorously derived from some theory. The construction of weapon models is not a clean-cut process. No models would exist if one insisted on complete rigor for all complex processes in a weapon. In present models, a need exists to determine values of certain poorly understood variables such as friction. This is often done by adjustment of them within the bounds of reasonable values to force better agreement between model and experiment. The true values are unknown anyway, so choosing them to improve model predictions makes sense. Rigor in weapon models is merely one means to an end, not an end in itself.

2.1.3.2 Second Attitude

The second extreme attitude is also unjustified for reasons stated below:

(a) Mathematical modeling has many limitations. High confidence-levels in the accuracy of certain predictions are presently not within the realm of feasibility. For example, although manufacturing tolerances may be set on certain orifices, model capability in the tolerance area is virtually nonexistent. How often a weapon will malfunction; how a weapon will be affected by cold weather, sand, fouling, and moisture; how much rough usage a weapon can be subjected to before it malfunctions; and many other important areas of study cannot be defined on the basis of present models.

(b) Experimentation and experience are still essential in many areas of weapon design, effectiveness, and reliability. Models are merely useful tools. One of the areas of model development in need of further exploration is that of model validation. "Verification" or "validation" of a complex weapon model by experiment is a nebulous concept. The problem is complicated by (1) experimental uncertainty (2) random processes by which measurements are caused to change from round-to-round (3) an extremely large number of possible predictions from a model, some of which will be in better agreement with experiment than others, and most importantly, (4) a lack of criteria that can be applied to determine when experimental and theoretical curves are in satisfactory agreement, which curves, and how many such curves must be in satisfactory agreement before a model can be considered "verified".

2.2 Authors' Approach

The approach taken by the authors in modeling weapons is to use Newton's laws to describe the motion of each significant mass in the mechanism. Each force acting on each mass is individually analyzed, and mathematical expressions are obtained. Some of these forces do not act during various parts of the firing cycle. The logic capability of the computer is used to insert or remove these forces at the correct times. By this approach, the entire cycle is considered at once, and no "phases" are established.

The philosophy outlined above and followed in this report is not claimed to be either entirely unique or even best. It has produced successful results and was carefully and thoughtfully developed over a period of several years. Mr. Robert H. Coberly, Weapons Technology Division, RIA, made many significant contributions to this development.

2.3 Related Work

In addition to the mathematical computer model M16 of the M16A1 Rifle described in this volume, other models also exist. One of these was developed at Frankford Arsenal under the direction of F. Shinaly primarily for the study of malfunction dynamics. An early version of the Frankford model was used for the work described in Reference[1]. At BRL, H. Gay and E. Wineholt constructed an analog-computer model[5].

The advantages of an analog-computer are principally in short computation time and in ease of making parametric variations. However, one is usually much more limited in the size and complexity of a problem that can be solved on this computer than one is when using commonly available digital computers. Also, large modifications in the model can usually be more easily accomplished on a digital computer. Modeling work with a hybrid computer, in which analog and digital capabilities are combined, has also been done at BRL. The use of such a computer for weapon models appears to be very promising. One present disadvantage is that generally the memory capacity of the digital portion is considerably less than that of a computer such as the IBM 360/65.

Perhaps the greatest differences between the model described in this report and other existing models lie in the larger amount of detail incorporated into this model, the capacity of this model to approximately analyze weapon dispersion, the specific modeling approaches used, and the purposes for which the models were designed.

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[1] Grandy, A.J.; Duffy, J.A.; Horchler, M.H.; and Ehle, P.E.; "Investigation of Bolt/Bolt-Carrier Clearances in the M16A1 Rifle," Technical Note TN-1159, Frankford Arsenal, Philadelphia, Pa. (May 1971)

[5] Gay, H.P. and Wineholt, E.M., "Analog Simulation of the Mechanism of the M16A1 Rifle," BRL Report No. 1596, Aberdeen Proving Ground, Md. (Jun 1972)

3.0 M16A1 RIFLE MODEL

3.1 Brief Description of Weapon Operation

The M16A1 Rifle (Fig. 1) is a hand-held gas-operated 5.56mm weapon and has both full-automatic and semi-automatic modes of operation. Initially the bolt is locked to the barrel extension, and the hammer is in cocked position. When the trigger is pulled, the hammer is released by the sear and strikes the firing pin. The firing pin then strikes the primer, from which action the propellant is ignited. (The main components of the M16A1 are shown in Fig. 2). The pressure from the propellant gases acts on the main gun through the base of the cartridge case and the locked bolt. When the projectile reaches the gas port, which is located approximately 16 inches from the chamber, some gas is redirected rearwardly along a gas tube to a cavity in the bolt-carrier. The pressure in this cavity forces the bolt forward and the bolt-carrier

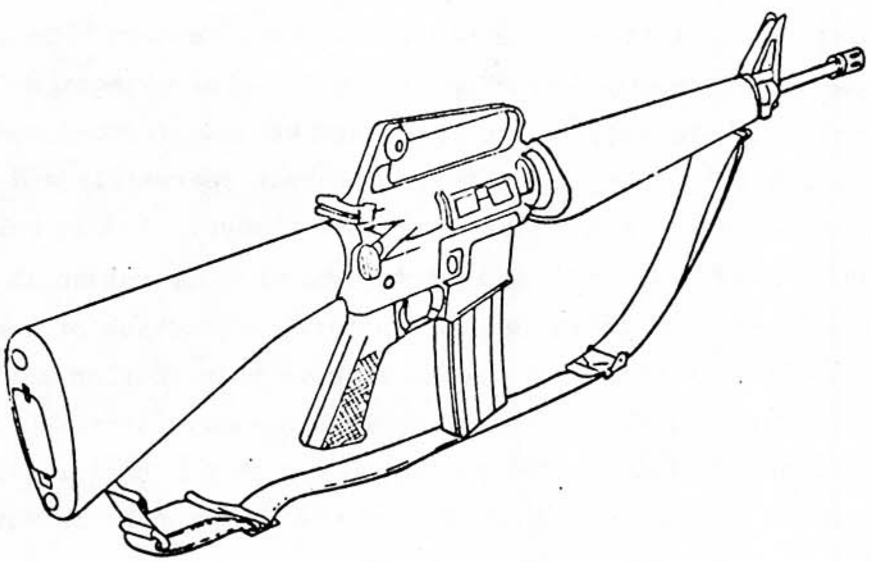

Figure 1 M16A1 Rifle

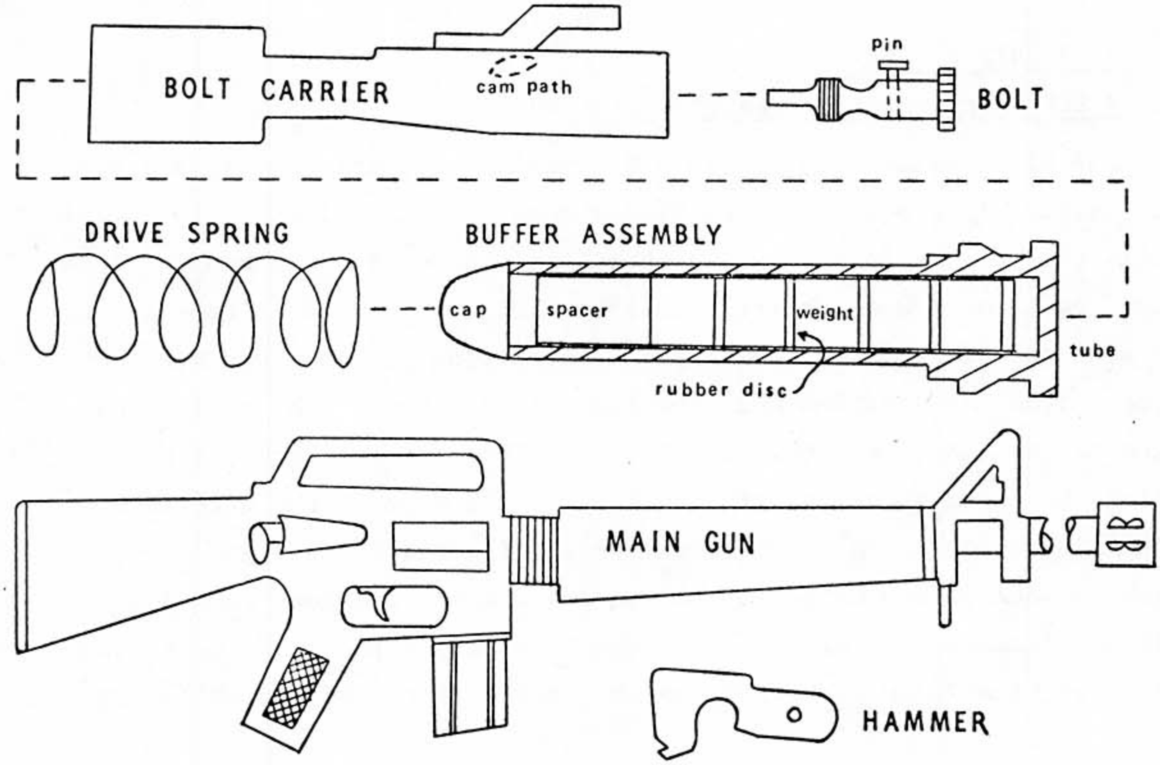

Figure 2 Components of M16A1 Rifle

rearward. As the bolt-carrier moves rearwardly, a cam path cut in the the carrier causes a pin rigidly connected to the bolt to turn. This action causes the bolt to become unlocked from the barrel extension. Just after the bolt is unlocked, the bolt pin strikes the forward end of the cam path. The bolt-carrier now pulls the bolt rearwardly and the bolt pulls the empty cartridge case from the chamber. As the bolt, bolt-carrier, and buffer assembly continue rearward, their motion is resisted by the drive spring acting on the buffer, the cocking of the hammer, and the various frictional forces. At some point during this rearward travel, a spring-loaded ejector expels the empty cartridge case. A few milliseconds later, a polyurethane cap on the buffer assembly usually comes in contact with the rear of the buffer tube in which the entire buffer assembly and drive spring are housed; then the counter recoil begins. Part of the energy of the bolt, bolt-carrier, and buffer is imparted to the main gun and mount, part is lost to permanent deformations and heat, and part is retained by the drive spring, which must push the bolt, bolt-carrier, and buffer assembly forward again.

In counter recoil, the bolt impacts against the face of the top cartridge in the magazine, strips it, and loads it into the chamber. The bolt-carrier continues forward and, by means of the cam path, causes the bolt to become locked to the barrel extension once again. Weights in the buffer body continue to move forward a small distance and to strike the end of the body. This impact counteracts a tendency for the bolt-carrier to rebound and thus prevents premature unlocking. In the full-automatic mode, just prior to completion of locking, the bolt-carrier trips the sear by which action the hammer is released from its cocked position. The hammer moves forward and strikes the firing pin; the operating cycle is repeated. In the semi-automatic mode, the sear is not tripped, and the hammer does not move forward until the trigger is depressed again.

3.2 General Equations of Motion for the M16A1 Rifle

In writing the equations of motion, the authors followed a Newtonian rather than a Lagrangian approach. For the present problem, where there were not a large number of irrelevant constraint forces, the equations could be found more simply through Newton's laws. Both approaches ultimately yield the same results.

Separate coordinate systems are established for each component mass. One coordinate system is constructed for rifle rotation, and the rotation angle is defined as zero when the barrel is horizontal. The origin of each coordinate system is fixed with respect to the ground. Positive axial directions are in the direction of initial bullet velocity. Positive rotation is that which causes the muzzle to move away from the ground.

The equations of motion and their initial conditions are listed below. These are written with only general expressions for the forces. A summary of forces is given (after this list of equations of motion) in which a brief physical description of each is provided (Table 1). The detailed mathematical descriptions of both axial and rotational forces are included in Appendix A.

a. Bolt

$$M_{B(E)} \ddot{X}_B = FB(1) + FB(2) + FB(3) + FB(4) + FB(5) + FB(6)$$
$$+ FB(7) + FB(8) + FB(9) + FB(11) + FB(12)$$

At $t = 0$: $\dot{X}_B = 0$, $X_B = X_{B_o}$

X_B = position of center of gravity of bolt

$FB(i)$ = ith force acting on bolt as discussed in Appendix A.

$M_{B(E)}$:
- = effective mass of bolt = .156 lb after a new round has been stripped from the magazine, but before the round is fired
- = .143 lb after the round has been fired, but before the case is ejected
- = .130 lb after the cartridge case has been ejected, but before a new round is obtained from the magazine

$X_{B_o} = 0$

b. Bolt-Carrier

$$M_{BC} \ddot{X}_{BC} = FBC(1) + FBC(2) + FBC(3) + FBC(4) + FBC(5) + FBC(6)$$
$$FBC(7) + FBC(8) + FBC(9) + FBC(10) + FBC(11) + FBC(12)$$

At $t = 0$: $\dot{X}_{BC} = 0$, $X_{BC} = X_{BC_o}$

X_{BC} = position of center of gravity of bolt-carrier

$FBC(i)$ = ith force acting on bolt-carrier as discussed in Appendix A.

M_{BC} = mass of bolt-carrier = .586 lb

$X_{BC_o} = .7 \times 10^{-3}$ inch

c. Main gun structure

$$M_{MG(E)} \ddot{X}_{MG} = FMG(1) + FMG(2) + FMG(3) + FMG(4) + FMG(5) + FMG(6)$$
$$+ FMG(7) + FMG(8) + FMG(9) + FMG(10) + FMG(11)$$
$$+ FMG(12) + FMG(13) + FMG(14) + FMG(15)$$

At $t = 0$: $\dot{X}_{MG} = 0$, $X_{MG} = X_{MG_o}$

X_{MG} = position of center of gravity of main gun

$FMG(i)$ = ith force acting on main gun as discussed in Appendix A.

$M_{MG(E)}$ = Effective mass of main gun = 6.2 + .0252(N-19) lb

N = Number of rounds of ammunition in magazine at any given time

$X_{MG_o} = 0$

d. Buffer tube

$$M_{BUFF}\ddot{X}_{BUFF} = FBUFF(1) + FBUFF(2) + FBUFF(3) + FBUFF(4) + FBUFF(5) + FBUFF(6)$$

At $t = 0$: $\dot{X}_{BUFF} = 0$, $X_{BUFF} = X_{BUFF_o}$

X_{BUFF} = position of center of gravity of empty buffer tube

FBUFF(i) = ith force acting on buffer tube

M_{BUFF} = mass of empty buffer tube =.0987 lb

$X_{BUFF_o} = 1.4 \times 10^{-3}$ inch

e. Buffer weights

Option 1: Lumped mass system

$$M_{TOTAL}\ddot{X}_{WTS} = F_{WTS}$$

At $t = 0$: $\dot{X}_{WTS} = 0$, $X_{WTS} = X_{WTS_o}$

X_{WTS} = position of center of gravity of buffer weight system

F_{WTS} = force acting on buffer weights as described in Appendix A.

M_{TOTAL} = Total mass of all buffer weights = .224 lb

X_{WTS_o} = .0014 inch when weights are completely forward in tube
= -.129 inch when weights are completely rearward in tube

Option 2: Individual masses

$$M_i\ddot{X}_i = FM_{i,1} + FM_{i,2} + FM_{i,3} + FM_{i,4} \quad (i = 1,2,3,4,5)$$

At $t = 0$: $\dot{X}_i = 0$, $X_i = X_{i_o}$

X_i = position of center of gravity of ith mass

$FM_{i,j}$ = jth force acting on ith mass as described in Appendix A.

M_i = mass of ith buffer weight ($M_1 = M_2 = M_3 = M_4 = .0406$ lb, $M_5 = .0616$ lb)

X_{i_o} = .0014 inch when weights are completely forward in tube
=-.129 inch when weights are completely rearward in tube

f. Hammer

$$I_H \ddot{\theta} = T_{SPRING} - r_2 \cos(\alpha_2 + \theta)\ FMG(7) - r \cos(\alpha + \theta)\ FBC(8) - M_H[r_o \cos\theta]\ \ddot{X}_H$$

At $t = 0$: $\dot{\theta} = 0$, $\theta = \theta_o$

θ = rotational position of hammer

T_{SPRING} = torque exerted by hammer spring = 6 in-lb

FMG(7) and FBC(8) are forces acting between hammer and main gun and between hammer and bolt-carrier as discussed in Appendix A.

I_H = moment of inertia of hammer about an axis through the pivot point = .000015 slug-ft^2

$\theta_o = -70°$

M_H = mass of hammer = .0683 lb

r_o = distance from c.g. of hammer to pivot point of hammer = .75 inch

g. Rifle rotation

$$I_G \ddot{\theta}_G = T_{SHOULDER} + T_{BODY} + T_{\substack{INTERNAL \\ GUN\ MOTION}}$$

At $t = 0$ $\dot{\theta}_G = 0$, θ_G = any angle of elevation, typically 0°

$T_{SHOULDER}$, T_{BODY}, and $T_{\substack{INTERNAL \\ GUN\ MOTION}}$ are torques acting on the rifle as described in Appendix A.

I_G = Moment of inertia of the rifle about its center of gravity = .135 slug-ft^2

h. Exterior ballistics (neglecting drag)

$$Z = -\frac{1}{2} g \frac{(R-y_i)^2}{\dot{y}_i^{\,2}} + \dot{Z}_i \frac{R-y_i}{\dot{y}_i} + Z_i$$

z = vertical coordinate of projectile impact point at range R

R = horizontal coordinate of projectile impact point

z_i = vertical coordinate of projectile at exit from barrel

$\dot{z}_i$ = vertical velocity of projectile at exit from barrel

$\dot{y}_i$ = horizontal velocity of projectile at exit from barrel

See Appendix H for derivation.

3.3 A Note on Initial Conditions

Initially, unbalanced forces must not be present on any of the weapon components. Forces that must be balanced are the preload force from the drive spring that acts on the buffer tube and gravity forces that act on the masses when the weapon is elevated. These forces are balanced by the allowance of extremely small penetrations of the bolt pin into the forward end of the cam path, and the buffer tube into the bolt-carrier. Thus, if $F_{PRELOAD}$ = drive spring preload force, then

$$X_{BC_o} = [F_{PRELOAD} - g(\sin\theta_E)(M_{BC} + M_{BUFF})] / K_{BC} = .7 \times 10^{-3} \text{ inch}$$

and

$$X_{BUFF_o} = X_{BC_o} + [F_{PRELOAD} - g(\sin\theta_E)M_{BUFF}] / K_{BUFF} = 1.4 \times 10^{-3} \text{ inch}$$

TABLE 1 SUMMARY OF FORCES

	BRIEF DESCRIPTION OF FORCE	MASS RECEIVING THE FORCE									
		BOLT	BOLT CARRIER	MAIN GUN	BUFF BODY	HAMMER	BUFF. MASS 1	BUFF. MASS 2	BUFF. MASS 3	BUFF. MASS 4	BUFF. MASS 5
1.	Breech force	FB(1)									
2.	Friction between bolt and ammo	FB(2)		FMG(2)							
3.	Impact between bolt and barrel and engagement of extractor	FB(3)		FMG(3)							
4.	Friction between bolt and bolt carrier	FB(4)	FBC(1)								
5.	Force at end of cam path when rifle is not locked	FB(5)	FBC(2)								
6.	Unlocking force	FB(6)	FBC(3)								
7.	Locking force	FB(7)	FBC(5)								
8.	Force at end of cam path when rifle is locked	FB(8)	FBC(4)								
9.	Stripping of ammo from magazine	FB(9)		FMG(15)							
10.	Pressure in bolt carrier cavity	FB(10)	FBC(9)								
11.	Extraction force	FB(11)		FMG(10)							
12.	Gravity force on bolt	FB(12)									
13.	Interaction between bolt carrier and buffer body		FBC(6)	FBUFF(1)							
14.	Friction between bolt carrier and main gun		FBC(7)	FMG(4)							
15.	Constraint force between hammer and bolt carrier		FBC(8)			FHAM(1)					
16.	Friction between bolt carrier and ammo		FBC(10)	FMG(12)							
17.	Friction between hammer and bolt carrier		FBC(11)	FMG(13)							
18.	Gravity force on bolt carrier		FBC(12)								
19.	Mount force			FMG(1)							
20.	Drive spring force			FMG(5)	FBUFF(3)*						
21.	Bore friction			FMG(6)							
22.	Impact between hammer and main gun			FMG(7)		FHAM(2)					
23.	Constraint force between hammer and main gun			FMG(8)		FHAM(3)					
24.	Impact between buffer tube and backplate			FMG(9)	FBUFF(2)						
25.	Gravity force on main gun			FMG(11)							
26.	Friction between buffer tube and main gun			FMG(14)	FBUFF(6)						
27.	Gravity force on buffer tube				FBUFF(4)						

* NOTE: FBUFF(3) is equal and opposite to FMG(5) under static conditions

TABLE 1 SUMMARY OF FORCES (cont'd)

BRIEF DESCRIPTION OF FORCE	MASS RECEIVING THE FORCE									
	BOLT	BOLT CARRIER	MAIN GUN	BUFF BODY	HAMMER	BUFF. MASS 1	BUFF. MASS 2	BUFF. MASS 3	BUFF. MASS 4	BUFF. MASS 5
28. Interaction between buffer and all buffer weights						$FM_{1,3}$ & $FM_{1,1}$	$FM_{2,3}$	$FM_{3,3}$	$FM_{4,3}$	$FM_{5,3}$ & $FM_{5,2}$
29. Spring interaction between tube and 1st buffer weight				$FBUFF(5)_1$		$FM_{1,1}$				
30. Spring interaction between 1st and 2nd buffer weights						$FM_{1,2}$	$FM_{2,1}$			
31. Friction between tube and 1st buffer weight				$FBUFF(5)_2$		$FM_{1,3}$				
32. Gravity force on 1st buffer weight						$FM_{1,4}$				
33. Spring interaction between 2nd and 3rd buffer weights							$FM_{2,2}$	$FM_{3,1}$		
34. Friction between tube and 2nd buffer weight				$FBUFF(5)_3$			$FM_{2,3}$			
35. Gravity force on 2nd buffer weight							$FM_{2,4}$			
36. Spring interaction between 3rd and 4th buffer weights								$FM_{3,2}$	$FM_{4,1}$	
37. Friction between tube and 3rd buffer weight				$FBUFF(5)_4$				$FM_{3,3}$		
38. Gravity force on 3rd buffer weight								$FM_{3,4}$		
39. Spring interaction between 4th and 5th buffer weights									$FM_{4,2}$	$FM_{5,1}$
40. Friction between tube and 4th buffer weight				$FBUFF(5)_5$					$FM_{4,3}$	
41. Gravity force on 4th buffer weight									$FM_{4,4}$	
42. Spring interaction between tube and 5th buffer weight				$FBUFF(5)_6$						$FM_{5,2}$
43. Friction between tube and 5th buffer weight				$FBUFF(5)_7$						$FM_{5,3}$
44. Gravity force on 5th buffer weight										$FM_{5,4}$

NOTE: That $FBUFF(5) = \sum_{i=1}^{7} FBUFF(5)_i$

3.4 Examples of Model Predictions

A number of predictions are now presented as examples of data that can be generated from the model. These predictions include peak forces, impulses, energies, force shapes, and component motions.

3.4.1 Summary of Predicted Peak Forces, Impulses, and Energies

Predicted peak forces that act on the bolt-carrier, bolt, main gun, and buffer tube are contained in Tables 2 thru 5. Computed impulses and energies associated with these forces are represented by bar graphs in Figures 3 to 6. To evaluate the significance of a particular force, one must consider all three indicators. A force can have a high value in one category, but a low value in another. Regardless of impulse or energy value, a high peak force can be significant for material strength and fatigue considerations.

From these tables and figures, one can draw quantitative conclusions about the influence of each force in the model. A number of qualitative conclusions are mentioned below:

a. Forces applied at the ends of the cam paths and the gas pressure in the bolt-carrier cavity are of greatest significance to the motion of the bolt-carrier. In the nominal case, friction forces are the least significant, although in some circumstances they can, in practice, be large enough to stop weapon operation. The most likely areas for fatigue are at the ends of the cam paths.

b. The motion of the bolt is strongly influenced by the force between the bolt pin and the end of the cam path, and the gas pressure in the bolt-carrier cavity.

c. The mount, breech force, bore friction, drive spring, and the impact between buffer tube and backplate are most significant to the motion of the main gun. Thus, these are factors that affect accuracy.

d. The motion of the buffer tube is strongly affected by interaction with the bolt-carrier, impact with the backplate, the drive spring, and interaction with the buffer weights.

Although the forces mentioned are those that are most influential from the point of view of magnitude, the weapon could not be operated without the many small forces that cause events to occur in the right sequence at the right time.

TABLE 2 CALCULATED PEAK FORCES IMPARTED IN THE FIRST CYCLE TO THE BOLT-CARRIER

FORCE SYMBOL AND BRIEF DESCRIPTION		PEAK FORCE (lb)
FBC(1)	Friction between bolt and bolt-carrier	.26
FBC(2)	Force at end of cam path when rifle is not locked	440.
FBC(3)	Unlocking force	35.
FBC(4)	Force at end of cam path when rifle is locked	308.
FBC(5)	Locking force	18.4
FBC(6)	Interaction between bolt-carrier and buffer tube	310.
FBC(7)	Friction between bolt-carrier and main gun	.37
FBC(8)	Constraint force between hammer and bolt-carrier	52.
FBC(9)	Pressure in bolt-carrier cavity	520.
FBC(10)	Friction between bolt-carrier and ammo	2.67
FBC(11)	Friction between hammer and bolt-carrier	1.21
FBC(12)	Gravity force on bolt-carrier	0.0

TABLE 3 CALCULATED PEAK FORCES IMPARTED IN THE FIRST CYCLE TO THE BOLT

FORCE SYMBOL AND BRIEF DESCRIPTION		PEAK FORCE (lb)
FB(1)	Breech force	2185.
FB(2)	Friction between bolt and ammo	2.67
FB(3)	Impact between bolt and barrel and engagement of extractor	158.
FB(4)	Friction between bolt and bolt-carrier	.26
FB(5)	Force at end of cam path when rifle is not locked	440.
FB(6)	Unlocking force	35.
FB(7)	Locking force	18.4
FB(8)	Force at end of cam path when rifle is locked	308.
FB(9)	Stripping of ammo from magazine	6.91
FB(10)	Pressure in bolt-carrier cavity	520.
FB(11)	Extraction force	.94
FB(12)	Gravity force on bolt	0.0

TABLE 4 CALCULATED PEAK FORCES IMPARTED IN THE FIRST CYCLE TO THE MAIN GUN

FORCE SYMBOL	AND BRIEF DESCRIPTION	PEAK FORCE (lb)
FMG(1)	Mount force	44.3
FMG(2)	Friction between bolt and ammo	2.67
FMG(3)	Impact between bolt and barrel and engagement of extractor	158.
FMG(4)	Friction between bolt-carrier and main gun	.37
FMG(5)	Drive spring force	22.
FMG(6)	Bore friction	325.
FMG(7)	Impact between hammer and main gun	258.
FMG(8)	Constraint force between hammer and main gun	85.5
FMG(9)	Impact between buffer tube and backplate	318.
FMG(10)	Extraction force	.94
FMG(11)	Gravity force on main gun	0.0
FMG(12)	Friction between bolt-carrier and ammo	2.67
FMG(13)	Friction between hammer and bolt-carrier	1.21
FMG(14)	Friction between buffer tube and main gun	.33
FMG(15)	Stripping of ammo from magazine	6.91

TABLE 5 CALCULATED PEAK FORCES IMPARTED IN THE FIRST CYCLE TO THE BUFFER TUBE

FORCE SYMBOL	AND BRIEF DESCRIPTION	PEAK FORCE (lb)
FBUFF(1)	Interaction between bolt carrier and buffer tube	310.
FBUFF(2)	Impact between buffer tube and backplate	318.
FBUFF(3)	Drive spring force	19.0
FBUFF(4)	Gravity force on buffer tube	0.0
FBUFF(5)	Interaction between buffer tube and all buffer weights	144.
FBUFF(6)	Friction between buffer tube and main gun	.33

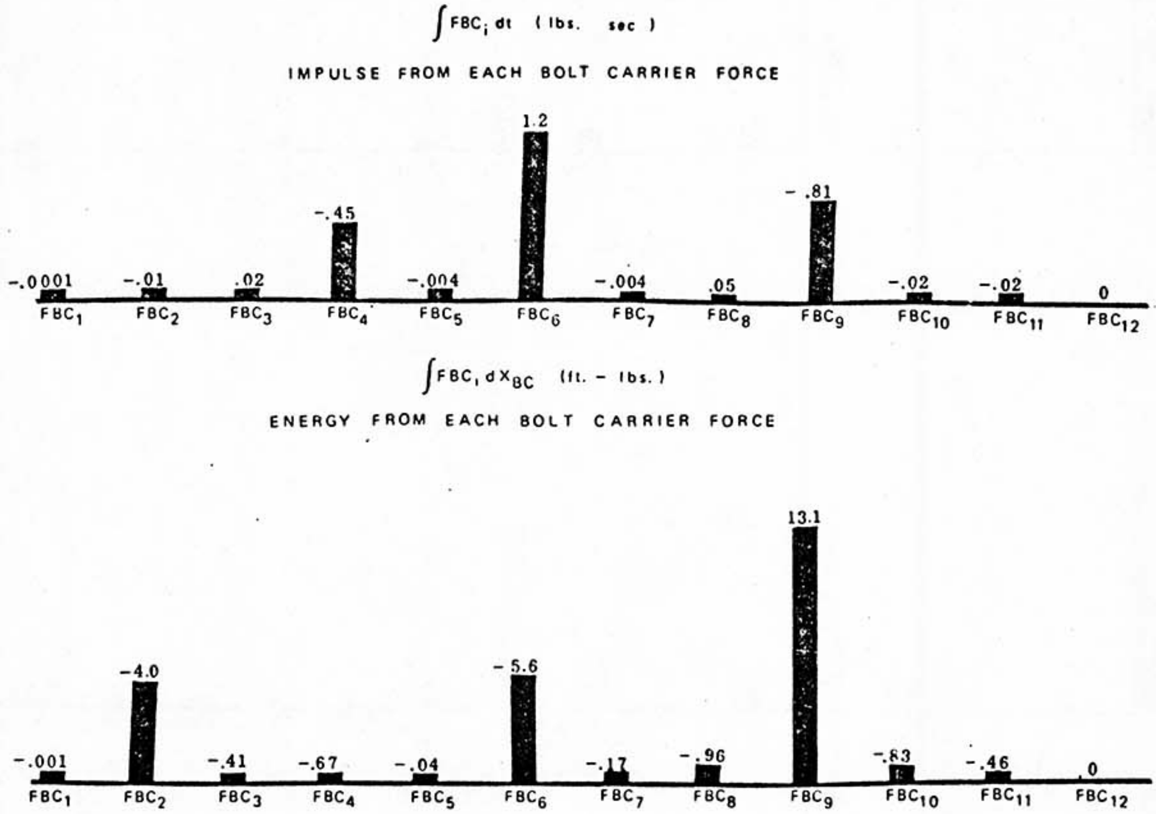

Figure 3 Computed Impulse and Energy Imparted by Each Bolt-Carrier Force

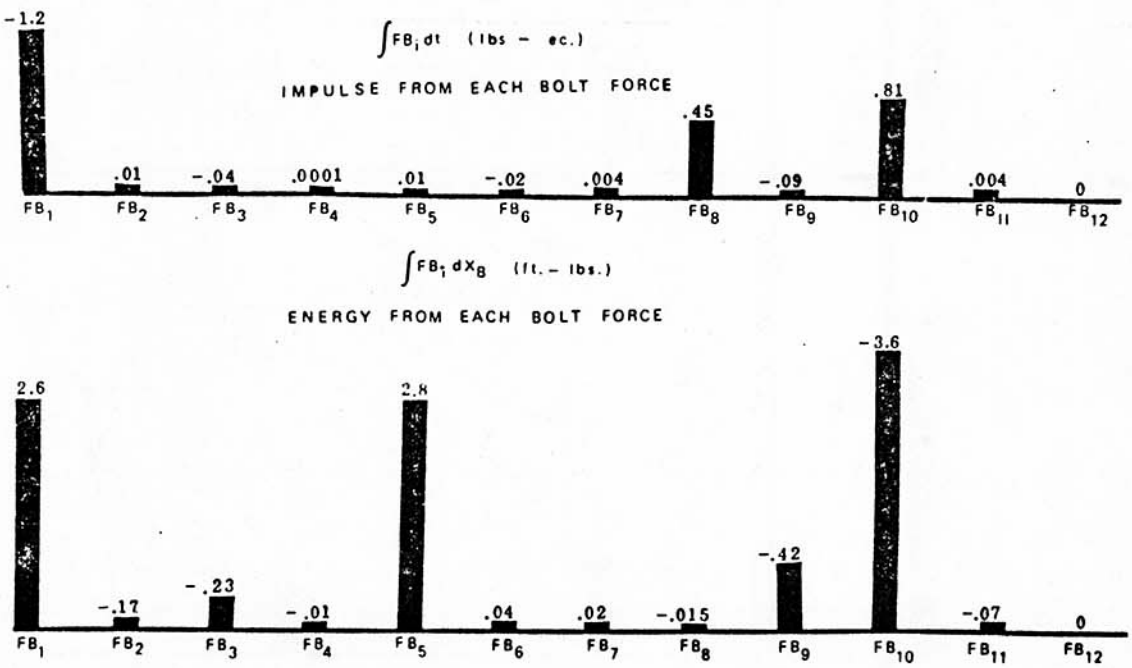

Figure 4 Computed Impulse and Energy Imparted By Each Bolt Force

NOTE: All integrals are for the first cycle, and rifle is rigidly mounted.

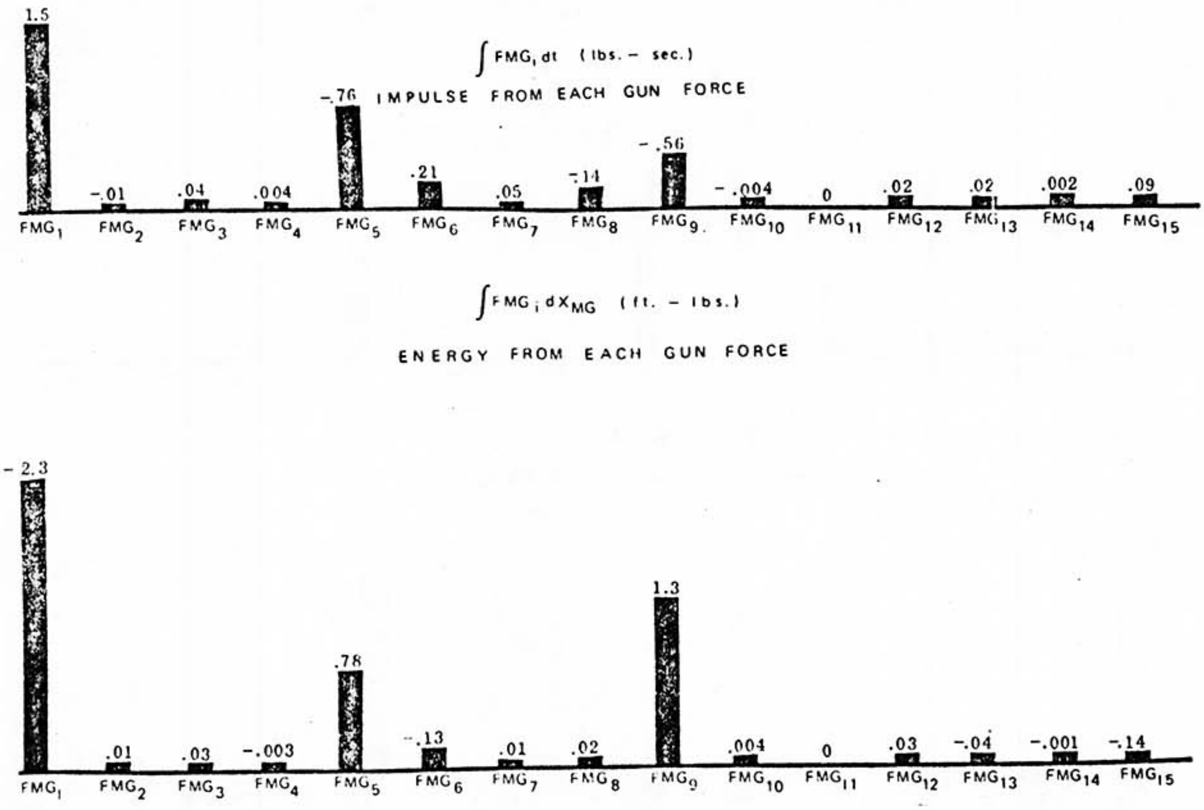

Figure 5 Computed Impulse and Energy Imparted By Each Main-Gun Force

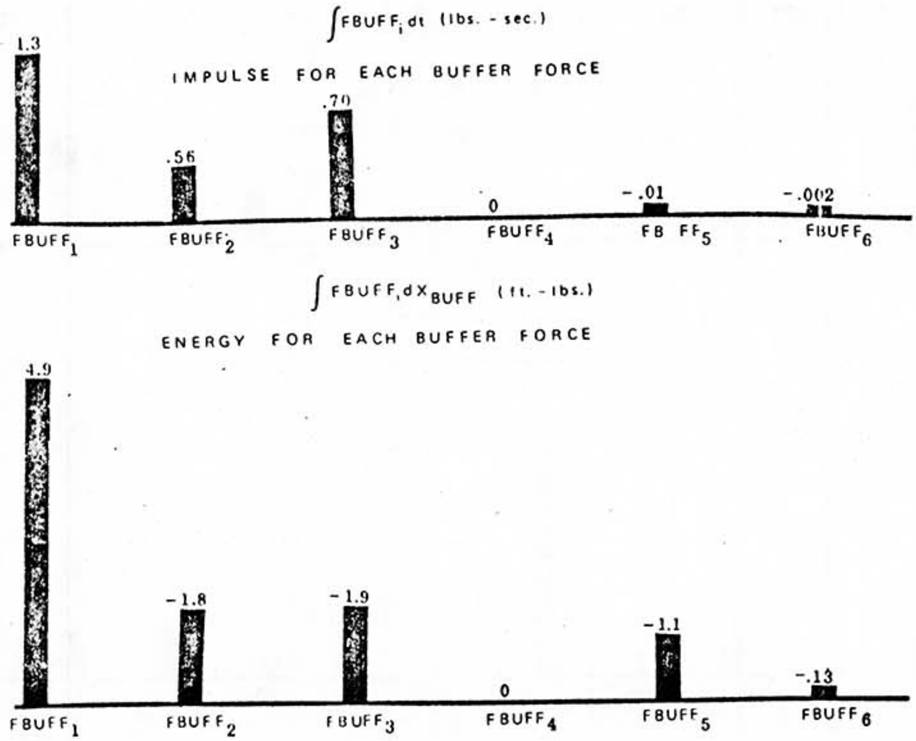

Figure 6 Computed Impulse and Energy Imparted By Each Buffer-Tube Force

NOTE: All integrals are for the first cycle, and rifle is rigidly mounted

3.4.2 Predicted Force Shapes (Cocking, Unlocking, Mount, Drive-Spring)

Cocking and unlocking forces on the bolt-carrier are shown in Figure 7. With this model, a very sharp rise and a gradual decay over approximately two milliseconds are predicted for both forces.

The force felt at the shooter's shoulder during one cycle is shown in Figure 8. Peaks occur as results of the breech pressure and the impact of the buffer tube on the backplate.

Drive-spring forces are shown in Figure 32. Note the delay in the response of the rearward end of the drive-spring after the forward end is displaced.

3.4.3 Predicted Component Motions

Displacement and velocity are predicted as functions of time for the bolt, bolt-carrier, main gun, and hammer (Figures 9-12). Key events are also noted on these graphs. Accelerations have such large fluctuations that they cannot be plotted effectively on graphs of this size. All graphs shown begin with hammer release at time equal to zero.

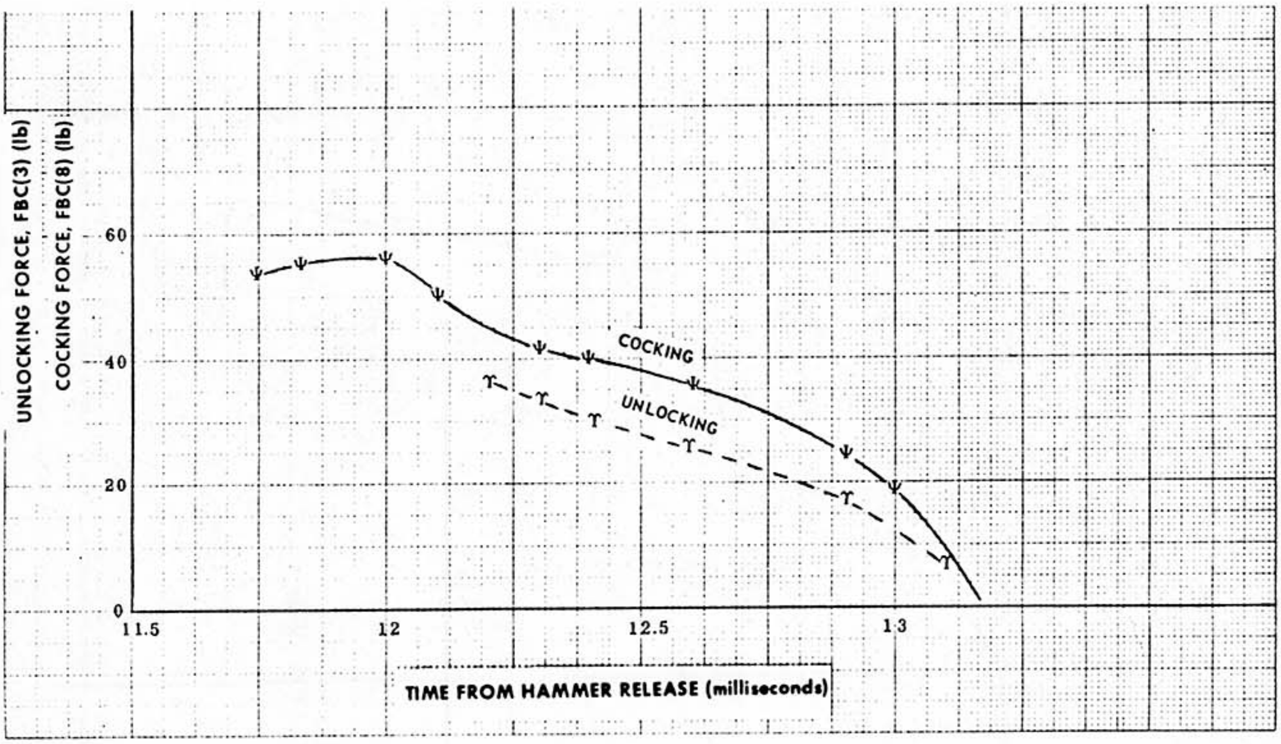

Figure 7 Computed Cocking and Unlocking Forces on Bolt-Carrier vs Time

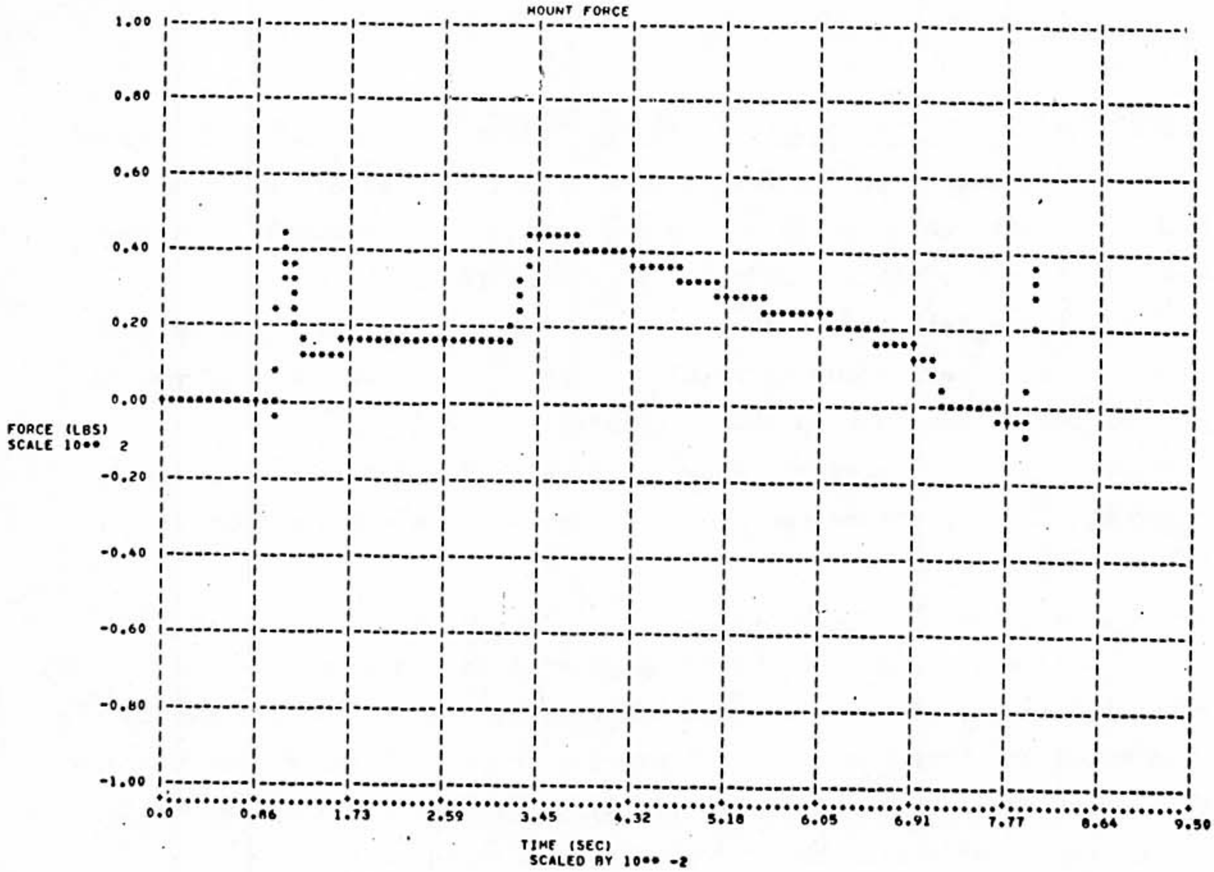

Figure 8 Computed Force at Shooter's Shoulder

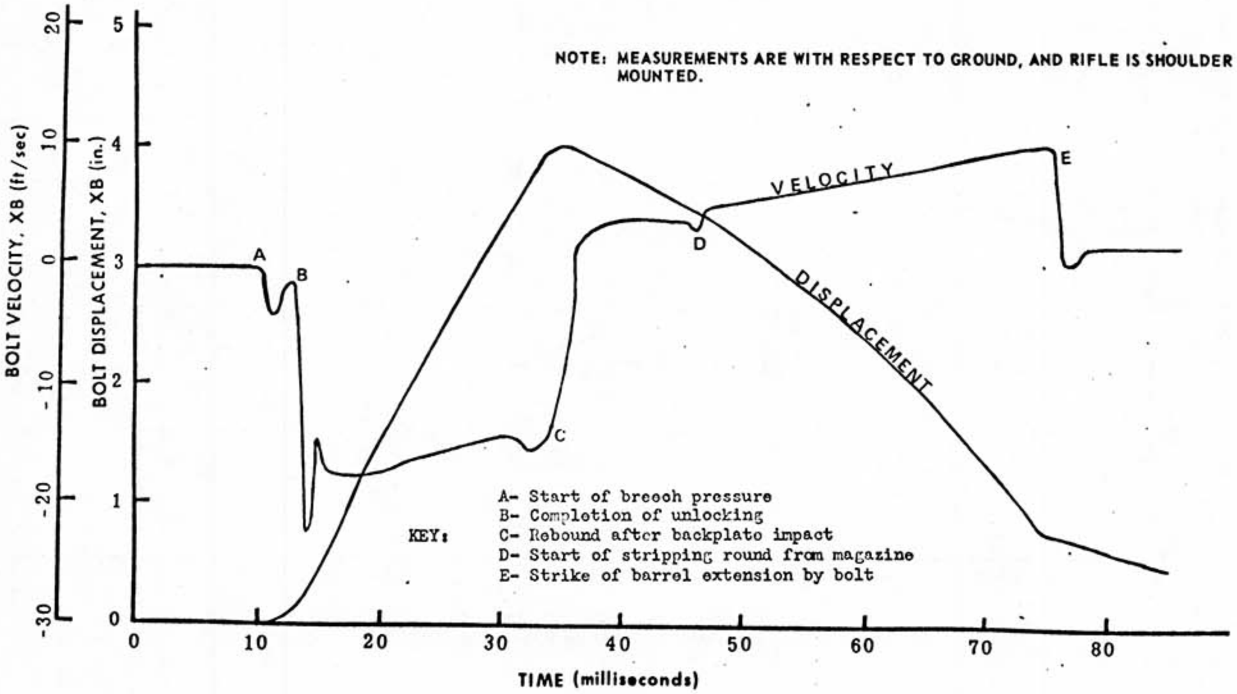

Figure 9 Computed Displacement and Velocity of Bolt vs Time

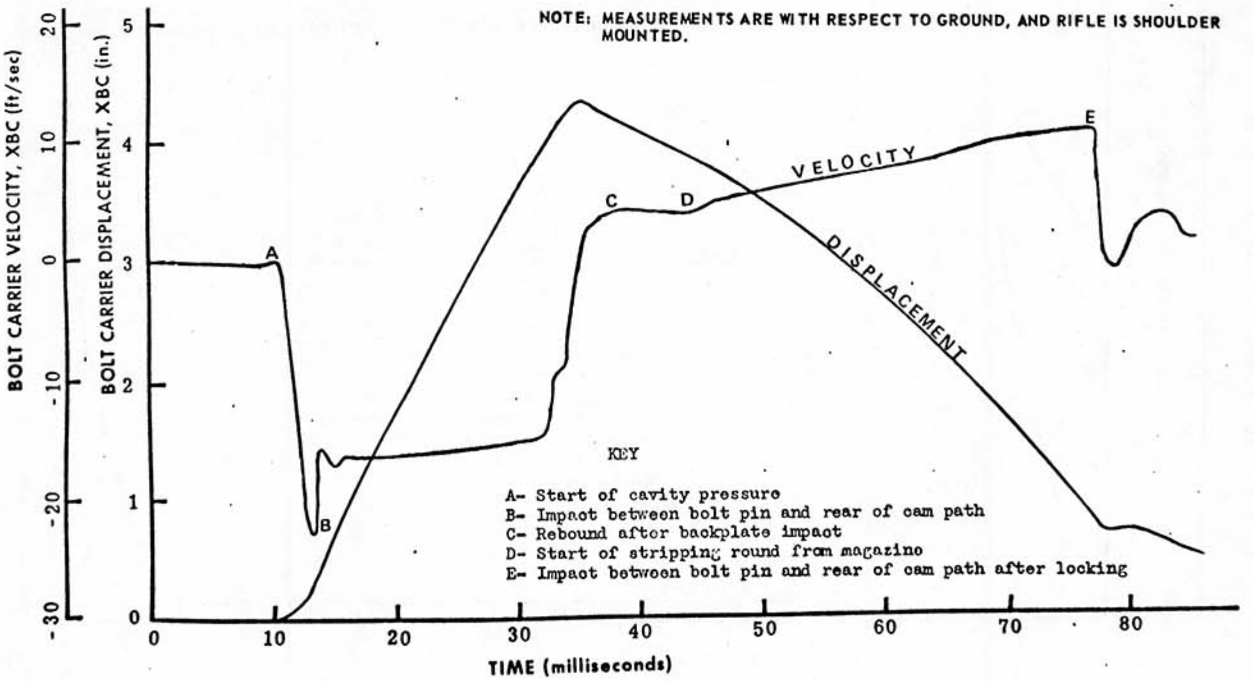

Figure 10 Computed Displacement and Velocity of Bolt-Carrier vs Time

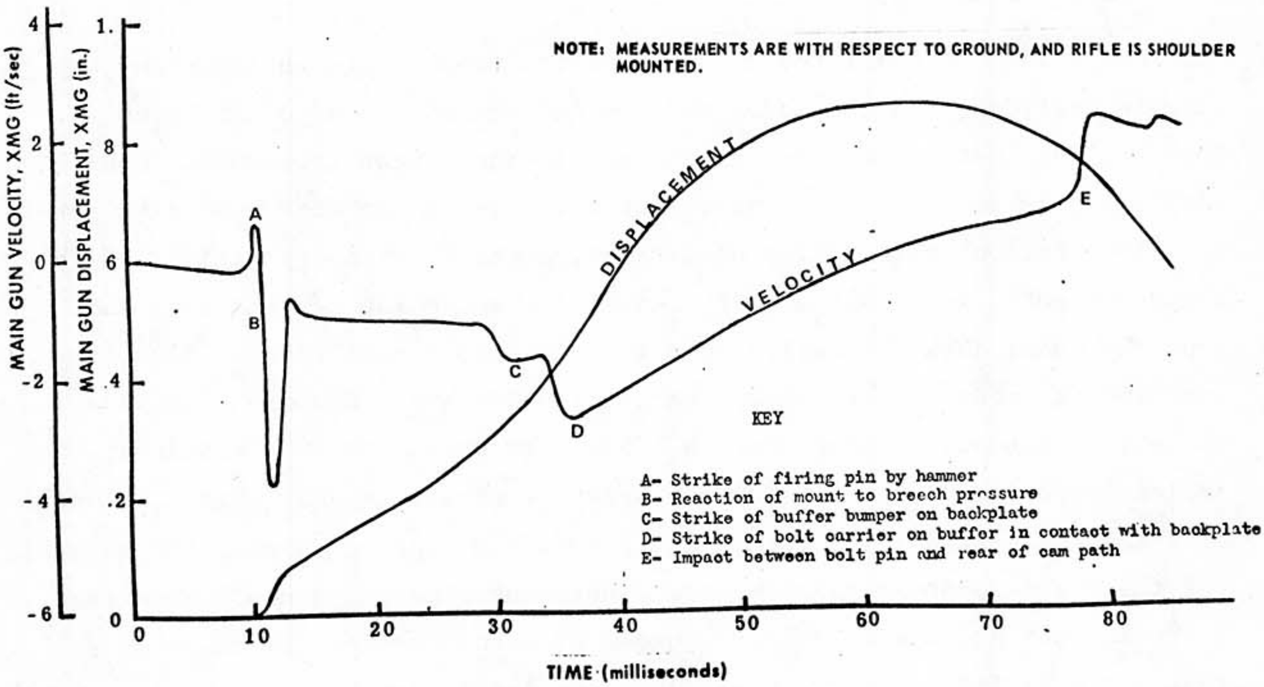

Figure 11 Computed Displacement and Velocity of Main Gun vs Time

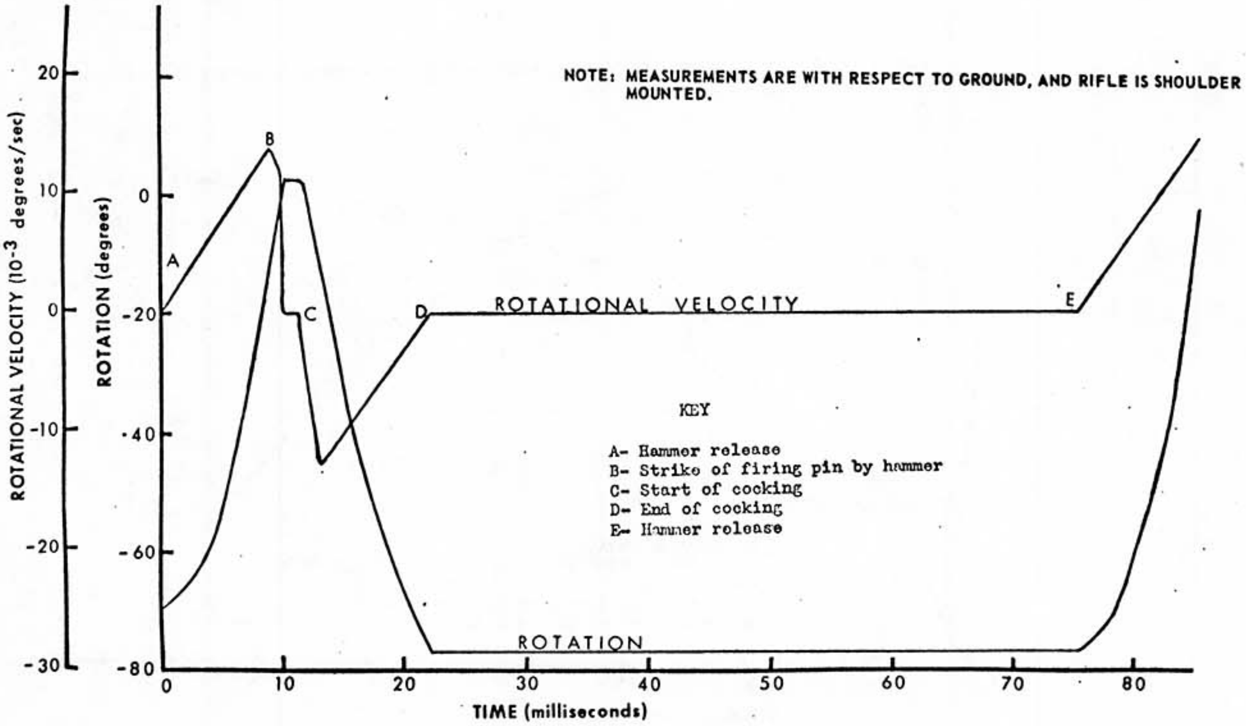

Figure 12 Computed Rotational Displacement and Velocity of Hammer vs Time

3.5 Agreement with Experiments

Comparison of predicted and experimental results is an important way to gain confidence in the model and to find areas in need of improvement. However, one must be cautious in the conclusions drawn from such a comparison. One should keep in mind that the experiments were made with certain lubrication and mounting conditions, certain initial positions of the buffer weights, etc. Not all of these factors are known precisely, and thus the model does not reflect exactly these same conditions. Also, the experimental measurements have a range of uncertainty because of a wide variety of potential error sources. Some phenomena are very difficult to isolate experimentally. One of the greatest advantages and strong points of a model is its ability to predict trends (as opposed to absolute values), and these are seldom available from experimental measurements. Note that virtually any model with a large number of parameters can be adjusted so that any experimental trace desired can be duplicated.

The key question is whether, once the parameter values are set on the basis of one curve, these values remain valid when other curves are predicted. Thus, the validity and usefulness of a model is not determined by whether cyclic time, extraction force, or other measured values coincide exactly with predicted values. The whole area of model validation is presently a nebulous one. No established criteria exist that, if met, render the model "verified." This aspect of weapon modeling deserves much additional study.

A number of model predictions will now be compared with published experimental data. These predictions include bolt-carrier velocities, cycle time, time for hammer fall, cocking-and-friction energy, drive-spring energy, friction energies, and time-displacement data.

3.5.1. Bolt-carrier velocities for rigid mount

TABLE 6 COMPARISON OF PREDICTED AND EXPERIMENTAL BOLT-CARRIER VELOCITIES

POSITION OF BOLT-CARRIER	COMPUTED VALUE ft/sec	EXPERIMENTAL VALUE ft/sec	EXPERIMENTAL RANGE ft/sec
.64 in from locked position (recoil stroke)	17.8	18.4	(16.6 to 20.2)
.64 in. forward of position at time of buffer/backplate impact (recoil stroke)	15.7	15.2	(13.7 to 16.7)
.64 in. forward of position at time of buffer/backplate impact (counterrecoil stroke)	6.26	7.9	(7.1 to 8.7)
.64 in. rearward of position at which impact with bolt pin occurs (counterrecoil stroke)	10.6	10.8	(9.7 to 11.9)

The source of the experimental values in Table 6 is Ref [1]. The experimental accuracy stated in this report is ±10%.

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[1]Grandy, A.J.; Duffy, J.A.; Horchler, M.H.; and Ehle, P.E.; "Investigation of Bolt/Bolt-Carrier Clearances in the M16A1 Rifle," Technical Note TN-1159, Frankford Arsenal, Philadelphia, Pa. (May 1971)

3.5.2. Cycle time

Note that recoil time is defined here as the time from when the bolt-carrier first begins rearward motion until the buffer body rebounds from the closed end of the buffer tube. Counterrecoil time is the time from when the bolt-carrier begins to move forward until the end of the cam path strikes the bolt pin after locking. The dwell time is defined as the time from when the end of the cam path strikes the bolt pin until the bolt-carrier begins to move rearward again. Cycle time is the sum of recoil, counterrecoil, and dwell times. These times are compared in Table 7.

TABLE 7 COMPARISON OF PREDICTED AND EXPERIMENTAL RECOIL, COUNTERRECOIL, DWELL, AND CYCLE TIMES

	Recoil Time	Counterrecoil Time	Dwell Time	Cycle Time
Experimental	23	38	10	71
Computed	21	39	10	70

NOTE: All times are in milliseconds

3.5.3. Hammer-fall time

In the automatic mode of fire, the time for hammer-fall is computed to be 10.3 milliseconds. Experimental measurements of the time for hammer-fall after sear release varied between 10 and 11 milliseconds during 20 trials. See page 13 of Ref[1]. (In the semi-automatic mode, the computed time is 9.9 milliseconds.)

3.5.4. Cocking-and-friction energy

The subject of this comparison is the integral, over bolt-carrier displacement, of the cocking force acting on the bolt-carrier and of the frictional force acting between the bolt-carrier and the hammer as the bolt-carrier is recoiling. Some of the difference between computed and experimental values can be attributed to the fact that the experimental measurements were not under realistic dynamic conditions; quasi-steady measurements were made with an Instron testing machine.

---

[1]Grandy, A.J.; Duffy, J.A.; Horchler, M.H.; and Ehle, P.E.; "Investigation of Bolt/Bolt-Carrier Clearances in the M16A1 Rifle," Technical Note TN-1159, Frankford Arsenal, Philadelphia, Pa. (May 1971)

The results[6]:

Experimental	14.0 in-lb
Computed	17.7 in-lb

Computer results indicate that more than enough energy is imparted to the hammer to fully cock it.

3.5.5. Drive-spring energy

The computed integral of spring force over buffer-body displacement is compared to quasi-steady experimental results obtained with an Instron testing machine[6].

Computed	45.5 in-lb
Experimental	36.2 in-lb

3.5.6. Frictional energies

a. Integral, over bolt-carrier displacement, of the drag force acting on the bolt-carrier in counterrecoil.[1]

Computed	3.2 in-lb
Experimental	3.0 in-lb

b. Integral, over bolt-carrier displacement, of the drag force acting on the bolt-carrier as it moves rearward over the uppermost round in the magazine[5].

19 rounds in magazine:	Computed	8.1 in-lb
	Experimental	8.1 in-lb
10 rounds in magazine:	Computed	6.3 in-lb
	Experimental	7.1 in-lb
2 rounds in magazine:	Computed	4.7 in-lb
	Experimental	5.5 in-lb

c. Integral, over bolt displacement, of the stripping force required to remove a round from the magazine[5].

---

[1]Grandy, A.J.; Duffy, J.A.; Horchler, M.H.; and Ehle, P.E.; "Investigation of Bolt/Bolt-Carrier Clearances in the M16A1 Rifle", Technical Note TN-1159, Frankford Arsenal, Philadelphia, Pa. (May 1971)
[5]Gay, H.P. and Wineholt, E.M., "Analog Simulation of the Mechanism of the M16A1 Rifle,"BRL Report No. 1596, Aberdeen Proving Grd, Md. (Jun 1972) Pg 23
[6]Brosseau, T.L., "Kinematic Study of the M16A1 Rifle," Ballistic Research Laboratories Report No. 2153, Aberdeen Proving Grd, Md. (Jan 1972)

19 rounds in magazine:	Computed	7.4 in-lb
	Experimental	7.4 in-lb
10 rounds in magazine:	Computed	5.7 in-lb
	Experimental	6.3 in-lb
2 rounds in magazine:	Computed	4.2 in-lb
	Experimental	3.2 in-lb

3.5.7. Time-displacement data

In Figure 13, one can compare theoretical and experimental time-displacement curves and the times of occurrence for various events. The exact conditions under which the experimental curve was established are not known. The curve was simply matched with a typical predicted curve that had about the same cycle time. Note that the cycle time varies widely, depending on the number of shots previously fired, the number of rounds in the magazine, the initial buffer weight positions, etc. Such a comparison cannot show that either the experimental or the theoretical results are correct, but only that they are in general agreement. Nothing more could be said even if the curves were to match perfectly.

3.5.8. Effect of mounting conditions

Experimental results indicate that cycle time is somewhat longer for soft mounting conditions than for stiff mounting conditions. The reason is that, when the recoiling parts strike the backplate of a soft-mounted weapon, they impart more of their momentum to the main gun. Thus, the initial counterrecoil velocity of the operating parts is slower, and more time is required for these parts to return to battery. For a stiff-mount spring constant of 10,000 pounds per foot, and a soft-mount spring constant of 300 pounds per foot, the difference in cycle time ranged from 3 to 4 milliseconds. Reference [1] indicates an experimental variation in cycle time of 1.7 to 4.7 milliseconds.

[1] Grandy, A.J.; Duffy, J.A.; Horchler, M.H.; and Ehle, P.E.; "Investigation of Bolt/Bolt-Carrier Clearances in the M16A1 Rifle", Technical Note TN-1159, Frankford Arsenal, Philadelphia, Pa. (May 1971)

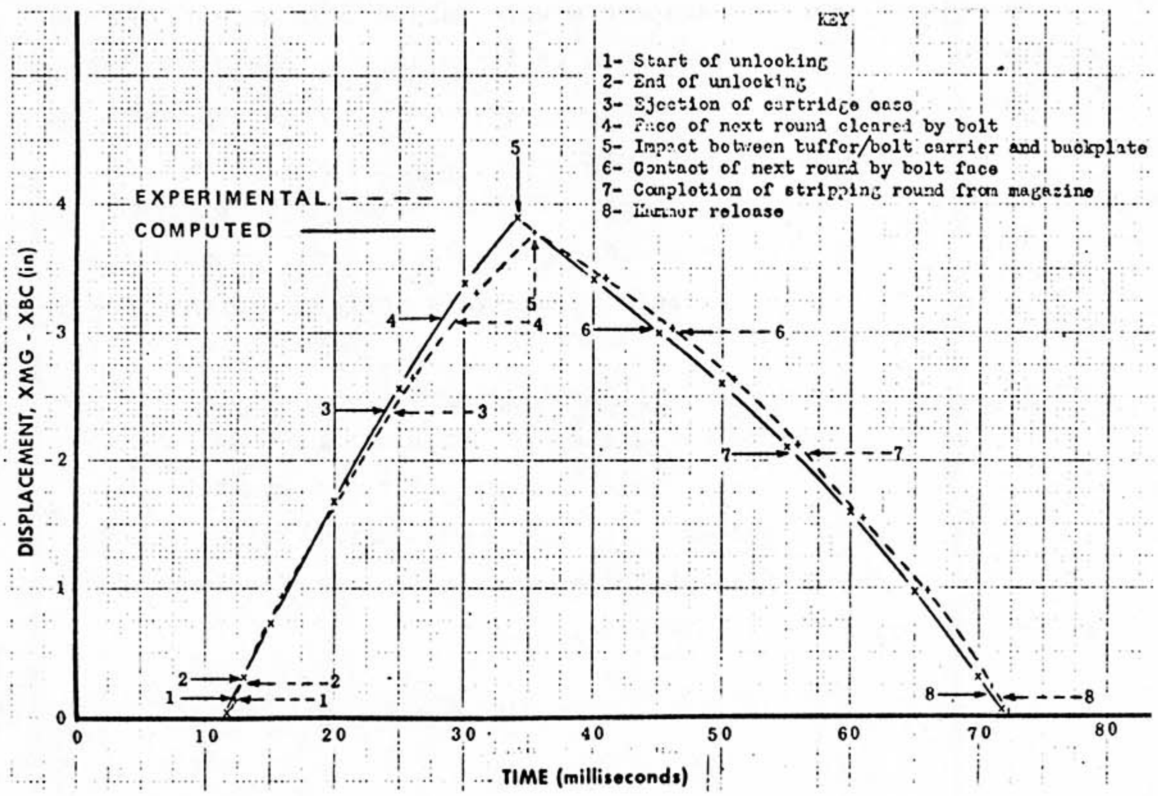

Figure 13 Computed and Experimental Bolt-Carrier Displacement vs Time

3.6 Sensitivity Analysis

One important use of a math model of a weapon is the conduct of a sensitivity analysis through parametric variations. Such a study is often very costly and difficult to perform experimentally because of the need to physically alter weapon components and the need to repeat many tests a large number of times to remove random variations from causes other than part modification. A sensitivity analysis is a very helpful tool with which to uncover potential malfunctions. In practice, small variations in parameters such as drive spring stiffness actually occur, and a model can be used to determine just how great these variations can be before performance is adversely affected. Also, one acquires a good idea of what particular design change might improve performance.

A sensitivity analysis was performed with the use of the M16A1 model. In this study, single parameters were varied independently, except in Figure 21. A much wider area of investigation would involve the simultaneous variation of a number of parameters. Accuracy and cycle time were studied as the parameters were varied. The effect on several peak forces was also predicted. The variations were plotted, and observations on each graph are presented below. Also, comparisons are made with results from a sensitivity analysis performed with a model of the XM19 Rifle.

3.6.1 <u>Sensitivity Analysis Graphs (Figures 14-30)</u>

a. Computed Accuracy and Cycle Time vs Drive Spring Constant (Fig. 14). Neither accuracy nor cycle time appear to be greatly affected by reasonable variations in spring constant that might occur in practice. Roughly, a 6 percent change in spring constant is needed to get a 1 percent change in cycle time.

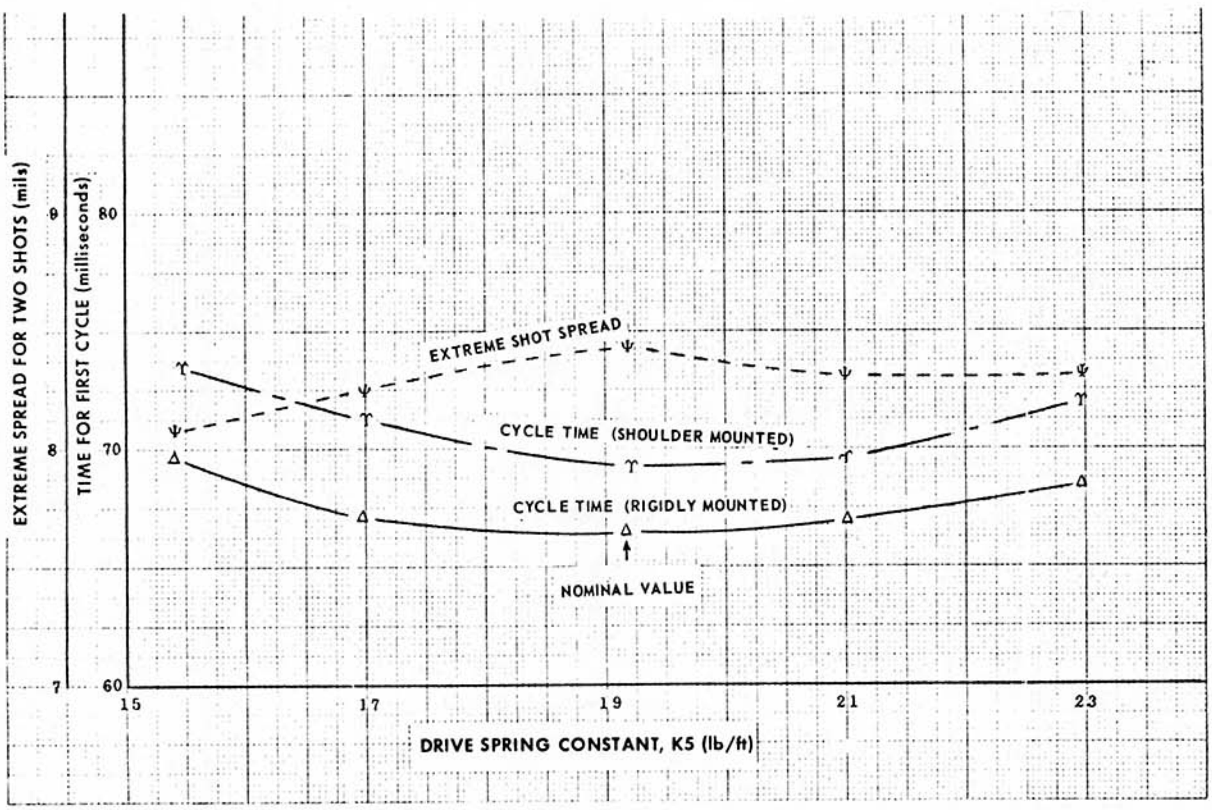

Figure 14 Computed Accuracy and Cycle Time vs Drive Spring Constant

b. Computed Accuracy and Cycle Time vs Linear Mount Spring Constant (Fig. 15). The spring referred to here represents the shooter's shoulder and should be distinguished from the torsional spring that represents the rest of the body. In the lower range of spring stiffnesses, characteristic of most men's shoulders, cycle time is moderately sensitive to spring stiffness and tends to decrease as stiffness increases. At higher values, sensitivity is low. Accuracy is also moderately sensitive to the spring constant in the lower range of values and tends to increase with spring stiffness. Accuracy tends to decrease with stiffness in the higher range, but this range is not realistic for a shoulder mount.

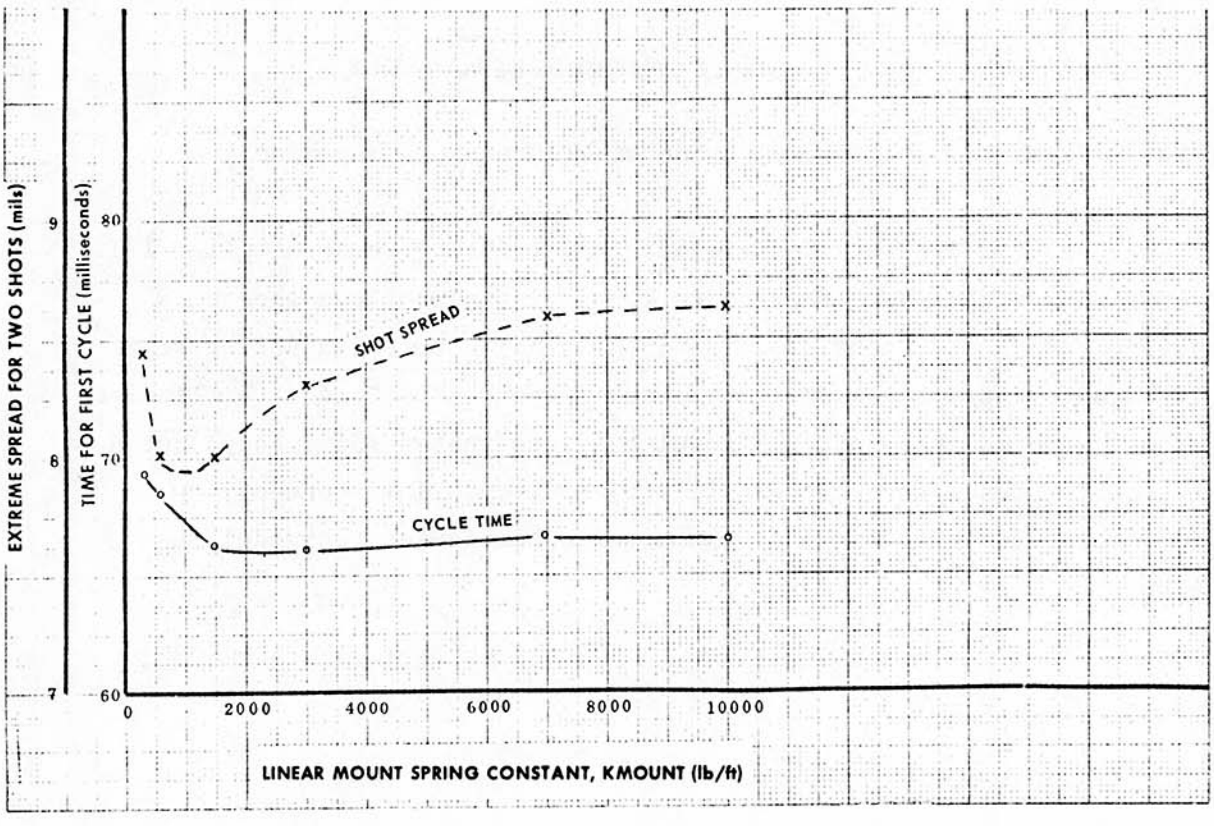

Figure 15 Computed Accuracy and Cycle Time vs Linear Mount Spring Constant

c. Computed Cycle Time vs Number of Rounds in Magazine (Fig. 16). Cycle time shows a slight increase with number of rounds. Variation up to about 1/4 millisecond can be expected from a 20-round magazine.

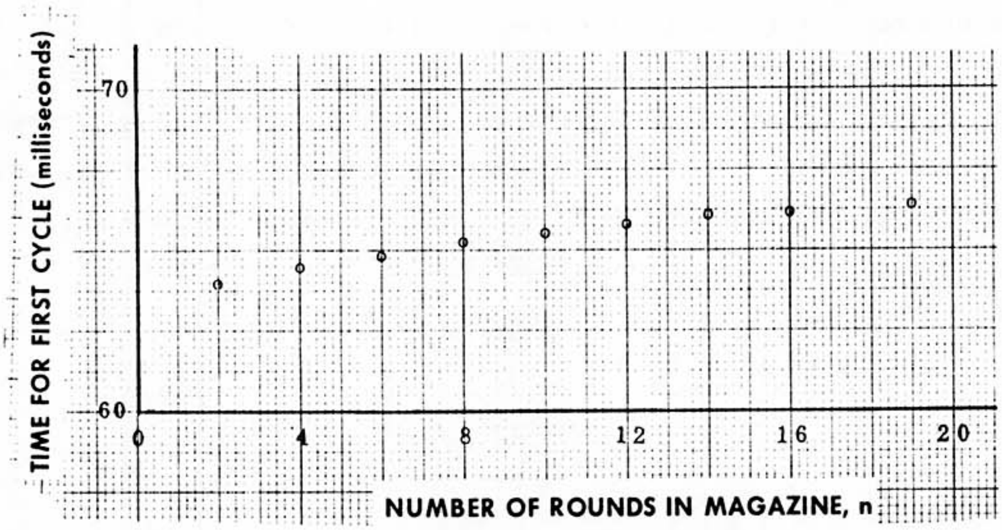

Figure 16 Computed Cycle Time vs Number of Rounds in Magazine

d. Computed Accuracy vs Torsional Mount Spring Constant (Fig. 17). The torsional spring here represents the resistance of the shooter's body to pivoting about the hips. A 33 percent increase in spring stiffness tends to increase accuracy by about 13 percent. Thus, from Figures 15 and 17, one can see that the model predicts that a stiff body, but a soft shoulder tends to be beneficial to accuracy. This tendency should be further explored when the more sophisticated man-model is incorporated.

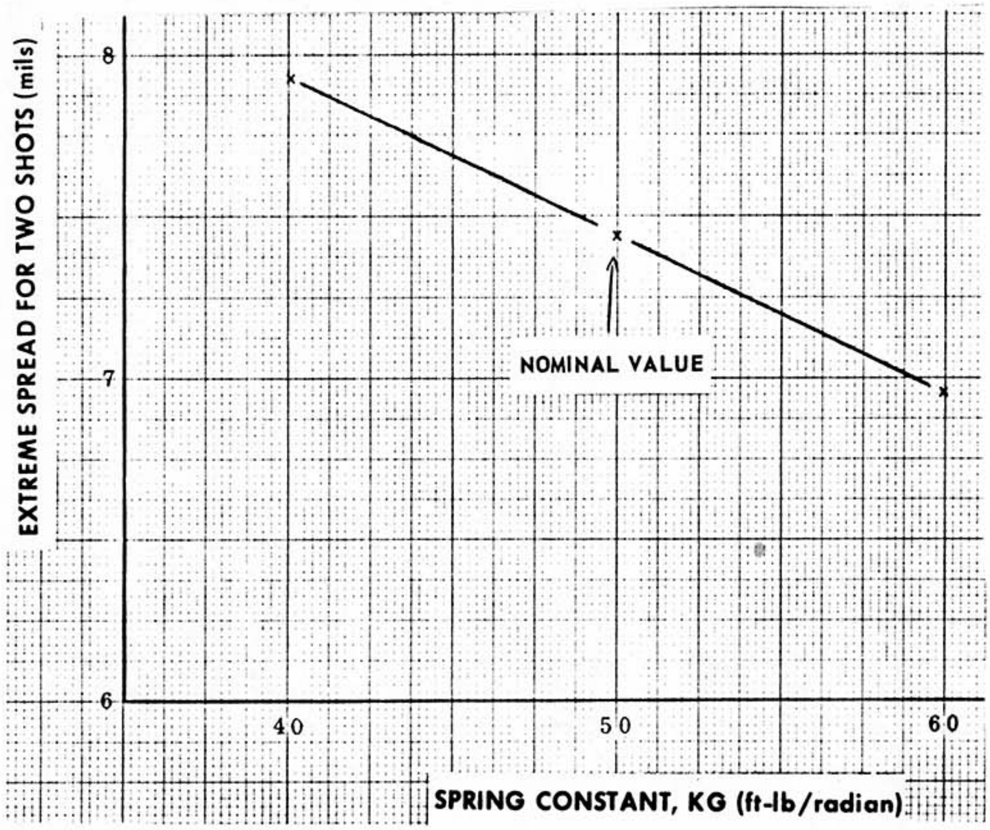

Figure 17 Computed Accuracy vs Torsional Mount Spring Constant

e. Computed Cyclic Time vs Cavity Area (Fig. 18). This variation in area is equivalent to a uniform increase in pressure over the nominal cavity area in the bolt-carrier. Cycle time is sensitive to this variation. An 18 percent increase in cavity area (or pressure) causes about a 16 percent decrease in time for the first cycle. Thus, one can expect that this weapon is sensitive to round-to-round variations in ammunition.

f. Computed Stripping and Friction Forces vs Number of Rounds in Magazine (Fig. 19). One can expect that stripping force may vary about three pounds, and that friction between bolt-carrier and ammunition may vary about one pound as rounds are removed from the magazine.

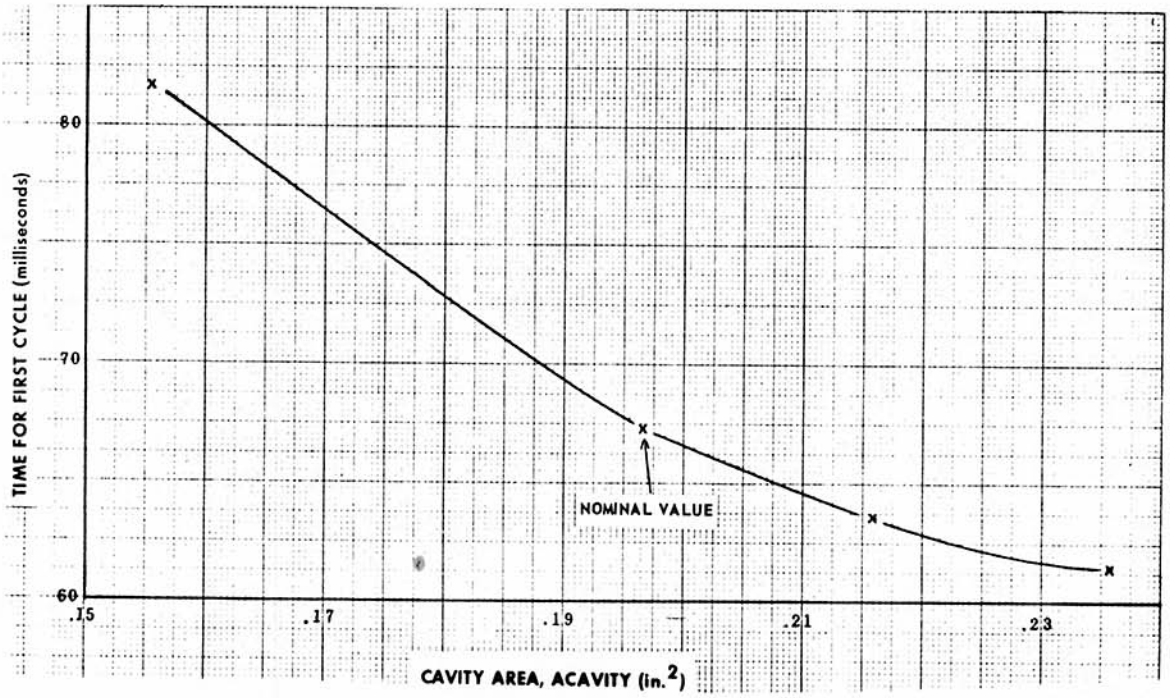

Figure 18 Computed Cyclic Time vs Cavity Area

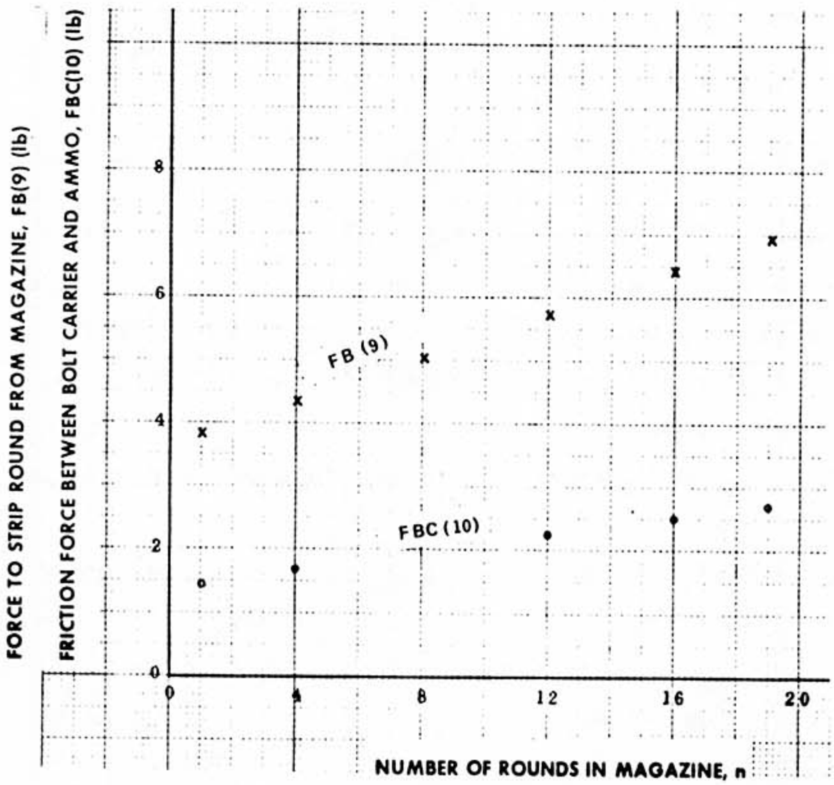

Figure 19 Computed Stripping & Friction Forces vs Number of Rounds in Magazine

g. Computed Accuracy and Cycle Time vs Breech Pressure Impulse (Fig. 20). This weapon exhibits a very high sensitivity of accuracy to breech pressure impulse. A 10 percent increase in breech pressure impulse can result in an accuracy loss of roughly 17 percent and a decrease in cycle time of about 4 percent. Thus, round-to-round variations in ammunition can be expected to significantly affect performance.

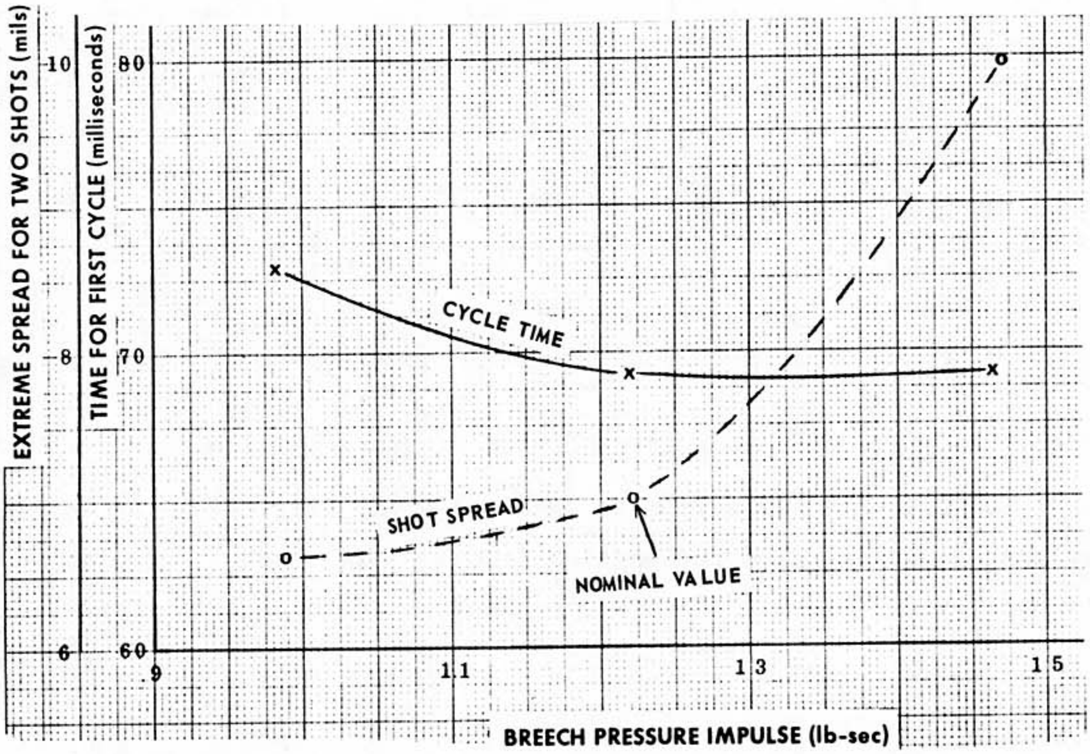

Figure 20 Computed Accuracy and Cycle Time vs Breech Pressure Impulse

h. Computed Accuracy vs Breech Pressure Impulse (Fig. 21). The difference between Figure 21 and Figure 20 is that here the pressure in the bolt-carrier cavity is allowed to change with breech pressure. A 1 percent increase in breech pressure impulse accompanied by a 1 percent increase in cavity pressure impulse results in about a 1 percent decrease in accuracy.

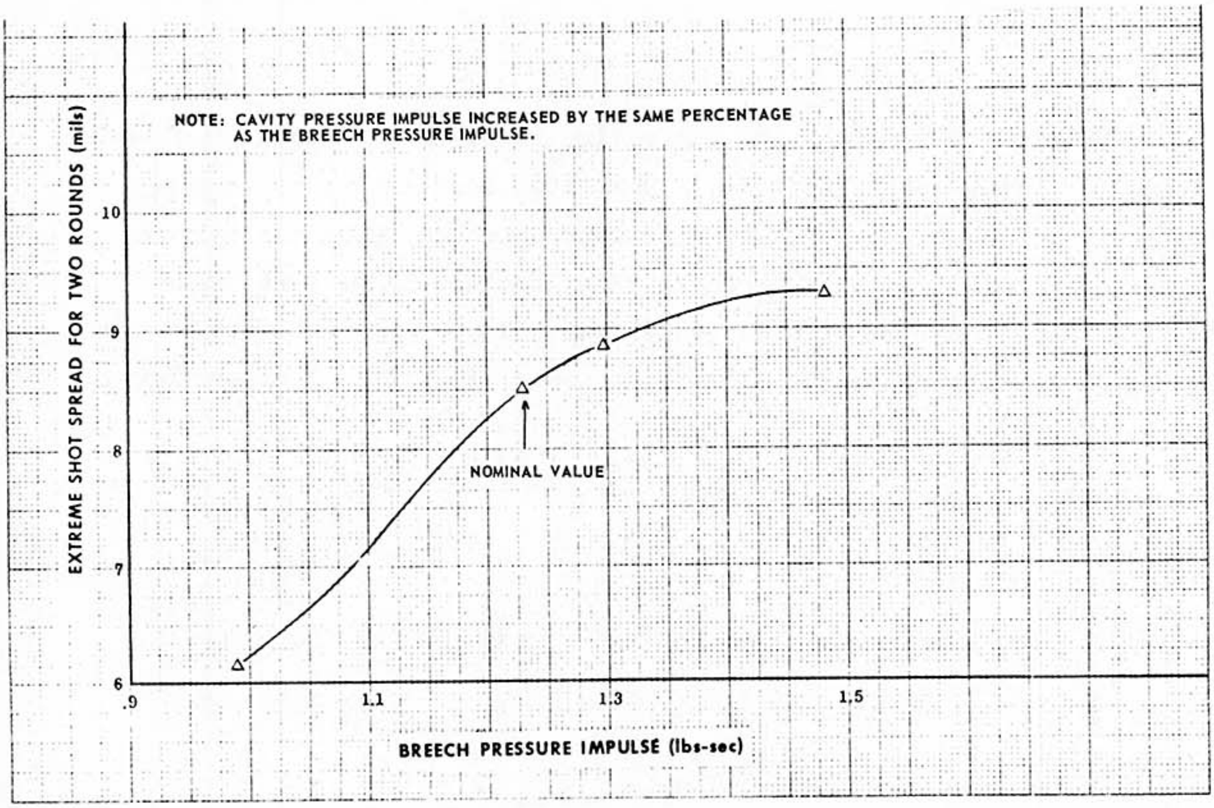

Figure 21 Computed Accuracy vs Breech Pressure Impulse

i. Computed Cycle Time vs Ignition Delay (Fig. 22). Cycle time is relatively insensitive to ignition delay. A 14 percent increase in ignition delay causes only about a 1 percent increase in cycle time.

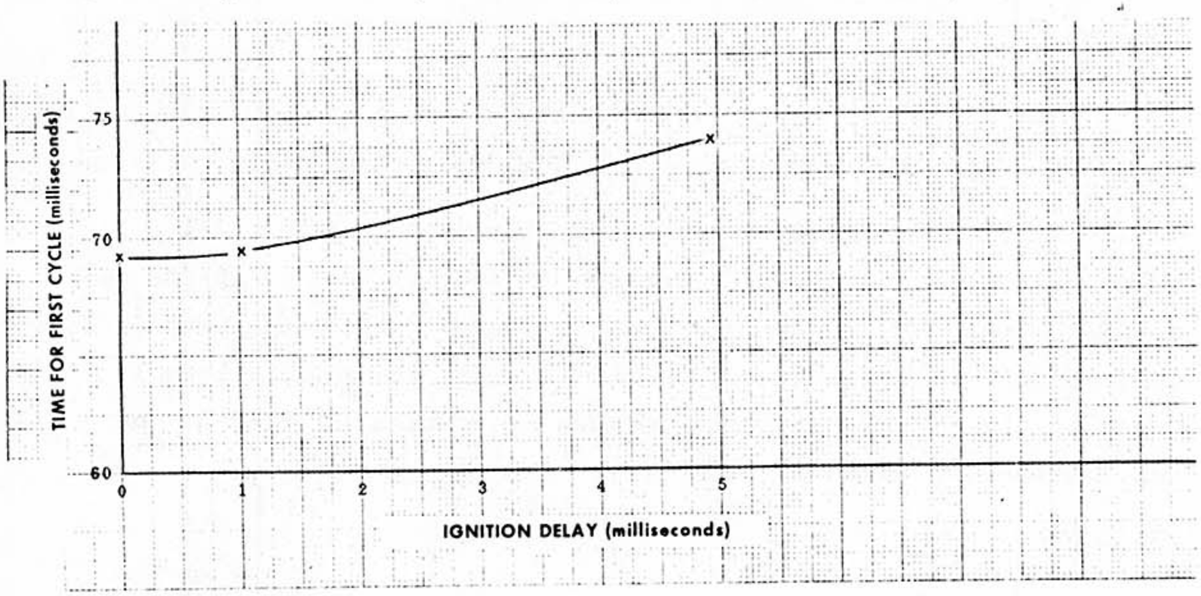

Figure 22 Computed Cycle Time vs Ignition Delay

j. Computed Cycle Time vs Ignition Delay (Fig. 23). The difference between this graph and Figure 20 is that here the ignition delay is in a more realistic range. Very little effect on cycle time is observed. This behavior is in contrast with that of the XM19 Rifle where the effect is much more significant. (See Part II of this report).

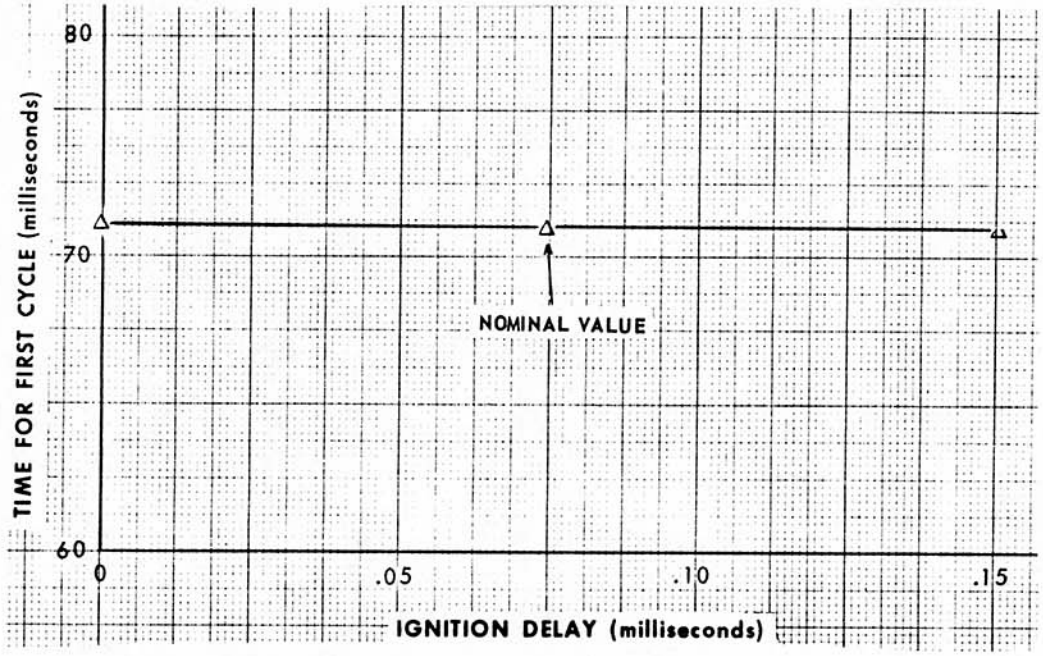

Figure 23 Computed Cycle Time vs Ignition Delay

k. Computed Accuracy vs Moment of Inertia of Rifle (Fig. 24). Accuracy of the M16A1 does not appear to be highly sensitive to the moment of inertia of the rifle about an axis through the center of gravity and perpendicular to the bore and gravity axes. A 10 percent increase in moment of inertia increases accuracy by about 3 percent.

l. Computed Unlocking Force and Cycle Time vs Length of Cam Path (Fig. 25). The peak unlocking force and the cycle time have a definite tendency to decrease as the length of the cam path in the bolt is increased. The force is more sensitive than is cycle time.

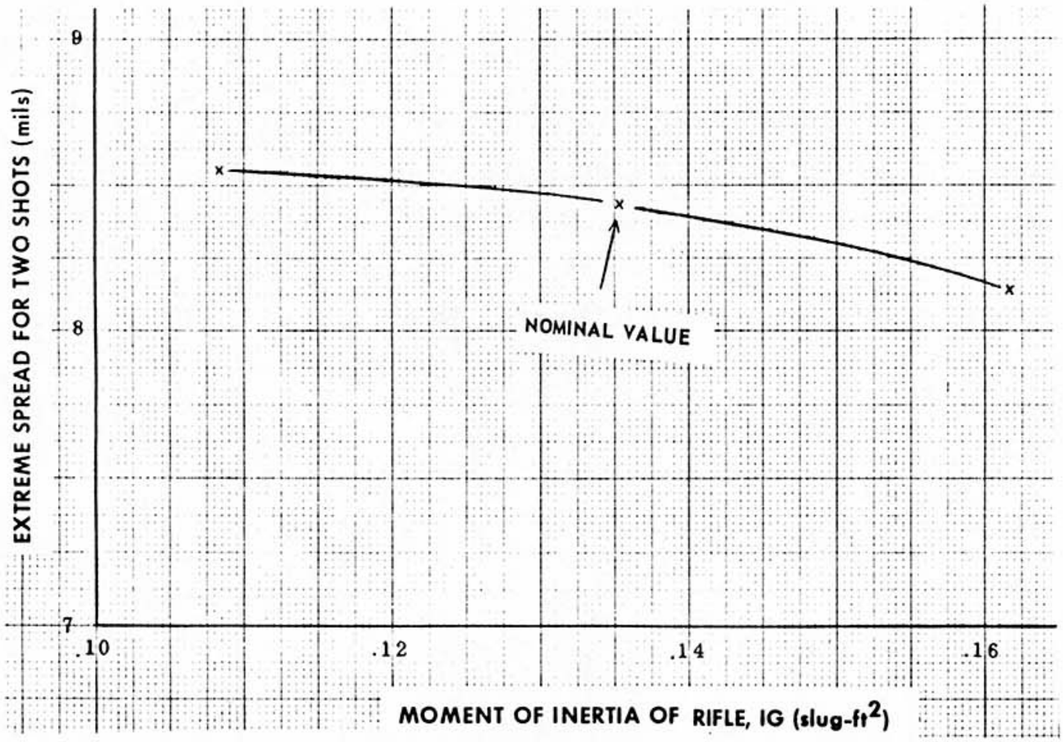

Figure 24 Computed Accuracy vs Moment of Inertia of Rifle

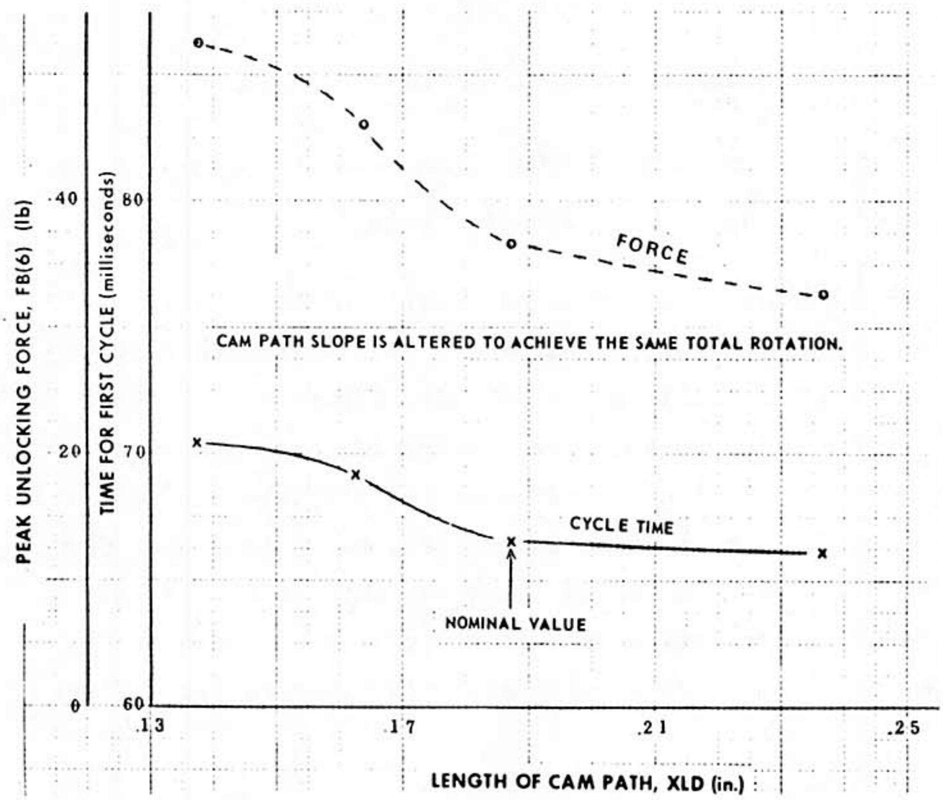

Figure 25 Computed Unlocking Force and Cycle Time vs Length of Cam Path

m. Computed Accuracy and Cycle Time vs Mass of Bolt (Fig. 26). A 10 percent increase in the mass of the bolt leads to only a 2 percent increase in cycle time and less than a 1 percent change in accuracy. Accuracy both increases and decreases with bolt mass. Thus, the mass of the bolt does not appear to be highly critical.

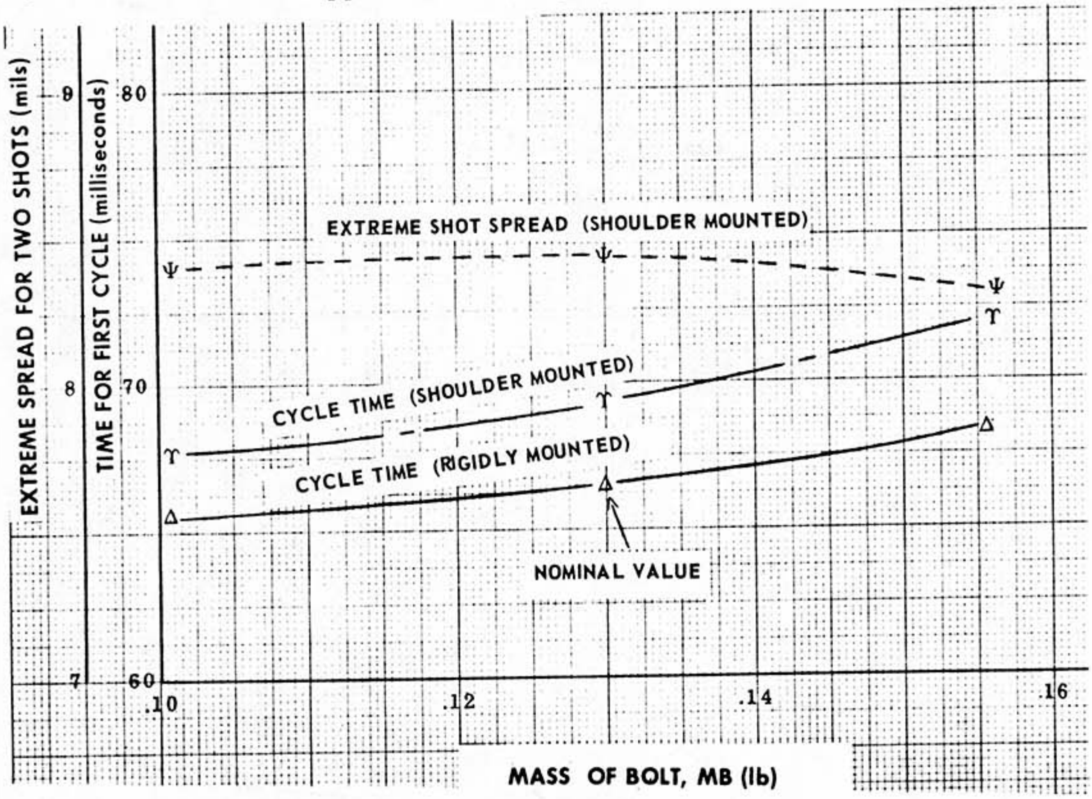

Figure 26 Computed Accuracy and Cycle Time vs Mass of Bolt

n. Computed Accuracy and Cycle Time vs Mass of Bolt-Carrier (Fig. 27). A 10 percent increase in the mass of the bolt-carrier leads to roughly a 4 percent increase in cycle time. No definite trend is apparent for accuracy. Performance appears to be only slightly more sensitive to variations in bolt-carrier mass than to variations in bolt mass.

o. Computed Accuracy and Cycle Time vs Drive Spring Preload (Fig. 28). Cycle time and accuracy appear to be more sensitive to reductions in preload rather than to increases in preload. A 10 percent reduction in preload leads to a 3 percent increase in cycle time and a 7 percent increase in accuracy.

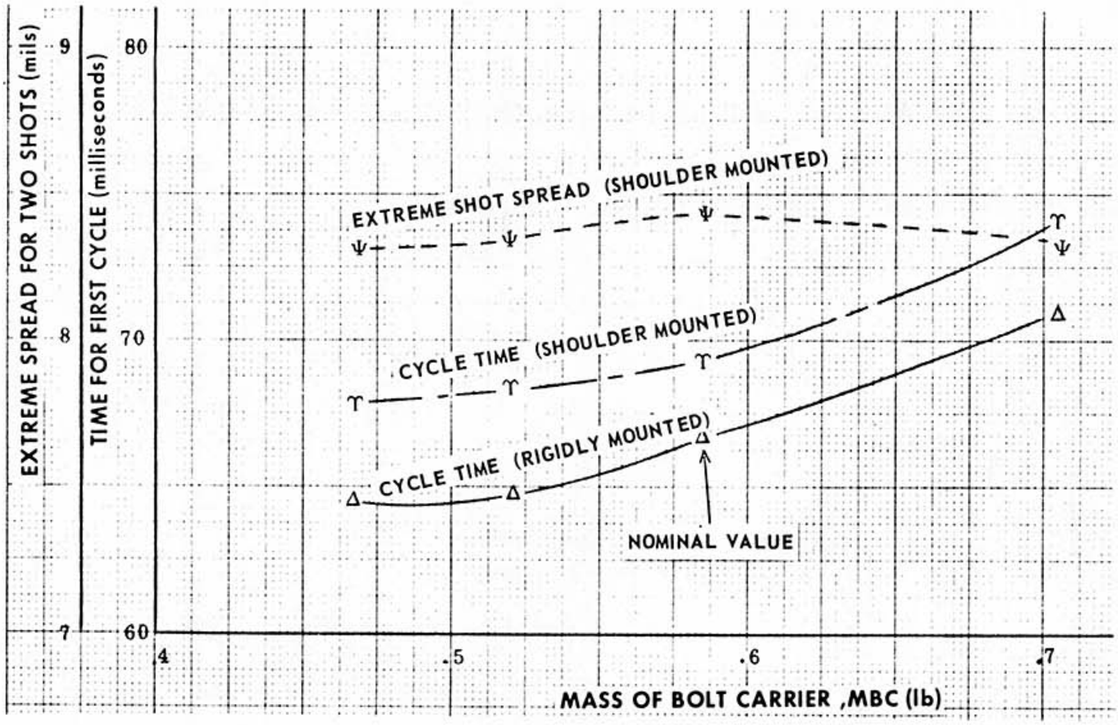

Figure 27 Computed Accuracy and Cycle Time vs Mass of Bolt-Carrier

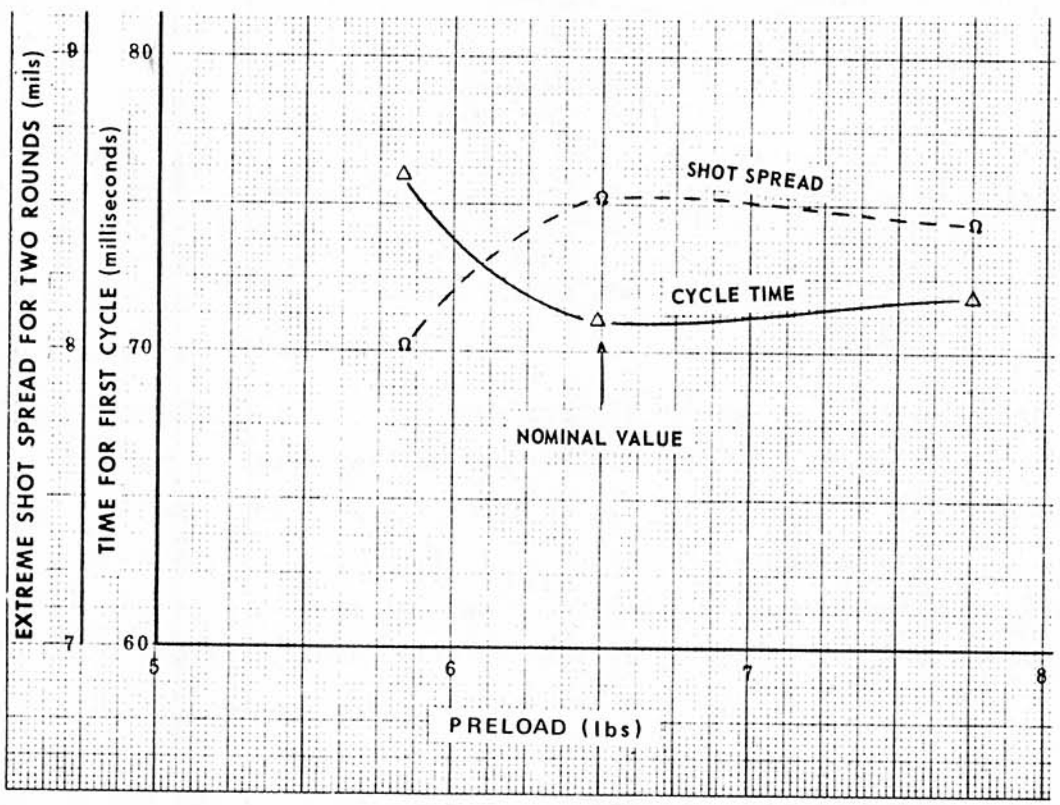

Figure 28 Computed Accuracy and Cycle Time vs Drive Spring Preload

p. Computed Accuracy and Cycle Time vs Initial Buffer Weight Positions (Fig. 29). According to the model, cycle time for the first shot is highly dependent on the initial positions of the buffer weights and can vary as much as 10 milliseconds. However, the effect tends to quickly diminish in succeeding shots. Shot spread for two rounds can vary by as much as 1 milliradian.

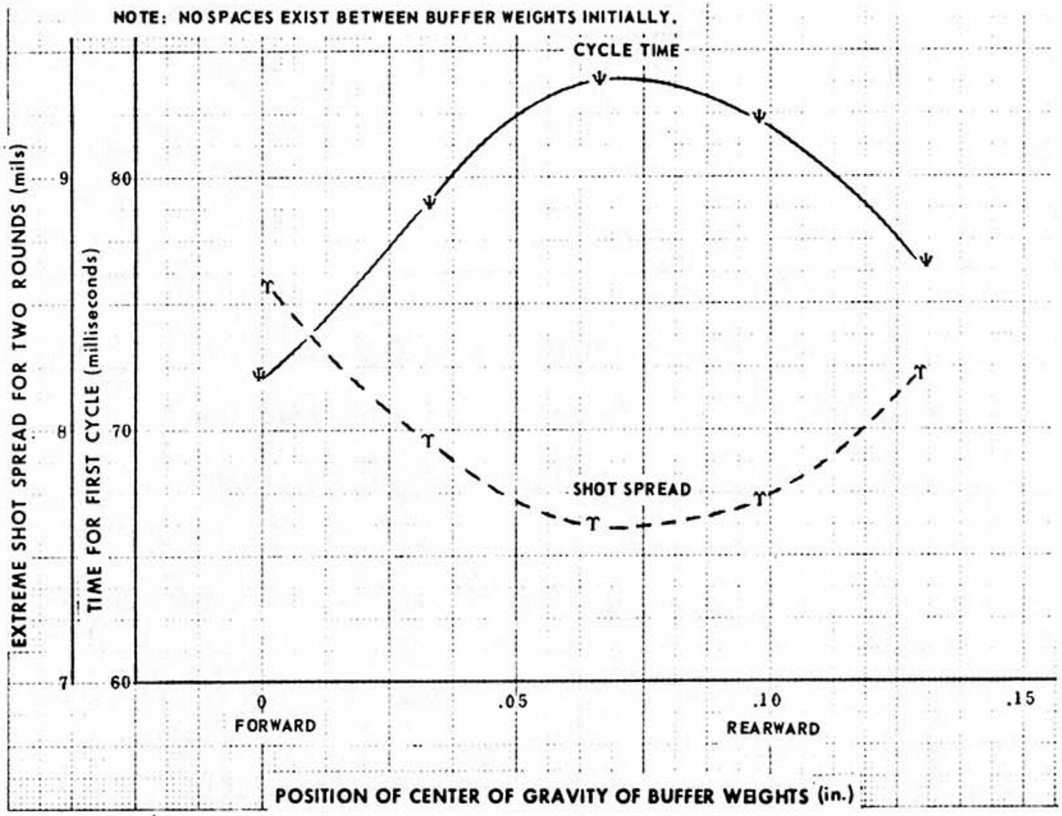

Figure 29 Computed Accuracy & Cycle Time vs Initial Buffer Weight Positions

q. Computed Accuracy and Cycle Time vs Bore Friction Impulse (Fig. 30). Cycle time is not affected by bore friction for the first shot. However, a 5 percent increase in bore friction impulse increases accuracy by about 1 percent. The reason is that bore friction tends to counteract the breech impulse.

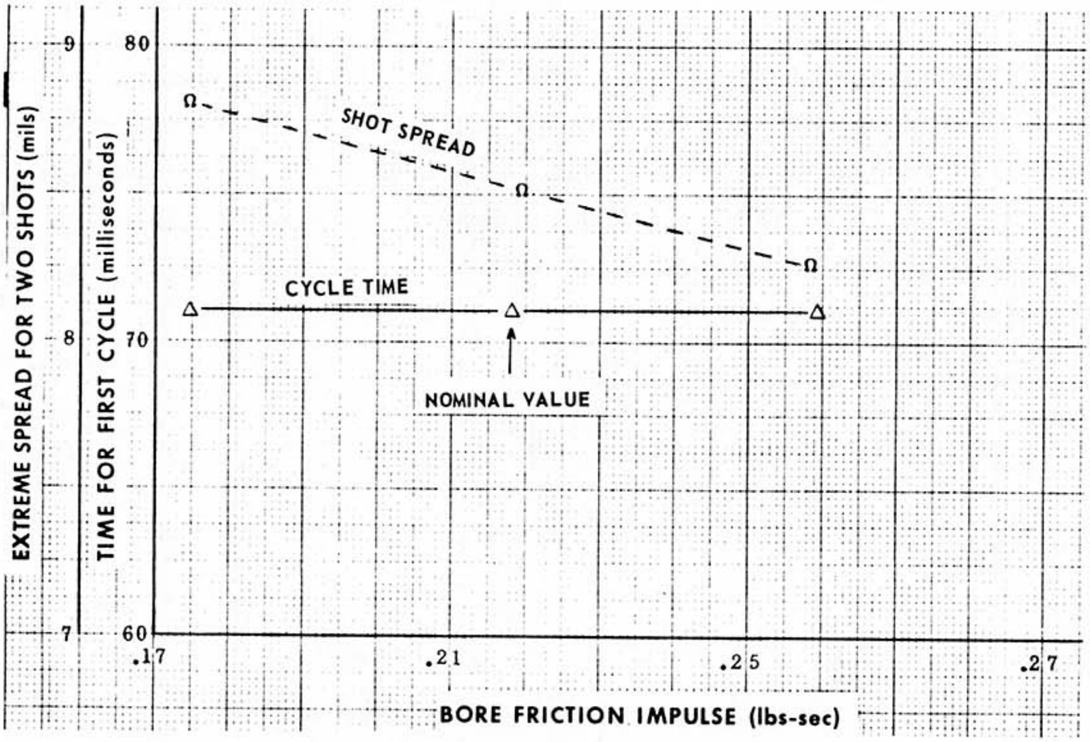

Figure 30 Computed Accuracy and Cycle Time vs Bore Friction Impulse

3.6.2 Sensitivity Analysis Graphs (Figures 14-30) Observations

The most significant parameter of those examined that affect accuracy is breech-pressure impulse. A ± 20 percent variation (.99 to 1.47 lb-sec) of impulse leads to a variation of shot spread from 6.6 to 10 mils. Changes of ± 20 percent in the following parameters have little effect (less that .5 milliradian) on the accuracy of the weapon: drive spring constant, mass of bolt moment of inertia of the rifle, mass of the bolt-carrier, and bore-friction impulse. Mechanical properties of the shooter are seen to have a significant effect on accuracy. Drive spring preload has a moderate effect.

The most significant parameter of those examined that affect cycle time is the cavity area in the bolt-carrier. A 20 percent variation about the nominal area causes cycle time to vary from 61 to 82 milliseconds. The initial positions of the buffer weights appear to be highly significant for the first round. The number of rounds in the magazine appears to have little effect. With two rounds in the magazine, the cycle time is 64 milliseconds, whereas the cycle time is 66 milliseconds with nineteen rounds. A decrease in the length of the cam path by 20 percent causes the cycle time to change from 66.5 to 70.5 milliseconds. The reason is that the gas forces have a shorter time over which to impart momentum to the bolt-carrier.

In Part II of this two-report series, a complete discussion appears in which performance of the M16A1 Rifle is compared with that of the XM19 Rifle. The results are based on mathematical models of both weapons. From this study, accuracy appears to be slightly more sensitive to changes in moment of inertia of the rifle in the M16A1. Also, the M16A1 cycle time is more sensitive to changes in the drive spring constant, in mount stiffness, and in ignition delay. However, sensitivity to changes in drive spring preload appears to be somewhat greater in the XM19.

3.7 Additional Comments on the Buffer and Drive Spring

The buffer assembly in the M16A1 was designed to provide a secondary impulse to the bolt-carrier. This impulse is needed to prevent the bolt-carrier from bouncing rearward just after locking. If the bounce is too great, a misfire can result. With the use of the model, a check was made to determine if the buffer does indeed perform this function. By setting the spring constants and damping coefficients between the buffer weights to zero, one effectively removes the influence of the buffer weights on the weapon. When this is done, the model predicts a bolt-carrier rebound of about 1/16th of an inch. The weapon begins to unlock, but the drive spring on the buffer body prevents complete unlocking. When the influence of the buffer weights is included in the model, virtually no rebound occurs. The reason is attributed to the secondary impact of the buffer weights on the forward end of the buffer body. The model predicts that this impact occurs 1.2 milliseconds after the bolt-carrier strikes the bolt pin at the end of locking.

The inertial effects of the drive spring have a very significant effect on the operation of the M16A1 Rifle. Initially, the drive spring was considered to behave linearly, and inertial effects were accounted for by the addition of one-third of its mass to the mass of the buffer body. With this kind of model, the bolt-carrier and buffer body were observed to separate just after unlocking and the buffer continued rearwardly at a faster rate than the bolt-carrier. The reason is that the picking up of the bolt by the bolt-carrier decreases the rearward acceleration of the bolt-carrier while the buffer is free to continue rearwardly since only compressive forces can act between the buffer and the bolt-carrier. The buffer was found to rebound from the closed end of the buffer body, strike the bolt-carrier, and bounce back toward the closed end again. Large variations in cyclic rate occurred. The operating parts entered the counterrecoil phase with less velocity than if all parts were together at the time of impact with the closed tube end. No experimental evidence could be found to indicate that the buffer rebounds from the closed end of the buffer body more than once per cycle. Thus, a more sophisticated model of the drive spring seemed to be required. As described in Appendices A and C, a finite-element method was used. With even a small number of elements, the inertia effects from this model were sufficient to cause the bolt, bolt-carrier, and buffer to strike the backplate together, and thus to eliminate the problem. In some instances, cycle time was reduced by as much as nine milliseconds. Two principal effects result from this more realistic model. The first is that the effective drive spring seen by the buffer body is initially greatly stiffened. The second is that the main gun is unaffected by the buffer body motion until a stress wave travels the entire length of the spring. The M16A1 drive spring can, for most purposes, be modeled with as few as five elements. To consider coil clashing in a detailed evaluation of the spring itself, one can use a number of elements equal to the number of coils as discussed in Appendix C. The difference in spring behavior with various numbers of elements is shown in Figures 31 to 33.

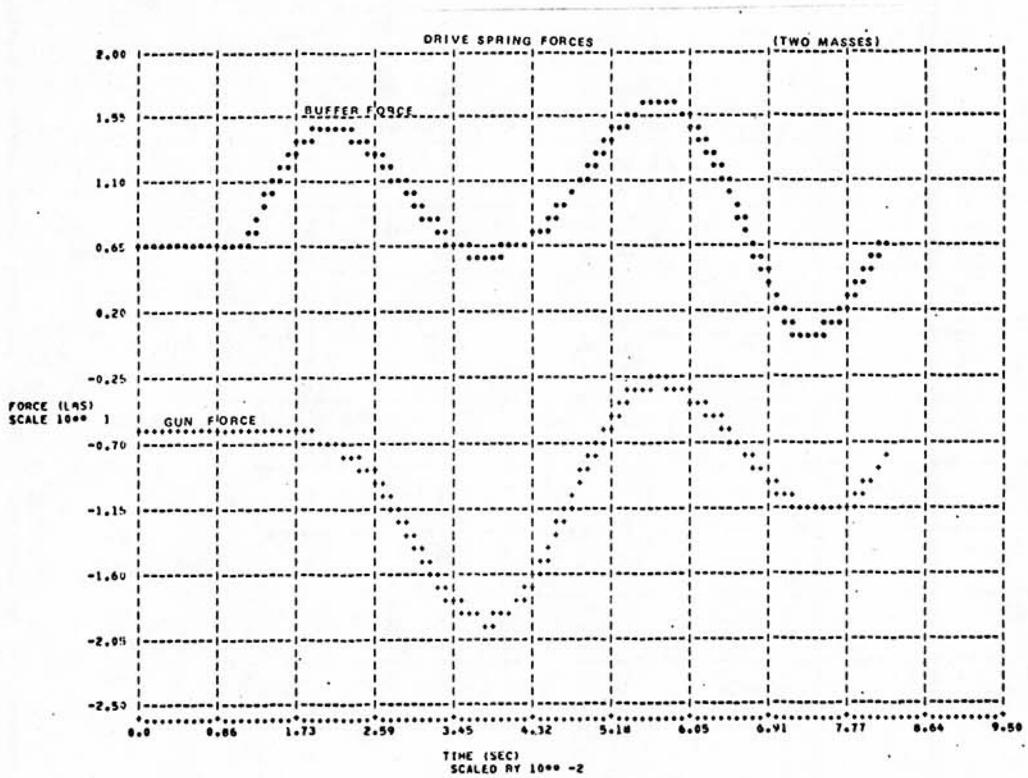

Figure 31 Drive Spring Forces (2 - Element Model)

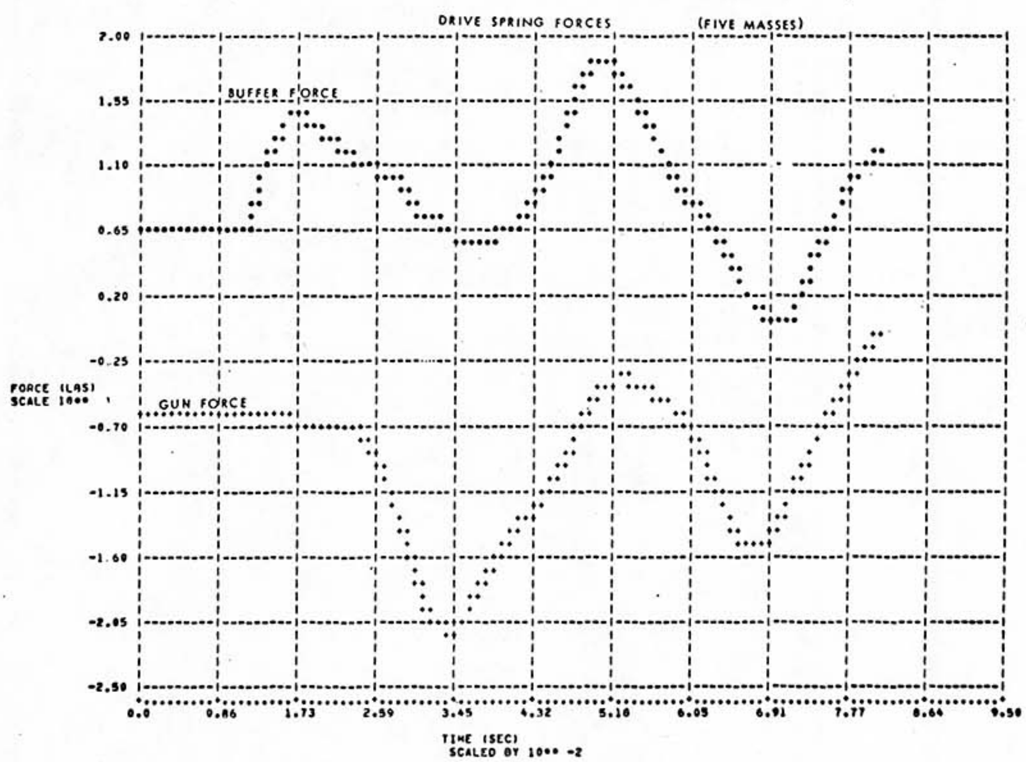

Figure 32 Drive Spring Forces (5 - Element Model)

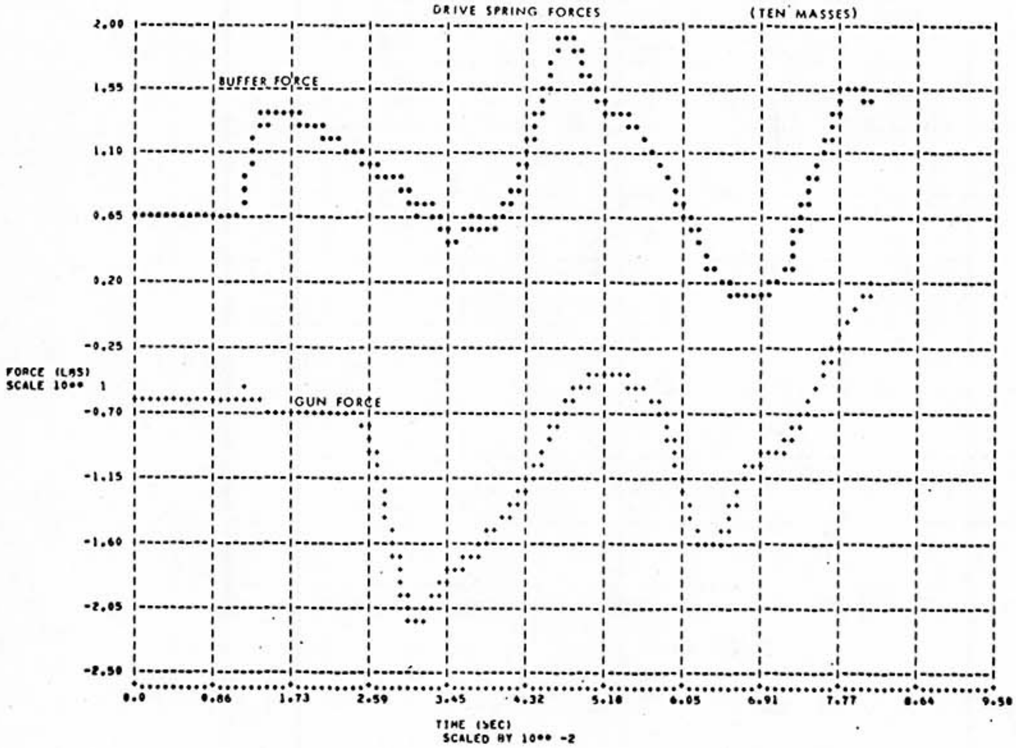

Figure 33 Drive Spring Forces (10 - Element Model)

4.0 CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

a. A digital-computer model of the M16A1 Rifle has been constructed and is suitable for the study of effects on mechanical performance and weapon dispersion as changes are made in various weapon parameters. From this model, information about the M16A1 can be obtained that would be extremely difficult and costly to determine by experiment.

b. As a result of the work associated with the development of this model, modest extensions of the state of the art in weapon modeling have been realized. For example, both continuum and finite-element approaches to impact have been explored (Appendix F), and three approaches for the analysis of cam forces are presented in Appendix A. A new area of study, the coupling of complex computer models, is presented in Appendix F. Another new approach to the modeling of weapons is that of the extension of deterministic models to the probabilistic regime (Appendix G). The concept of finite elements in solid mechanics has been extended to fluid mechanics, and the result is a unique approach to gas flow (Ref[7]). For the first time, very detailed small-caliber weapon models have been developed by which weapon dispersion can be approximately analyzed. Also, detailed models of spring surging, coil clashing, and buffer-weight behavior have become integral parts of the weapon model. Hammer and magazine dynamics have been extensively analyzed. However, much work remains in these and other areas of weapon modeling such as dynamic friction.

c. Accuracy of the M16A1 Rifle can be improved, in accordance with the model, by a stiffening of the shoulder and upper torso of the shooter, a decrease in breech-pressure impulse, and increase in moment of inertia of the rifle, an increase in bore-friction impulse (provided this increase is obtained without a corresponding increase in the time that the projectile is in the barrel), decreases in both drive spring constant and preload, and by the initiation of the firing cycle with the buffer weights in their most central location. Of course, these changes also have other ramifications that must be explored, and not all of these

[7] Ehle, P.E. and Rahe, A.E., "Development of a Finite Element Approach for Approximate Analysis of Unsteady Compressible Fluid Flow," WECOM Technical Report, SWERR-TR-72-36, AD 746234, Rock Island, Ill (Jun 1972)

may be beneficial. Also, the combined effect of the simultaneous changing of two or more parameters was not explored, although this could be done.

d. An increase in cyclic rate can be achieved in accordance with the model, by a stiffening of the shooter's shoulder, an increase in gas pressure in the cavity and at the breech, decreases in bolt and bolt-carrier masses, and the initiation of the firing cycle with the buffer weights at either extreme end of their allowed positions. Again, these changes would have other ramifications, none of which were explored in this study.

e. Techniques developed for the M16A1 Rifle model are applicable to the analyses of other shoulder-fired automatic weapons. Examples of these techniques are those used to model the drive spring, magazine, cam forces, weapon dispersion, and hammer dynamics. The preliminary results from the investigations of impact, stochastic methods, coupling of complex models, and fluid flow are also applicable.

f. Detailed analyses of the buffer assembly, shooter, and drive spring are important factors for accuracy and realism in an M16A1 Rifle model. These factors have important influences on weapon operation and dispersion.

g. The model provides a suitable base for the comparison of performance of other weapons.

h. In the section of this report on modeling philosophy, "verification" of a complex weapon model by experiment is mentioned as a concept not well defined. One often uses the term without a clear idea of its meaning. Since no established verification criteria exist, the present model will not be presented as either verified or unverified. Many of the model predictions are in excellent agreement with experiment; this can be seen in the section on comparison with experiment. Without doubt, other predictions such as a peak force value may be found that are not as good. Some uses for the model require better accuracy in certain areas than in others. All that can be said about validation is that the model is sufficiently accurate that it can be used with a high degree of confidence for many important studies, but that the model is by no means infallible.

4.2 Recommendations

a. Use of the M16A1 Rifle model should be continued as a vehicle for the development of modeling techniques.

b. A number of important areas exist in which development work should be emphasized. First, a detailed mathematical model of the shooter should be added to the M16A1 model. This man-model is already available at RIA and comprises springs, levers, masses, and dampers to simulate the passive response of a man's body to recoil. The sensitivity analysis in this report demonstrates that such response significantly affects both weapon operation and accuracy. Second, better models of friction should be developed with an instrument designed to simulate and measure friction forces typical of many different weapons. Cams capable of providing time-displacement histories found in a given weapon can be manufactured and inserted in this machine. Normal force, temperature, humidity, velocity, displacement, acceleration, and lubricant are some of the parameters that can be varied. Delivery of this instrument to the Weapons Laboratory, RIA, is expected during FY73. Third, an analysis of gas transmission should be included. A relatively simple analysis is needed that can be run with an already complex model without excessive computation time. Development of such an analysis is nearing completion (Ref[7]). Fourth, stochastic techniques should be developed and incorporated into the M16A1 Rifle model so that probabilistic data can be accepted as input and confidence levels can be established for predicted results (Appendix D). Fifth, improved models of impact should be developed that more accurately reflect the true stress levels involved (Appendix F). Sixth, methods should be explored for the direct coupling of complex computer models (Appendix G). Seventh, optimization and design methodologies should be developed so that models will have a more significant place in the design process. Eighth, the development of reliability data obtained from a model before a record of performance is established should be emphasized. Ninth, cartridge case

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[7]Ehle, P.E. and Rahe, A.E., "Development of a Finite Element Approach for Approximate Analysis of Unsteady Compressible Fluid Flow," WECOM Technical Report, SWERR-TR-72-36, AD 746234, Rock Island, Ill (Jun 1972)

extraction and bore friction are study areas important for the construction of accurate models and should become the subjects of in-depth studies.

c. Special emphasis should be focused upon the semiphilosophical problem of model verification and the development of suitable validation criteria.

d. The M16A1 Rifle model should be used as a basis for the comparison of performance of other automatic shoulder-fired weapons and for the improvement of the M16A1 itself.

APPENDIX A

DERIVATION OF FORCE EXPRESSIONS

A.1 BASIC NOMENCLATURE

1. X_{MG} = position of center of gravity (cg) of main gun structure with no operating parts
2. X_B = position of cg of bolt
3. X_{BC} = position of cg of bolt-carrier
4. X_{BUFF} = position of cg of buffer body
5. X_{WTS} = position of cg of buffer weight system
6. θ_B = angle through which bolt rotates during locking and unlocking
7. θ_H = angle through which hammer rotates
8. θ_E = angle of elevation of Rifle
9. $M_{B(E)}$ = effective mass of the bolt at time t
10. g = acceleration of gravity
11. t = time

A.1.1 Coordinate System

Each mass has an associated coordinate system fixed with respect to the ground. These are such that when $t = 0$, the coordinate values are all zero if the masses are in the locked position and completely forward.

A.2 Force acting on the bolt

A.2.1 FB(1) Breech force - For several milliseconds after propellant ignition, the bolt is locked to the main gun and is considered a part of it; at this time only, FB(1) is considered to act on the main gun. After unlocking occurs, the FB(1) acts only on the bolt. FB(1) is a digitized function of time and becomes nonzero when the hammer strikes the firing pin. At this time, the mass of the bolt is reduced by an amount equal to the mass of the projectile plus the mass of the propellant.

Then,

$$FB(1) = -P_{BREECH}(t)\, A_{BORE} \tag{A-1}$$

Nominal values are: $A_{BORE} = .0377\ in^2$,

$P_{BREECH}(t)$ = See Table 8 below.

TABLE 8 NOMINAL BREECH PRESSURE vs TIME

t (sec)	$P_{BREECH}(t)$ (lbs/in^2)
0	0
.0001	1000
.0002	3000
.0003	5000
.0004	11000
.0005	36000
.0006	50000
.0007	58000
.0008	50000
.0009	35000
.0010	25000
.0011	19000
.0012	15000
.0013	12000
.0014	10000

A.2.2 FB(2) Friction between bolt and ammo - This force is caused by the friction between the top round of ammunition in the magazine and the bolt during the recoil stroke. It becomes nonzero just after the bolt-carrier has passed over the ammunition and the bolt contacts the top round in the magazine. Three conditions exist that must be met for the force to continue to be nonzero:

a. The bolt has not reached the position where it "picks up" a new round,

b. The weapon is still in recoil,

c. The bolt-carrier is not in contact with the ammunition.

Then,

$$FB(2) = 2\mu_1 \operatorname{sgn}(\dot{X}_B - \dot{X}_{MG}) \left((M_E g \cos \theta_E) + K_M (Y - Y_o) \right) \quad \text{(A-2)}$$

if

the weapon is in recoil,

if

$$X_{BC} - X_{MG} \leq - 2.34 \text{ in.},$$

And if

$$X_B - X_{MG} \geq - 2.81 \text{ in.}$$

FB(2) is zero otherwise.

The derivation of the expression used for this force is the same as the derivation of FBC(10). Nominal values of the parameters are also chosen to be the same.

An equal and opposite force FMG(2) acts on the main gun.

A.2.3 FB(3) Impact between bolt and barrel and engagement of extractor

The contribution to FB(3) from the engagement of the extractor is assumed to be a constant 7.5 pounds. The contribution to this force from the impact between bolt and barrel is determined by a stiff spring and damper hypothetically placed between these two parts. When, during impact, the spring is about to decompress, the bolt and main gun are considered to be a single mass since locking has just occurred. This is a completely inelastic collision between bolt and barrel.

Then,

$$FB(3) = -E_{XTF} + K_{\substack{BARREL\\IMPACT}} (X_{MG} - X_B) + C_{\substack{BARREL\\IMPACT}} (\dot{X}_{MG} - \dot{X}_B) \qquad \text{(A-3)}$$

E_{XTF} is nonzero if weapon is in counterrecoil and $-.13" \leq X_B - X_{MG} \leq 0$.

$K_{\substack{BARREL\\IMPACT}}$ and $C_{\substack{BARREL\\IMPACT}}$ are nonzero if weapon is in counterrecoil,

if

$$X_{MG} - X_B \leq 0,$$

and if

$$\dot{X}_B \geq \dot{X}_{MG}.$$

Nominal values are:

$E_{XTF} = 7.5$ lbs

$K_{\substack{BARREL\\IMPACT}} = 100{,}000$ lb/ft

$C_{\substack{BARREL\\IMPACT}} = 18$ lb-sec/ft

For the two-mass system consisting of the bolt and the main gun, 18 lb-sec/ft represents 45 percent of critical damping where

$$C_{CRITICAL} = 2\sqrt{\frac{K_{\substack{BARREL\\IMPACT}} M_{B(E)} M_{MG}}{M_{B(E)} + M_{MG}}}$$

This definition is made in analogy to the single mass-spring system where a mathematical change in the form of the solution for the governing equations occurs at this value of damping.

NOTE: FB(3) is zero after the bolt is locked, and an equal and opposite force FMG(3) acts on the main gun.

A.2.4 FB(4) Friction between bolt and bolt-carrier - The friction force between bolt and bolt-carrier acts continuously.

Then,

$$FB(4) = -C_B \left(\dot{X}_B - \dot{X}_{BC}\right) + \mu_B M_{B(E)} g \, \mathrm{sgn} \left(\dot{X}_{BC} - \dot{X}_B\right) \cos \theta_E \qquad \text{(A-4)}$$

C_B = .01 lb-sec/ft

μ_B = .2

The first term in this expression represents viscous damping and the second term is Coulomb friction due to the weight of the bolt.

NOTE: An equal and opposite force FBC(1) acts on the bolt-carrier.

A.2.5 FB(5) Force at the end of the cam path when weapon is not locked - This force is the means by which the bolt-carrier transmits motion to the bolt. A 1/16 inch play exists between the bolt and the bolt-carrier. A stiff spring is assumed to act on both sides of this 1/16 inch distance.

Thus,

$$FB(5) = -K_{BBC}\left(X_B - X_{BC} - X_{BBC}\right) - C_{BBC}\left(\dot{X}_B - \dot{X}_{BC}\right) \qquad \text{(A-5)}$$

if

$$\left(X_B - X_{BC} - X_{BBC}\right) \leq 0$$

and if

$FB(5) > 0.$

NOTE: X_{BBC} is the total distance the bolt moves relative to the bolt-carrier (5/16 inch).

Therefore,

$$FB(5) = -K_{BBC}\left[X_B - X_{BC} - \left(X_{BBC} - 1/16''\right) \right] - C_{BBC}\left(\dot{X}_B - \dot{X}_{BC}\right) \qquad \text{(A-6)}$$

if the bolt is not about to be locked or unlocked,

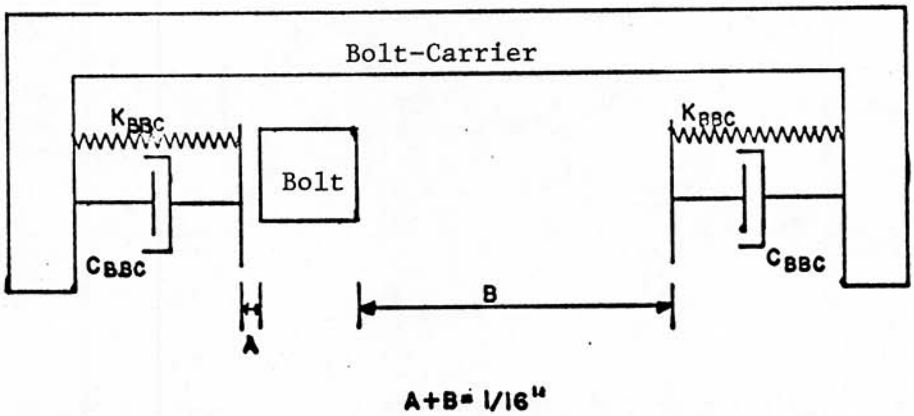
if

$$X_B - X_{BC} - (X_{BBC} - 1/16'') \geq 0,$$

and if

$FB(5) < 0.$

This situation is represented in Figure 34.



K_{BBC} = Assumed Spring Constant Between Bolt and Bolt-Carrier

C_{BBC} = Assumed Damping Constant Between Bolt and Bolt-Carrier

Figure 34 Schematic of Bolt/Bolt-Carrier Interaction (Unlocked Weapon)

Nominal values are: K_{BBC} = 100,000 lbs/ft,

C_{BBC} = 18 lb-sec/ft (49 percent of critical)

NOTE: An equal and opposite force FBC(2) acts on the bolt-carrier.

A.2.6 FB(6) Locking force, FB(7) Unlocking force

These are constraint forces that act between the bolt and bolt-carrier during locking and unlocking. They are the axial components of the forces needed to rotate the bolt in accordance with the cam path in the bolt-carrier. Two derivations of the force expressions are presented. In the first, the Newtonian approach is used; and in the second, the Lagrangian approach is followed. Finally, an alternate formulation of the problem is discussed.

A.2.6.1 Newtonian approach

Figures 35 illustrates the Newtonian approach to resolving the forces acting between bolt and bolt-carrier during locking and unlocking.

Newtonian approach:

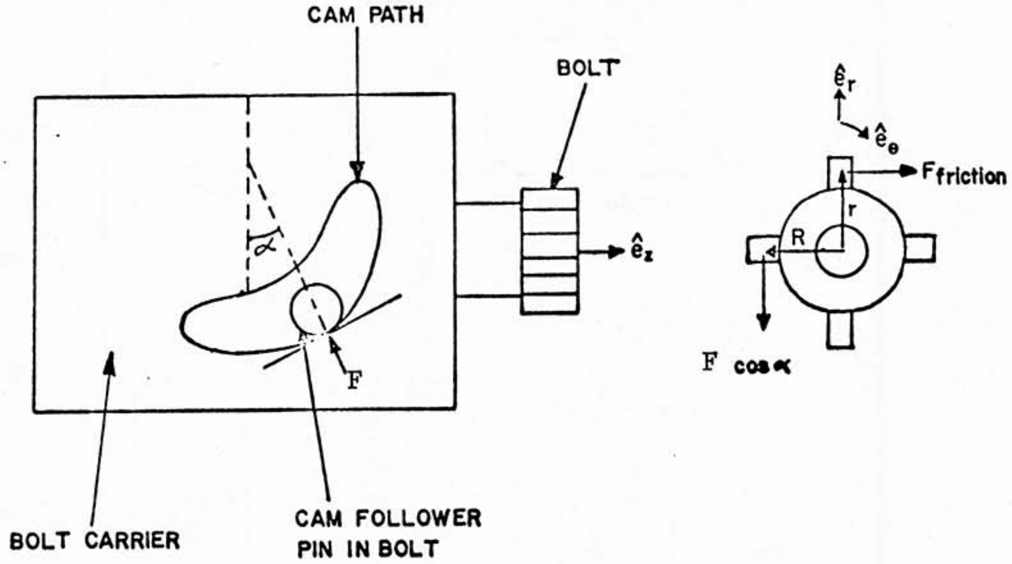


Figure 35 Schematic of Bolt/Bolt-Carrier Interaction during Locking and Unlocking

From Fig. 35, $\mathcal{F} \cos \alpha$ = the force that tends to rotate the bolt.

I_B = moment of inertia of bolt about longitudinal axis

$\vec{N} = \dot{\vec{L}}$ sum of torques equals rate of change of angular momentum

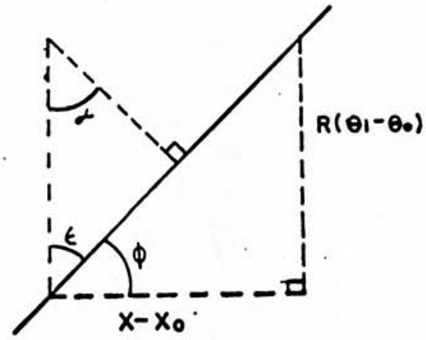
$$I_B \ddot{\theta}\, \hat{e}_z = r\, \hat{e}_r \times (-F_{friction})\, \hat{e}_\theta + R\, \hat{e}_r \times \mathcal{F} \cos \alpha\, \hat{e}_\theta$$

$$I_B \ddot{\theta} = -r F_{friction} + R \mathcal{F} \cos \alpha$$

$$\mathcal{F} = \frac{I_B \ddot{\theta} + r F_{friction}}{R \cos \alpha}$$

$$FB(6,7) = \mathcal{F} \sin \alpha \qquad (A-7)$$

$$FB(6,7) = \frac{I_B \ddot{\theta} + r F_{friction}}{R \cot \alpha} \qquad (A-8)$$



$$\begin{cases} \varepsilon = \frac{\pi}{2} - \phi \\ \alpha + \varepsilon = \frac{\pi}{2} \end{cases}$$

$$\therefore \alpha = \phi$$

$$\phi = \tan^{-1}\left(R\frac{d\theta}{dx}\right)$$

$$FB(6,7) = \frac{I_B\ddot{\theta} + r\,F_{friction}}{R\cot\left[\tan^{-1}(R\frac{d\theta}{dx})\right]} = \frac{I_B\ddot{\theta} + r\,F_{friction}}{R}\tan = \left[\tan^{-1}(R\frac{d\theta}{dx})\right] \quad \text{(A-9)}$$

$$FB(6,7) = \frac{d\theta}{dx}\left(I_B\ddot{\theta} + r\,F_{friction}\right) \quad \text{(A-10)}$$

From the chain rule of differentiation,

$$\ddot{\theta} = \frac{d^2\theta}{dx^2}\dot{X}^2 + \frac{d\theta}{dx}\ddot{x}$$

where

$$X = \left(X_{BC} - X_{BC_o}\right) - \left(X_B - X_{B_o}\right)$$

A.2.6.2 Lagrangian approach

The rotation is assumed to be a linear function of relative position between the bolt and bolt-carrier. For the M16A1 Rifle, this is an accurate assumption.

Let $q_i (i = 1,2, \cdots n)$ represent generalized coordinates.

Let $\sum_{i=1}^{n} a_i \, dq_i = 0$ be the constraint equation.

Let L be the Lagrangian, which is the kinetic energy minus the potential energy of the system.

Let F_i $(i = 1,2, \cdots n)$ represent dissipative forces in the system.

Then $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial \dot{q}_i} = \lambda \, a_i + F_i \qquad i = 1,2, \cdots n$

The constraint forces are $\lambda \, a_i$, where λ is a Lagrangian multiplier. (See, for example, pg 269 of Ref[8])

For the problem at hand, generalized coordinates are the rotation of the bolt, θ, the displacement of the bolt, X_B, and the displacement of the bolt-carrier, X_{BC}.

The constraint equation is then $\theta_B = -.131 + [\frac{d\theta_B}{d(X_B - X_{BC})}](X_B - X_{BC})$

This equation can be put into the form $\sum_{i=1}^{n} a_i \, dq_i = 0$ as follows:

$$- d\theta_B + \left[\frac{d\theta_B}{d(X_B - X_{BC})}\right] dX_B - \left[\frac{d\theta_B}{d(X_B - X_{BC})}\right] dX_{BC} = 0$$

The Lagrangian is $L = \frac{1}{2} I_B \, \dot{\theta}_B^2 + \frac{1}{2} M_B \, \dot{X}_B^2 + \frac{1}{2} M_{BC} \, \dot{X}_{BC}^2$

Let F_B and F_{BC} be other forces acting on the bolt and bolt-carrier.

The only forces acting on the bolt in the rotational direction are the constraint force and friction.

Substitution into $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \lambda \, a_i + F_i$ yields the following

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[8] Greenwood, D.T., Principles of Dynamics, Prentice-Hall, 1965

three equations of motion:

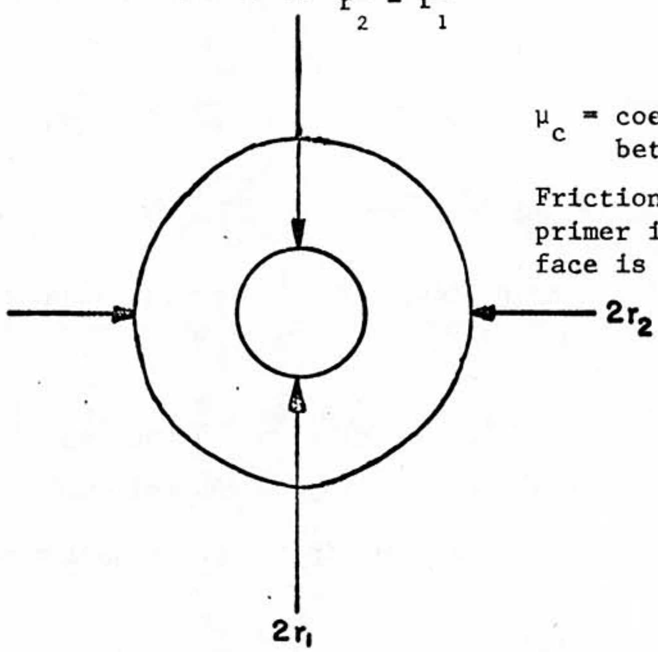
$$\begin{cases} I_B \ddot{\theta}_B = -\lambda + T_{FRICTION} & \text{(A-11)} \\ M_B \ddot{X}_B = \lambda \dfrac{d\theta_B}{d(X_B - X_{BC})} + F_B & \text{(A-12)} \\ M_{BC} \ddot{X}_{BC} = -\lambda \dfrac{d\theta_B}{d(X_B - X_{BC})} + F_{BC} & \text{(A-13)} \end{cases}$$

$T_{FRICTION}$ is the torsional friction moment due to the compression of the bolt on the base of the cartridge case. This force is assumed to be uniformly distributed over the base.

Let F_N denote the normal force between bolt and case that acts on the bolt. Then

$$T_{FRICTION} = + \int_{r_1}^{r_2} r \, \mu_c \frac{F_N}{\pi(r_2^2 - r_1^2)} 2\pi r \, dr$$

$$= + \frac{2}{3} \mu_c F_N \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$$



μ_c = coefficient of friction between bolt and case

Friction from portion of primer in contact with bolt face is neglected.

r_1 = radius of primer

r_2 = radius of base of cartridge case

Figure 36 Diagram of Cartridge Case Base

From the first equation of motion, $\lambda = -I_B [\frac{d\theta_B}{d(X_B - X_{BC})}] (\ddot{X}_B - \ddot{X}_{BC})$ (A-14)

$+ T_{FRICTION}$

$FB(6) = \lambda a_2$ where $a_2 = \frac{d\theta_B}{d(X_B - X_{BC})}$ (A-15)

$$FB(6) = \frac{d\theta_B}{d(X_B - X_{BC})} T_{FRICTION} + I_B [\frac{d\theta_B}{d(X_B - X_{BC})}]^2 (\ddot{X}_{BC} - \ddot{X}_B) \quad \text{(A-16)}$$

$$FB(6) = + \frac{d\theta_B}{d(X_B - X_{BC}} [\frac{2}{3} \mu_c \frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)}] F_N + I_B [\frac{d\theta_B}{d(X_B - X_{BC})}]^2 (\ddot{X}_{BC} - \ddot{X}_B) \quad \text{(A-17)}$$

$= FB(7)$

if $-.0625 \text{ inch} \geq X_{BC} - X_B \geq - X_{LD} = -.25$ inch where X_{LD} is the distance the bolt and bolt-carrier must move relative to each other to achieve locking or unlocking

Nominal values are: $\frac{d\theta_B}{d(X_B - X_{BC})} = 2.09$ radians/inch

$I_B = 10.5 \times 10^{-7}$ slug-ft^2

$r_1 = .0938$ inch

$r_2 = .1875$ inch

$\mu_c = .2$ for FB(6) and 0 for FB(7)

Friction on unlocking is very high because of gas pressure, but is considered insignificant on locking. See Figure 37.

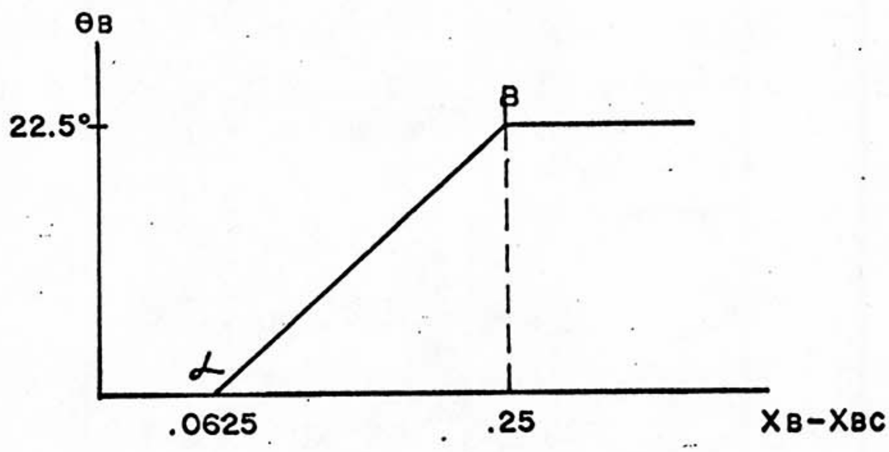


Figure 37 Rotation of Bolt vs Separation Distance Bolt and Bolt-Carrier

Note that θ_B as a function of $X_B - X_{BC}$ is discontinuous in the first derivative just prior to rotation and after rotation is completed. The points of discontinuity are at α and B in Fig. 37. Prior to rotation, $\dot{\theta}_B = 0$; on the other side of the discontinuity, $\dot{\theta}_B = [\frac{d\theta_B}{d(X_B - X_{BC})}] (\dot{X}_B - \dot{X}_{BC})$. This instantaneous change in angular momentum can be considered to be caused by impulsive constraint forces acting between the bolt and bolt-carrier. The instantaneous changes of bolt and bolt-carrier momenta will be computed from conservation of linear momentum and conservation of energy equations. This situation is unlike impact with a spring where a decrease in time interval leads to a smaller jump in force.

Now, make the following definitions:

- $M_{B(E)}$ is the effective mass of the bolt when the discontinuity is encountered.
- M_{BC} is the mass of the bolt-carrier.
- $\dot{X}_{B(I)}$ is the velocity of the bolt prior to the discontinuity.
- $\dot{X}_{B(F)}$ is the computed velocity of the bolt after the discontinuity.
- $\dot{X}_{BC(I)}$ is the velocity of the bolt-carrier prior to the discontinuity.
- $\dot{X}_{BC(F)}$ is the velocity of the bolt-carrier after the discontinuity.

Here, only the case where the bolt begins to unlock will be considered. The other three cases, unlocking completed, beginning of locking, and locking completed, are treated similarly.

From the conservation of linear momentum:

$$M_{B(E)}\dot{X}_{BC(I)} + M_{BC}\dot{X}_{BC(I)} = M_{B(E)}\dot{X}_{B(F)} + M_{BC}\dot{X}_{BC(F)} \qquad \text{(A-18)}$$

From the constraint equation, $$\dot{\theta}_B = \frac{d\theta_B}{d(X_B - X_{BC})} (\dot{X}_B - \dot{X}_{BC}) \qquad \text{(A-19)}$$

and conservation of energy:

$$M_{B(E)} \frac{\dot{X}^2_{B(I)}}{2} + M_{BC} \frac{\dot{X}^2_{BC(I)}}{2} = M_{B(E)} \frac{\dot{X}^2_{B(F)}}{2} + M_{BC}\frac{\dot{X}^2_{BC(F)}}{2} + \frac{I_B}{2} [\frac{d\theta_B}{d(X_B - X_{BC})}]^2 (\dot{X}_{B(F)} - \dot{X}_{BC(F)})^2 \qquad \text{(A-20)}$$

The variables $\dot{X}_{BC(I)}$ and $\dot{X}_{B(I)}$ are known; and from the conservation of momentum and energy equations, $\dot{X}_{BC(F)}$ and $\dot{X}_{B(F)}$ can be computed. In doing so, one must solve a quadratic algebraic equation and the appropriate sign of the discriminate must be chosen.

In obtaining the accelerations during locking and unlocking, one must solve simultaneously the equations of motion for the bolt and bolt-carrier.

NOTE: Equal and opposite forces, FBS(3) and FBC(5), act on the bolt-carrier.

The inclusion of the analysis of the discontinuity is an important addition. If it is not included, an implicit assumption exists in the remaining analysis that, when unlocking begins, the bolt is already rotating. In reality, a relatively large impulse must be provided to cause the bolt to overcome its initially zero rotational velocity. The value of the force required to accomplish this momentum change depends on the time it is assumed to act. If the change is instantaneous, an infinite force is needed. Another approach is to replace the discontinuity with a smooth curve of small radius. In this analysis, no attempt was made to calculate the force, but the impulse was calculated. In comparing the results of measurements of unlocking force with the computer results, one must remember that what is labeled as unlocking force does not include the force needed to put the bolt into rotation initially.

A.2.6.3 <u>Alternate formulation of the problem</u>

Now, an alternative formulation of the locking and unlocking problem will be discussed. The strong point of the formulation is that it does not require the treatment of discontinuities. It also can be modified to include chatter. In the math model of the XM19 Rifle, Part II of this report, the following approach was used.

F = [Spring Constant] x [Penetration of bolt pin into bolt-carrier cam path in direction perpendicular to cam path surface]

F = K [distance bolt pin moves relative to bolt-carrier C.G. in direction normal to cam path surface minus distance point of contact between bolt pin and cam path moves in direction normal to cam path surface]

$$F = K\,[(X_B - X_{BC} - a)\sin\alpha - R\theta\cos\alpha] \quad \text{(A-21)}$$

$a = X_B - X_{BC}$ at start of rotation

K = spring constant for hypothetical stiff spring whose location is at the point of contact between cam path and bolt pin

$$I\ddot{\theta} = RF\cos\alpha, \quad \dot{\theta}(0) = \theta(0) = 0 \quad \text{(A-22)}$$

Solve for θ and substitute in the following equation:

$$FB(6,7) = F\sin\alpha = K[(X_B - X_{BC} - a)\sin\alpha - R\theta\cos\alpha]\sin\alpha \quad \text{(A-23)}$$

A.2.7 FB(8) Force between bolt and bolt-carrier while weapon is locked

The interface between the bolt and bolt-carrier while the weapon is locked is assumed to be characterized by a stiff spring.

$$FB(8) = -K_{BC}(X_B - X_{BC}) - C_{BC}(\dot{X}_B - \dot{X}_{BC}) \text{ if } X_B - X_{BC} \le 0 \text{ and } FB(8) \ge 0. \quad \text{(A-24)}$$

Nominal values are:

K_{BC} = 100,000 lbs/ft

C_{BC} = 37 lb-sec/ft = 45% critical

A.2.8 FB(9) Stripping force used to remove a round of ammo from the magazine

The collision between the bolt and the uppermost round in the magazine is assumed to be inelastic. At impact, a simultaneous change in the mass and velocity of the bolt is assumed to occur. Make the following definitions:

$\dot{X}_{MG(I)}$ is the velocity of the rifle just prior to the collision between bolt and cartridge

$\dot{X}_{B(I)}$ is the velocity of the bolt just prior to the collision between bolt and cartridge

$\dot{X}_{B(F)}$ is the velocity of the bolt just after collision between bolt and cartridge

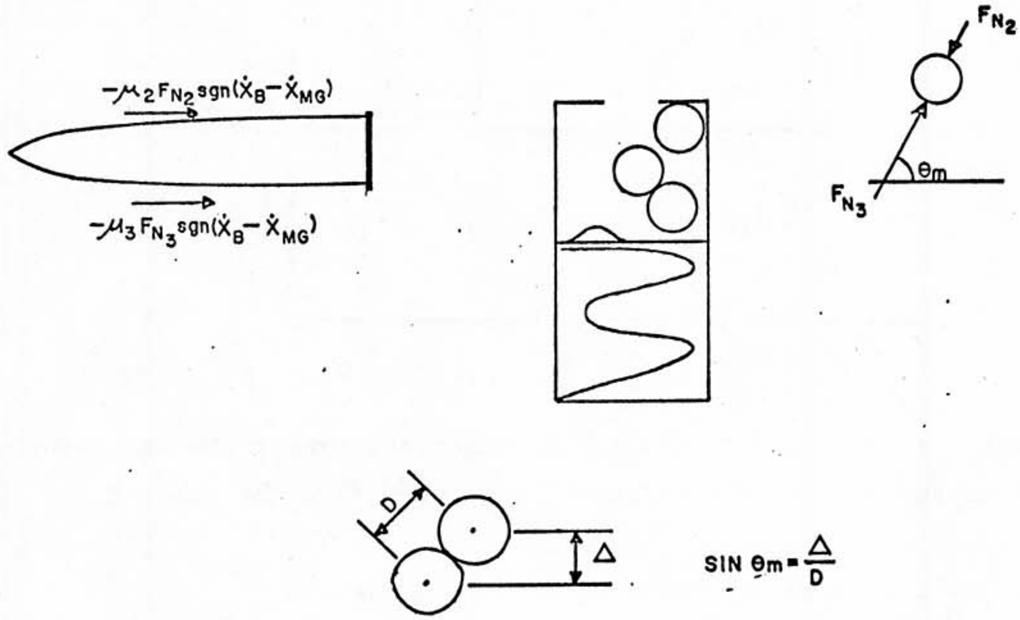
M_B is the mass of the bolt

M_R is the mass of the cartridge

From conservation of momentum, $M_B \dot{X}_{B(I)} + M_R \dot{X}_{MG(I)} = (M_B + M_R)\dot{X}_{B(F)}$ (A-25)

$$\therefore \dot{X}_{B(F)} = \frac{M_B \dot{X}_{B(I)} + M_R \dot{X}_{MG(I)}}{M_B + M_R} \quad \text{(A-26)}$$

Note that, at the time of collision, the cartridge is considered to be part of the main gun and, therefore, the velocity of the cartridge is assumed to be that of the rifle. Also, at this instant, the effective mass of the bolt is increased by M_R and the mass of the rifle is decreased by M_R. The foregoing development provides the values for the simultaneous changes in mass and velocity. Now the stripping force itself will be found. Consider the geometry and forces shown in Fig. 38.



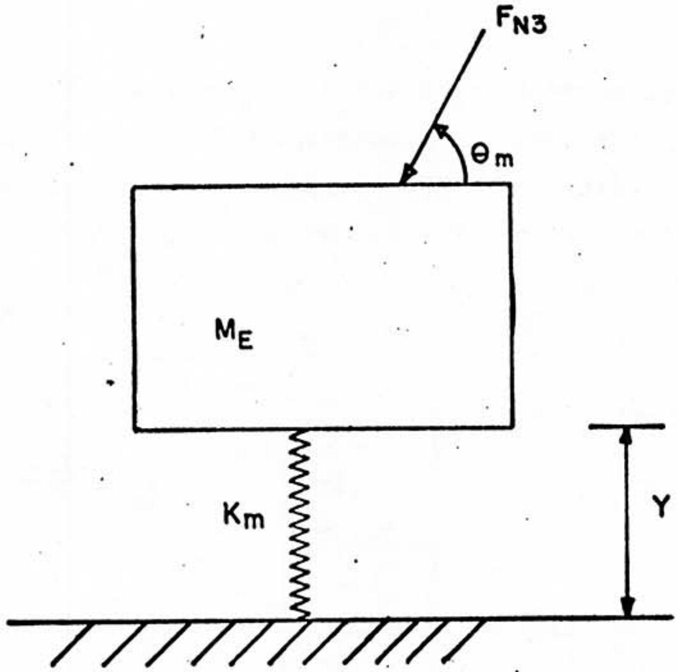
μ_2, μ_3 are coefficients of friction

For vertical equilibrium, $F_{N_2} = F_{N_3}$

Figure 38 Schematic of Magazine

Note that Δ is the displacement of the spring per round of ammo and D is the diameter of the cartridge case. To establish the value of Δ, one divides the difference between the height of the magazine spring when the magazine is full and the height when empty by the round capacity of the magazine.

To continue the analysis, one must compute F_{N_2} and F_{N_3}. Assume that the magazine can be idealized as follows:



M_E is the total mass of all rounds in the magazine, except the one being stripped, plus the mass of the follower, plus one-third the mass of the magazine spring.

Vertical equilibrium requires that

$$-F_{N_3} \sin \theta_M - M_E g \cos \theta_E - K_M(Y - Y_o) = 0 \qquad \text{(A-27)}$$

The free length of the magazine spring is Y_o, and K_M is the spring constant. The angle of elevation of the rifle is θ_E.

$$F_{N_2} = F_{N_3} = -\frac{1}{\sin\theta_M} [M_E g \cos \theta_E + K_M (Y - Y_o)] \qquad \text{(A-28)}$$

The stripping force is then:

$$FB(9) = \left(\frac{\mu_2 + \mu_3}{\sin\theta_M}\right) [M_E g \cos \theta_E + K_M (Y - Y_o)] \operatorname{sgn} (\dot{X}_B - \dot{X}_{MG}) \qquad \text{(A-29)}$$

This force is nonzero when the weapon is in counterrecoil and $-2.81 \text{ inch} \leq X_B - X_{MG} \leq -1.84 \text{ inch}$.

The parameters that are invariant during the firing of the weapon are Δ, D, $(M_F + \frac{1}{3} M_K)$ (mass of the follower plus 1/3 the mass of the magazine spring), M_R, μ_2, μ_3, K_M and Y_o. After each round is stripped, M_E and Y are recomputed. $Y = Y_E - n\Delta$, where Y_E is height of magazine spring when magazine is empty, and n is number of rounds in magazine.

Nominal values for the parameters of a twenty-round magazine are:

$\Delta = .18$ in

$D = .375$ in

$M_F + \frac{1}{3} M_K = .03$ lb

$M_R = .025$ lb

$\mu_2 = .30$

$\mu_3 = .30$

$K_M = 10.8$ lbs/ft

$Y_o = 7.77$ in

$Y_E = 4.5$ in

NOTE: An equal and opposite force, FMG(15), acts on the main gun.

A.2.9 FB(10) Force from bolt cavity pressure

This force is caused by the gases in the bolt-carrier cavity. These gases enter the cavity by means of a long tube connected to the gas port on the barrel. This pressure is the main source of energy for weapon actuation. This force can be calculated by the method of matched asymptotic expansions. (See Ref[9]) Finite element methods also show promise for such calculations. (See Ref[7]) For present work, the following nominal values were experimentally obtained for:

$$FB(10) = -P_{CAVITY}(t)\ A_{CAVITY}, \text{ where} \qquad (A-30)$$

$A_{CAVITY} = .1967$ in^2.

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[9]Spurk, J.H., "The Gas Flow in Gas-Operated Weapons," Ballistic Research Laboratories Report No. 1475, Aberdeen Proving Ground, Md. (Feb 1970)

[7]Ehle, P.E. and Rahe, A.E., "Development of a Finite Element Approach for Approximate Analysis of Unsteady Compressible Fluid Flow," WECOM Technical Report, SWERR-TR-72-36, AD 746234, Rock Island, Ill (Jun 1972)

TABLE 9 NOMINAL PRESSURE vs TIME IN BOLT-CARRIER CAVITY

Time (milliseconds	$P_{CAVITY}(t)$ (lb/in^2
0.0	0.0
.264	1250
.528	2500
.792	2650
1.056	2400
1.32	2000
1.584	1500
1.844	1100
2.112	600
2.376	400
2.64	300
2.904	250
3.168	200
3.432	150
3.696	100
3.96	100
4.224	50
4.488	50

The delay between the time that the breech pressure becomes nonzero and the time that the cavity pressure becomes nonzero is t_{port} + t_{cav}. Nominal values are t_{port} = 1.1 milliseconds = time required by projectile to reach gas port, and t_{cav} = .2 milliseconds = time required by the pressure wave to travel from the gas port to the bolt cavity.

NOTE: An equal and opposite force FBC(9) acts on the bolt-carrier.

A.2.10 FB(11) Extraction of the cartridge case from the chamber

This force becomes nonzero just after the bolt is unlocked, and the bolt and main gun are again two separate masses. The first problem is that of determining when the bolt becomes separated from the main gun. Once the bolt-carrier reaches the end of the cam path in the unlocking process, the accelerations of the rifle and the bolt are computed from the following equations:

$$M_{B(E)} \ddot{X}_B = \sum_{i=1}^{K} FB(I) + F_{CON} \qquad (A-31)$$

$$M_{MG(E)} \ddot{X}_{MG} = \sum_{i=1}^{n} FMG(I) - F_{CON} \qquad (A-32)$$

$$\ddot{X}_B = \ddot{X}_{MG} \qquad (A-33)$$

F_{CON} represents the constraint force that causes the bolt and main gun to move together. When this constraint force overcomes the static stiction force between cartridge case and chamber wall, then the bolt and main gun begin to separate. At this point, FB(11) becomes non-zero.

From the three equations given above,

$$F_{CON}\left[\frac{1}{M_{B(E)}} + \frac{1}{M_{MG(E)}}\right] = -\frac{1}{M_{B(E)}} \sum_{i=1}^{K} FB(I) + \frac{1}{M_{MG(E)}} \sum_{i=1}^{n} FMG(I) \qquad (A-34)$$

∴ Separation occurs when

$$\frac{1}{M_{B(E)}+M_{MG(E)}} \left[-M_{MG(E)} \sum_{i=1}^{K} FB(I) + M_{B(I)} \sum_{i=1}^{n} FMG(I) \right] > F_{STICTION} \qquad (A-35)$$

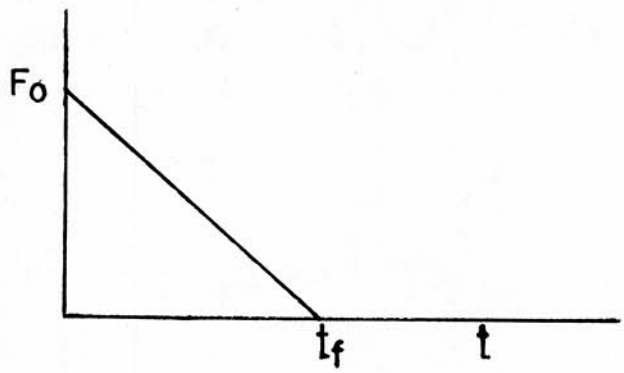
Model predictions are very insensitive to reasonable values of $F_{STICTION}$, therefore, $F_{STICTION}$ is set equal to zero. Experimentally, extraction forces have been obtained by placement of a strain gauge on the extractor. Such measurements include the force needed to impart the velocity of the bolt-carrier to the empty cartridge case in addition to the friction force between chamber wall and cartridge case. In the model, the force needed to impart the velocity of the bolt-carrier to the cartridge case is automatically taken into account by the addition of the cartridge case mass to that of the bolt. Thus, to use the experimental data, that component must be removed. The remainder will be an approximation of the friction force between chamber and cartridge case.

Let $\int F_{EXT}\, dt$ be the measured impulse on the extractor, $M_c V_c$ be the momentum of the cartridge case after it acquires the velocity of the bolt-carrier, and $F_{CHAMBER}$ be the friction force between cartridge case and chamber.

Then $\int F_{EXT}\,dt - \int F_{CHAMBER}\,dt = M_c\,(V_c - V_{c_o})$ (A-36)

Assume $V_{c_o} \simeq 0$ and $V_c \simeq \dot{X}_{MG} - \dot{X}_{BC} = 17$ ft/sec when case assumes bolt-carrier velocity. Further assume V_c is constant in time during extraction. This value for V_c obtained from a computer run is in agreement with Reference[10].

Assume $F_{CHAMBER}$ has the following shape:



$$F_{CHAMBER} = F_o \frac{t_f - t}{t_f} \qquad t_f = \frac{L}{V_c} \tag{A-37}$$

where L = length of cartridge case.

$$F_{CHAMBER} = F_o \left[1 - \frac{V_c}{L} t \right] \tag{A-38}$$

$$\int_o^{t_f} F_{CHAMBER}\,dt = \frac{1}{2} F_o \frac{L}{V_c} \tag{A-39}$$

From Eq. (A-36),

$$\int F_{EXT}\,dt - \frac{1}{2} F_o \frac{L}{V_c} = M_c V_c \tag{A-40}$$

$$F_o = \frac{2V_c}{L} \left[\int F_{EXT}\,dt - M_c V_c \right] \tag{A-41}$$

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[10] Werner, W.M., "Comparison of a Theoretical and Experimental Study of the Gas System in the M16A1 Rifle," BRL Report No. 1548, Aberdeen Proving Ground, Md., (Aug 1971) Pg 23.

V_c = 17 ft/sec

$\int F_{EXT}\, dt$ = .011 lb/sec

(See WECOM DF, 5 Jun 68, from SWERI-RDT 9280 to AMSWE-RDSR (C.F. Packard), Subject: "M16A1 Test Equipment Instrumentation.")

M_c = .000411 slug

L = 1.75 in

F_o = .94 lb

$$F_{CHAMBER} = FB(11) = F_o \left[1 - \frac{V_c}{L} t \right] \qquad \text{(A-42)}$$

$$FB(11) = .94 [1 - 116.6 t]$$

Time is zero when the criterion for separation of bolt and main gun is met.

FB(11) is zero after $t \geq \frac{L}{V_c}$

NOTE: An equal and opposite force FMG(10) acts on the main gun.

A.2.11 FB(12) Gravity force acting on bolt

This force acts continuously.

$$FB(12) = -M_{B(E)}\, g \sin \theta_E \qquad \text{(A-43)}$$

$$g = 32.2 \text{ ft/sec}^2$$

NOTE: $M_{B(E)}$ varies with time.

$M_{B(E)}$ = .156 lb after a new round has been stripped from the magazine, but before round is fired

.143 lb after the round has been fired, but before the case is ejected

.130 lb after the case has been ejected but before a new round is obtained from the magazine

Forces acting on the bolt-carrier

A.2.12 FBC(1) = -FB(4) (See previous derivation)

A.2.13 FBC(2) = -FB(5) (See previous derivation)

A.2.14 FBC(3) = -FB(6) (See previous derivation)

A.2.15 FBC(4) = -FB(8) (See previous derivation)

A.2.16 FBC(5) = -FB(7) (See previous derivation)

A.2.17 FBC(6) Interaction force between buffer tube and bolt-carrier

A hypothetical stiff spring is placed between the buffer tube and the bolt-carrier at their interface.

$$FBC(6) = K_{BUFF}\ (X_{BUFF} - X_{BC}) + C_{BUFF}\ (\dot{X}_{BUFF} - \dot{X}_{BC}) \quad \text{if } FBC(6) \geq 0. \qquad \text{(A-44)}$$

K_{BUFF} = 100,000 lb/ft

C_{BUFF} = 14.06 lb-sec/ft = 37% critical

NOTE: An equal and opposite force FBUFF(1) acts on the buffer.

A.2.18 FBC(7) Friction force between bolt-carrier and main gun

This force acts continuously.

$$FBC(7) = -C_{BC}\ (\dot{X}_{BC} - \dot{X}_{MG}) - \mu_{BC}(M_{BC} + M_{B(E)})\ g\ [\mathrm{sgn}(\dot{X}_{BC}-\dot{X}_{MG})]\ \cos\theta_E \qquad \text{(A-45)}$$

The first term represents viscous damping, and the second term represents Coulomb friction caused by the weight of the bolt-carrier.

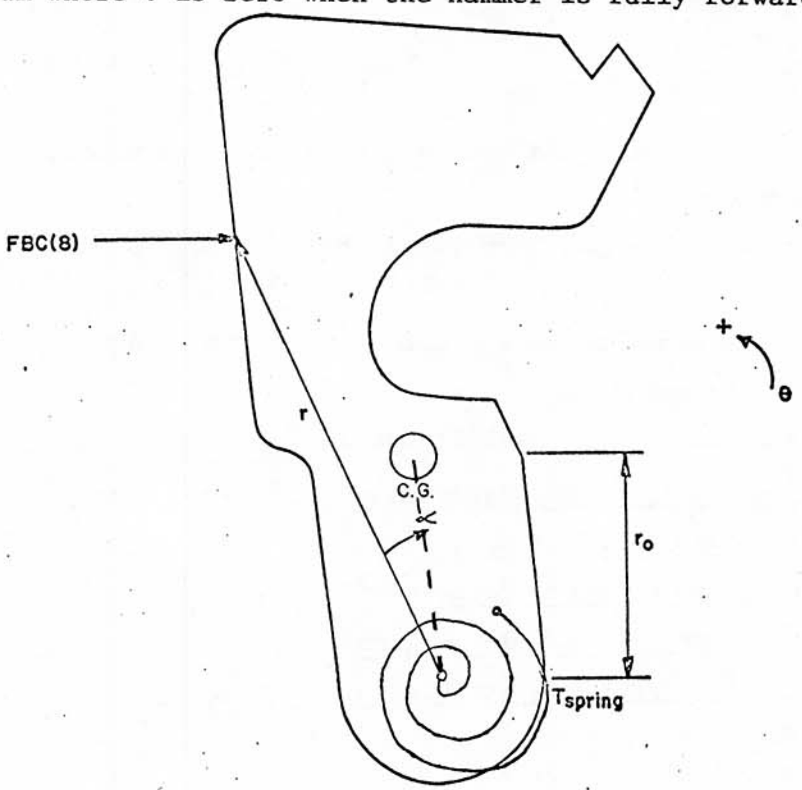
Nominal values are: C_{BC} = .01 lb-sec/ft

μ_{BC} = .2

NOTE: An equal and opposite force FMG(4) acts on the main gun.

A.2.19 FBC(8) Constraint force between hammer and bolt-carrier

This is the force that cocks the hammer. Consider the following diagram where θ is zero when the hammer is fully forward.



Equating rate of change of angular momentum with the sum of the moments, one finds:

$$I_H \ddot{\theta} = T_{SPRING} - r\ FBC(8) \cos(\theta + \alpha) - M_H[r_o \cos\theta]\ \ddot{X}_{MG} \qquad \text{(A-46)}$$

Ordinarily, in equating rate of change of angular momentum with the sum of moments, one must refer these quantities with respect to a fixed point on the hammer or to the center of mass of the hammer. Although the pivot point on the hammer is not fixed with respect to the ground because the main gun is in motion, reference of angular momentum and moments to this point would be convenient. This can be done if a correction factor is added to account for the motion of the pivot point. This term is $M_H\ [r_o \cos\theta]\ \ddot{X}_{MG}$, which is the last term in Equation (A-46). In general, the sum of the moments about an arbitrary point is equal to the vector cross product of the vector from the point to the C.G. and the vector from the body mass times the acceleration of the C.G. with respect to an inertial frame.

That is.

$$\Sigma \vec{M}_P - \dot{\vec{L}}_P = \vec{r}_{P\ \text{to}\ CG} \times m\ddot{\vec{r}}_{CG}\ . \qquad \text{(A-47)}$$

The truth is that all results derived from Newton's laws of motion relative to an inertial frame can be extended to an accelerating nonrotating frame if inertial forces associated with acceleration of the frame are treated as additional external forces on the system. (See Pg 146 of Ref[8])

NOTE: In Equation (A-46), M_H is the mass of the hammer, and r_o is the distance from the pivot point to the C.G.

I_H is the moment of inertia of the hammer about an axis through the pivot point. T_{SPRING} is the constant torque exerted by the hammer spring.

The constraint equation is given by the functional relation

$$\theta = f\ (X_{MG} - X_{BC}) \qquad \text{(A-48)}$$

whenever the hammer and bolt-carrier are in contact.

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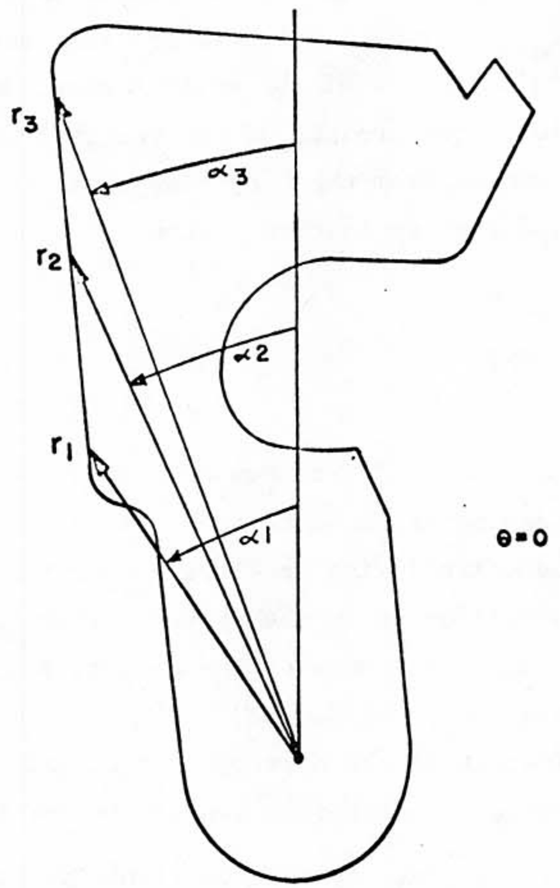
[8]Greenwood, D.T., Principles of Dynamics, Prentice-Hall, 1965

$$FBC(8) = \frac{T_{SPRING} - I_H\left[\frac{d^2 f}{d(X_{MG}-X_{BC})}(\dot{X}_{MG}-\dot{X}_{BC})^2 + \frac{d\ f}{d(X_{MG}-X_{BC})}(\ddot{X}_{MG}-\ddot{X}_{BC})\right]}{r \cos(f + \alpha)} \qquad \text{(A-49)}$$

Derivations for $f(X_{MG} - X_{BC})$, r, and α will now be given. These are all functions of the angle of rotation of the hammer. First make the following definitions:



r_1 = radial distance from the pivot point to the point on the hammer that contacts the bolt-carrier when the hammer is in the firing position($\theta = 0$)

r_2 = radial distance from the pivot point to the point on the hammer that contacts the firing pin when $\theta = 0$.

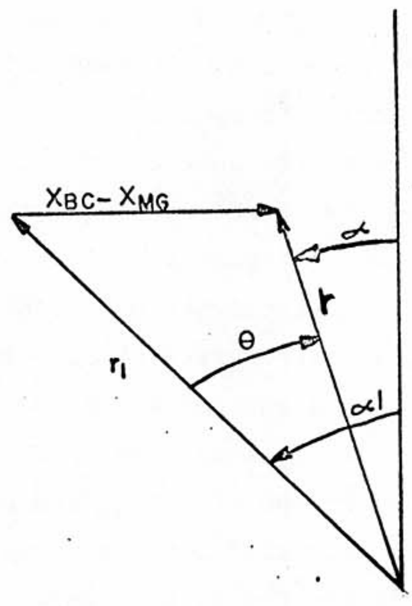
r_3 = radial distance from the pivot point to the highest point on the hammer that ever contacts the bolt-carrier (occurs when $\theta = 77°$)

α_1, α_2, and α_3 are the corresponding angles $\theta = 0$.

r = radial distance from the pivot point to the point on the hammer in contact with the bolt-carrier for any position of the bolt-carrier

α = angle between r and a line imbedded in the hammer that passes through the pivot point and is perpendicular to the bore axis when $\theta = 0$.

Note that the line joining points on the hammer that contact the bolt-carrier lie in a straight vertical line when $\theta = 0$. Using this fact, one can solve for θ as a function of $X_{MG} - X_{BC}$. Consider the following diagram:



$$X_{BC} - X_{MG} = -r_1 \sin \alpha_1 + \tan (\alpha_1 + \theta) r_1 \cos \alpha_1 \quad \text{(A-50)}$$

$$\theta = -\alpha_1 + \tan^{-1} \left[\frac{r_1 \sin \alpha_1 + (X_{BC} - X_{MG})}{r_1 \cos \alpha_1} \right] \quad \text{(A-51)}$$

$$\therefore f (X_{MG} - X_{BC}) = -\alpha_1 + \tan^{-1} \left[\frac{r_1 \sin \alpha_1 + (X_{BC} - X_{MG})}{r_1 \cos \alpha_1} \right] \quad \text{(A-52)}$$

This relationship applies only if $\theta \geq -70°$ because, after a 70° rotation, the hammer motion is determined by the geometry of the underside of the bolt-carrier.

From the previous diagram, one can also find r.

$$r = [(r_1 \cos \alpha_1)^2 + (r_1 \sin \alpha_1 + X_{BC} - X_{MG})^2]^{1/2} \quad \text{(A-53)}$$

One can solve for α using the fact that the distance is constant from any point on the hammer contacting the bolt-carrier to the line through the pivot point imbedded in the hammer.

Thus, $r_1 \sin \alpha_1 = r \sin \alpha$. (A-54)

$$\alpha = \sin^{-1} \left(\frac{r_1 \sin \alpha_1}{r}\right) \quad \text{(A-55)}$$

Equations (A-51),(A-53), and (A-55) are substituted in Equation (A-49).

Note that the derivation given above applied only while the hammer and bolt-carrier are in contact. After unlocking, a series of forces acting on the bolt-carrier causes the bolt-carrier velocity to fall below that of the hammer, and the hammer continues rearward apart from the bolt-carrier. These retarding forces are caused by the strike of the end of the cam path in the bolt-carrier by the bolt pin, and they are caused by forces associated with cartridge case extraction. This situation occurs when FBC(8) is calculated and found to be a negative number. Then FBC(8) is set equal to zero, and the hammer continues rearward with only the spring tension affecting its motion. At a later time, the hammer and bolt-carrier may again come in contact with each other. This encounter is treated as an impact problem where a hypothetical spring and damper are assumed to come between the two bodies. Thus,

$$FBC(8) = K_H \cos \theta \, [\theta - f(X_{MG} - X_{BC})] + C_H \cos \theta \, [\dot{\theta} - f(X_{MG} - X_{BC})(\dot{X}_{BC} - \dot{X}_{MG})] \quad \text{(A-56)}$$

if

$$\theta > f\,(X_{MG} - X_{BC}).$$

Once the hammer reaches 77° or if $f\,(X_{MG} - X_{BC}) < 70°$ and $\dot{\theta} > 0$, the hammer angular velocity and acceleration are set to zero. Thus, constrained motion from 70° to 77° is not treated in detail. To do so requires accounting for the slope cut in the underside of the bolt-carrier. This small distance is of little consequence to the model and the effort required for accurate treatment would not be worthwhile. Note that the full 77° rotation is fully treated if the hammer is not in contact with the bolt-carrier after 70°. Very little interaction occurs between the bolt-carrier and the hammer after 70°. Most of this interaction is due to friction, and this force is included in the model.

Nominal values are:

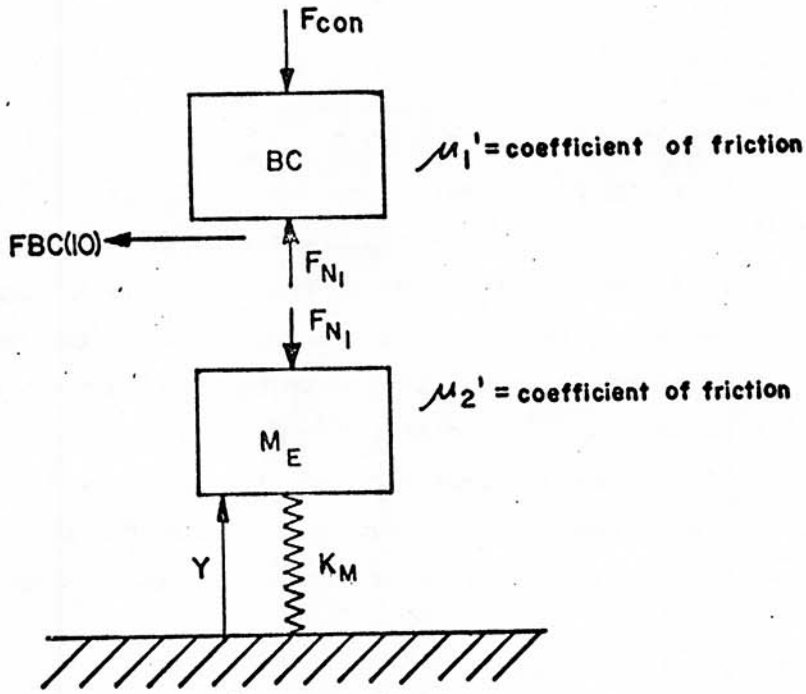
$r_1 = 1.08$ in
$r_2 = 1.33$ in
$r_3 = 1.58$ in
$r_o = .75$ in
$M_H = .0683$ lb
$\alpha_1 = 22°$
$\alpha_2 = 17.5°$
$\alpha_3 = 15°$
$K_H = 4500$ lb/rad
$C_H = .086$ lb-sec/rad
$I_H = .00002$ slug-ft^2
$T_{SPRING} = 6$ in-lb

FBC(8) becomes nonzero when the bolt-carrier cavity pressure is nonzero, $\dot{X}_{MG} - \dot{X}_{BC} \geq 0$, and $X_{MG} - X_{BC} \geq 0$.

A.2.20 FBC(9) = -FB(10) (See previous derivation)

A.2.21 FBC(10) Friction between bolt-carrier and cartridge case

This force is caused by friction between the bolt-carrier and the uppermost cartridge in the magazine. The bolt-carrier depresses the cartridge a small amount. The situation is depicted schematically in the following diagram:



F_{CON} is the constraint force that ensures vertical equilibrium of the bolt-carrier. $M_E = kM_R + M_F + M_S/3$ (A-57)

where

k is the number of rounds in the magazine

M_F is the mass of the magazine follower

and M_S is the mass of the magazine spring.

For vertical equilibrium of the bolt-carrier,

$$F_{CON} = F_{N_1} \quad \text{(A-58)}$$

$$FBC(10) = -2\mu_1 F_{N_1} \operatorname{sgn}(\dot{X}_{BC} - \dot{X}_{MG}) \quad \text{(A-59)}$$

where

$2\mu_1 \equiv \mu_1^1 + \mu_2^1$ = twice the average coefficient of friction.

Now F_{N_1} must be determined. For vertical equilibrium of the ammunition,

$$-F_{N_1} - M_E g - K_M (Y - Y_o) = 0. \quad \text{(A-60)}$$

Y_o is the free length of the magazine spring. Y is the length of the magazine spring at any time.

$$\therefore \quad FBC(10) = 2\mu_1 \; [M_E g + k_M \; (Y - Y_o)] \; \text{sgn} \; (\dot{X}_{BC} - \dot{X}_{MG}) \qquad (A-61)$$

One can determine exactly when the ammunition is in contact with the bolt-carrier. A number of milliseconds are required for the round just below the one being stripped to move upward into position. A derivation of the proper expression is on pg 7-5 of the "Engineering Design Handbook," Guns Series, Automatic Weapons, AMCP 706-260.

This expression is:

$$t_{DELAY} = \sqrt{\frac{M_E}{\varepsilon K_M}} \cos^{-1}\left(\frac{F_o}{F_M}\right). \qquad (A-62)$$

F_M is the maximum spring force (preceding one cartridge displacement), and F_o is the minimum spring force (following one cartridge displacement). The efficiency of the system is ε and is assumed to be 1.0. If k is the number of rounds in the magazine, Δ is the displacement per round, and Y_o is the length of the magazine spring when the magazine is empty, then

$$F_o = -K_M(Y_e - k\,\Delta - Y_o) \quad \text{where } Y_e = \text{current length} \qquad (A-63)$$

and

$$F_M = -K_M(Y_e - (k + 1)\,\Delta - Y_o). \qquad (A-64)$$

In summary, FBC(10) is nonzero when $X_{MG} - X_{BC} \geq 2.34$ inches and $t - t_{PICK} \geq t_{DELAY}$ where t_{PICK} is the time at which the topmost round is completely stripped from the magazine.

NOTE: An equal and opposite force FMG(12) acts on the main gun.

A.2.22 <u>FBC(11) Friction between bolt-carrier and hammer</u>

The bolt-carrier slides over the hammer after the hammer is fully cocked. Consider Figure 39:

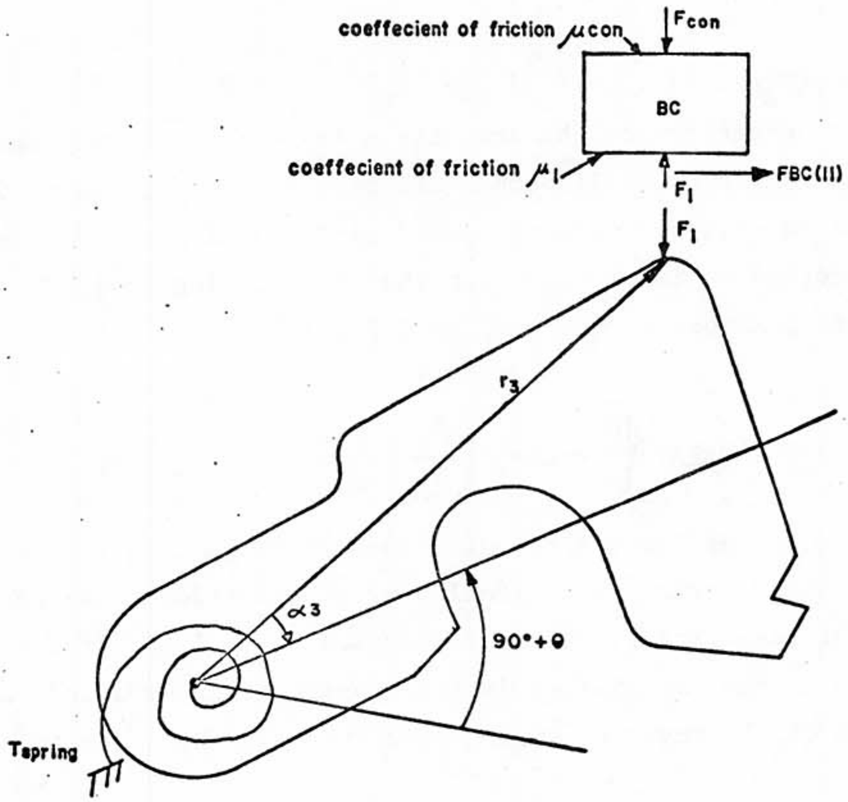


Figure 39 Schematic of Hammer - Bolt-Carrier Interaction

The constraint force F_1 that acts between the hammer and the bolt-carrier can be computed based on the assumption of rotational equilibrium. Equating moments, one finds that

$$T_{SPRING} = -F_1\, r_3 \cos \alpha_3 \sin \theta \qquad \text{(A-65)}$$

$$\therefore \quad F_1 = -\frac{T_{SPRING}}{r_3 \cos\alpha_3 \sin\theta} \qquad \text{(A-66)}$$

$$FBC(11) = -\left[\mu_{CON} F_{CON} + \mu_1 F_1\right] \operatorname{sgn} (\dot{X}_{BC} - \dot{X}_{MG}) \qquad \text{(A-67)}$$

Let $\mu_{eff} \equiv \frac{\mu_{CON}+\mu_1}{2}$

Vertical equilibrium of the bolt-carrier requires that $F_{CON} = F_1$

$$FBC(11) = +\, 2\, \mu_{eff} \frac{T_{SPRING}}{r_3 \cos\alpha_3 \sin\theta} \operatorname{sgn} (\dot{X}_{BC} - \dot{X}_{MG}) \qquad \text{(A-68)}$$

This force is nonzero only if the hammer is in rotational equilibrium, that is: $X_{MG} - X_{BC} \geq 1.36$ inches, and $\dot{\theta} = \ddot{\theta} = 0$. $\mu_{eff} = .15$.

NOTE: An equal and opposite force FMG(13) acts on the main gun.

A.2.23 FBC(12) Gravity force acting on the bolt-carrier

This force acts continuously. The angle of elevation is θ_E.

$$FBC(12) = -M_{BC} g \sin \theta_E \qquad (A\text{-}69)$$

$$M_{BC} = .0182 \text{ slug}$$

$$g = 32.2 \text{ ft/sec}^2$$

Forces Acting on Main Gun

A.2.24 FMG(1) Mount force

This is the force exerted by the shooter's shoulder acting on the butt of the weapon. Efforts are in progress to join the M16 Rifle mechanism model with a mechanical model of the shooter. Meanwhile, a simple spring-dashpot system will be used. Another approach by which FMG(1) is represented as a function of position, velocity, and acceleration, is detailed on Pg 44 of Ref[7].

$$FMG(1) = - K_{MOUNT} X_{MG} - C_{MOUNT} \dot{X}_{MG} \qquad (A\text{-}70)$$

where nominal values are:

$$K_{MOUNT} = 300 \text{ lb/ft}$$

$$C_{MOUNT} = 9.53 \text{ lb-sec/ft}$$

A.2.25 FMG(2) = -FB(2) (See previous derivation)

A.2.26 FMG(3) = -FB(3) (See previous derivation)

A.2.27 FMG(4) = -FBC(7) (See previous derivation)

A.2.28 FMG(5) = Drive spring force

This force acts continuously. It serves to retard the bolt-carrier on the recoil stroke and to provide energy for stripping and locking on the counterrecoil stroke.

The simplest model is based on the assumption that the drive spring is a massless, linear spring through which elastic waves travel with infinite velocity. One step higher in sophistication is the treatment of

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[7] Ehle, P.E. and Rahe, A.E., "Development of a Finite Element Approach for Approximate Analysis of Unsteady Compressible Fluid Flow," WECOM Technical Report, SWERR-TR-72-36, AD 746234, Rock Island, Ill (Jun 1972)

mass effects by the addition of one-third the spring mass to the buffer tube. The reasoning behind this approach is discussed at the end of Appendix C. Further sophistication can be achieved by either continuum or finite-element approaches. Incorporated in the M16 Rifle model is a finite-element approach by which the spring is divided into a series of point masses and ideal springs. The program allows for a maximum of ten elements, and as few as two elements can be used to obtain reasonable results. A further step in sophistication can be taken by the division of the spring into a large number of elements and by the detailed treatment of coil clashing. This approach is discussed in Appendix C. However, since running time is significantly increased, this model is not used unless a special study of the spring itself is desired. The analysis for the small number of masses normally used in the model closely follows the analysis for the large number of masses given in Appendix C.

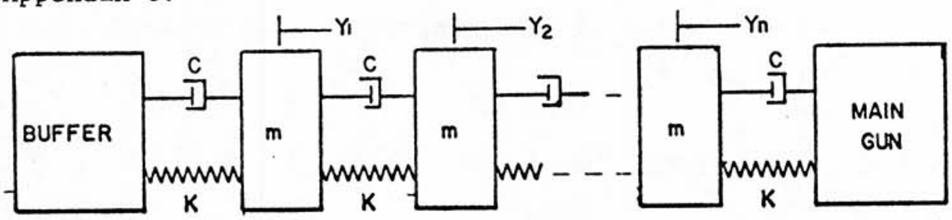


Figure 40 Finite-Element Model of Drive Spring

$$FMG(5) = -K(X_{MG} - Y_n) - C(\dot{X}_{MG} - \dot{Y}_n) - \beta K_5 \qquad \text{(A-71)}$$

where

β = preload distance = .337 ft

K = element spring constant = K_5 (n+1)

$m = m_s/n$

K_5 = drive spring constant = 19.2 lb/ft

m_s = mass of drive spring = .1336 lb

n = number of elements into which the drive spring is divided,

n = 5 normally, but can be up to 10

C = .01 lb-sec/ft

NOTE: Under static conditions, an equal and opposite force FBUFF(3) acts on the buffer.

A.2.29 FMG(6) Bore Friction

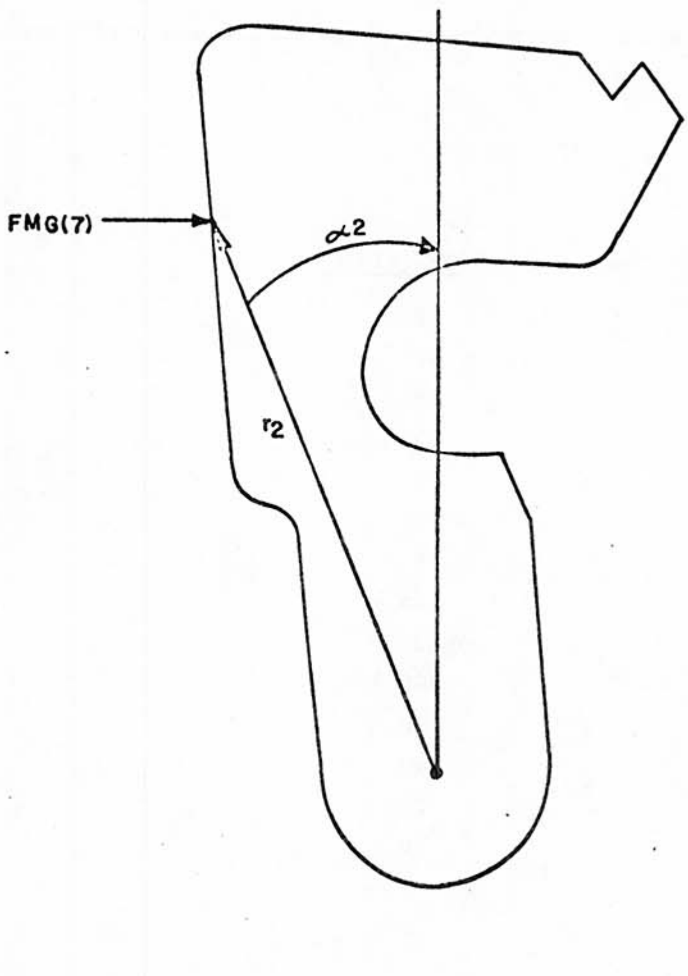
This force is caused by friction between the bullet and the bore as the projectile travels along the length of the barrel. To experimentally obtain values for this force, a bullet was pushed down the barrel at about two inches per second. A steel rod actually pushed the bullet, and the rod was pushed by a machine normally used to obtain load versus deflection curves for springs. Certainly, numbers obtained at such a low speed are questionable. However, the impulse represented by the force obtained is in good agreement with values obtained at BRL. This impulse is roughly 17 percent of the impulse of the breech pressure curve. Thus, for small caliber weapons, this force appears to be rather important; and efforts should be made to obtain more accurate numbers. The values used are:

TABLE 10 BORE FRICTION vs TIME

Time (sec)	FMG(6) (lb)
0	210
.0001	325
.0002	295
.0003	265
.0004	240
.0005	210
.0006	180
.0007	155
.0008	125
.0009	100
.0010	70
.0011	40
.0012	20
.0013	0

A.2.30 FMG(7) Impact between hammer and main gun

This is the force that the hammer exerts on the main gun at the time of firing. It includes the strike of the firing pin, the impact of the firing pin with a spring-like primer, and the final strike of the hammer on the main gun itself. This final strike causes all rotational motion to cease. The entire force is treated as an effective inelastic collision between the hammer and the main gun. If, just after impact, the hammer tends to bounce back and to rotate rearward, its velocity and acceleration are set to zero until the bolt-carrier begins to move rearward.



$$FMG(7) = K_H r_2 \sin\theta + C_H r_2 \dot{\theta} \cos\theta \qquad \text{(A-72)}$$

Nominal values are : K_H = 50000 lb/ft

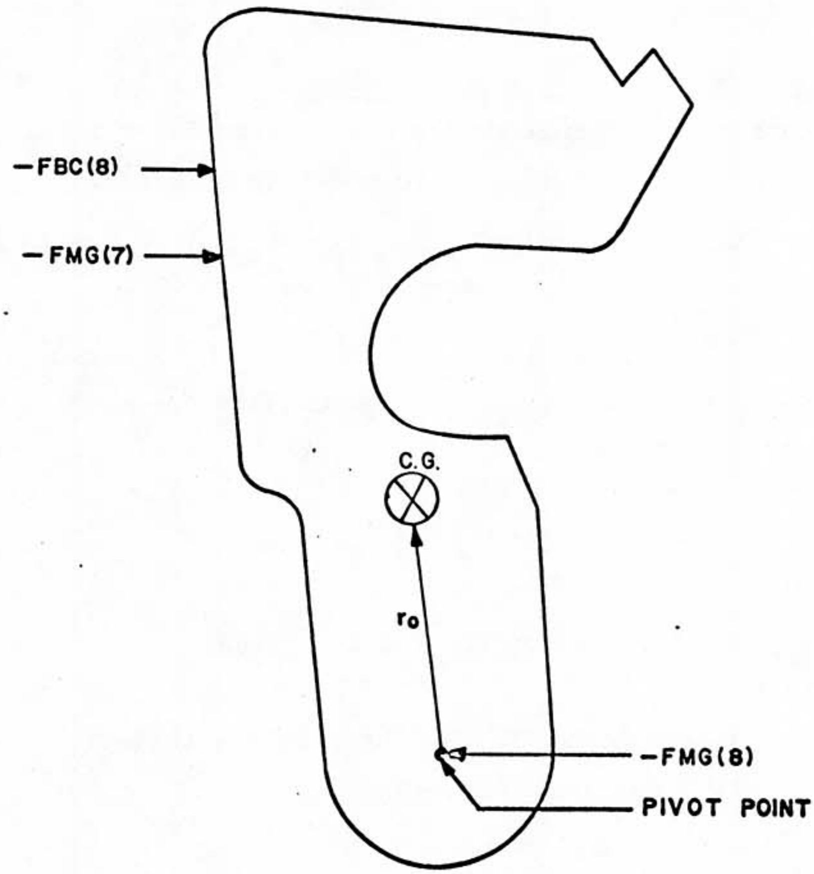
C_H = .278 lb-sec/ft

This force is nonzero only if $\theta > 0$ and $\dot{\theta} > 0$.

NOTE: An equal and opposite force FHAM(2) acts on the hammer.

A.2.31 <u>FMG(8) Constraint force between hammer and main gun</u>

This force acts at the pivot point of the hammer. Consider the following diagram and write the equation of motion of the hammer in the horizontal direction.



$$M_H \ddot{X}_H = -FMG(7) - FBC(8) - FMG(8) \quad \text{(A-73)}$$

$$\therefore FMG(8) = -[M_H \ddot{X}_H + FMG(7) + FBC(8)] \quad \text{(A-74)}$$

M_H = mass of hammer

$\ddot{X}_H$ = acceleration of the center of gravity of the hammer

The following constraint condition exists between the main gun and the hammer.

$$X_H = r_o \sin\theta + X_{MG} + \gamma \quad \text{where } \gamma \text{ is some constant} \quad \text{(A-75)}$$

$$\therefore \ddot{X}_H = r_o [\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta] + \ddot{X}_{MG} \quad \text{(A-76)}$$

$$\therefore FMG(8) = -M_H [r_o(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta) + \ddot{X}_{MG}] - FMG(7) - FBC(8) \quad \text{(A-77)}$$

FMG(7) and FBC(8) are defined elsewhere in this report.

Nominal values are M_H = .0683 lb and r_o = .75 in

A.2.32 FMG(9) Impact between buffer and main gun

This force occurs on the recoil stroke when the buffer strikes the backplate. A hypothetical spring is used to model this impact.

$$FMG(9) = -K_{\substack{BACK\\PLATE}} (X_{MG} - X_{BUFF} - \alpha) - C_{\substack{BACK\\PLATE}} (\dot{X}_{MG} - \dot{X}_{BUFF}) \quad \text{(A-78)}$$

This force is nonzero if: $X_{MG} - X_{BUFF} \geq \alpha$ and $FMG(9) < 0$.

Nominal values are:

α = 3.80 in

$K_{\substack{BACK\\PLATE}}$ = 60,000 lb/ft

$C_{\substack{BACK\\PLATE}}$ = 10.8 lb-sec/ft = 40% critical

NOTE: An equal and opposite force FBUFF(2) acts on the buffer.

A.2.33 FMG(10) = -FB(11) (See previous derivation)

A.2.34 FMG(11) Gravity force on main gun

$$FMG(11) = -M_{MG}\, g \sin\theta_E \quad \text{(A-79)}$$

θ_E = angle of elevation of the gun

M_{MG} is a function of the number of rounds in the magazine. With no ammunition, M_{MG} = 5.84 lb

A.2.35 FMG(12) = -FBC(10) (See previous derivation)

A.2.36 FMG(13) = -FBC(11) (See previous derivation)

A.2.37 FMG(14) Friction between buffer and main gun

A combination of Coulomb friction and viscous damping was assumed.

$$FMG(14) = -\mu_{BUFF} (M_{BUFF} + M_{TOTAL}) \cos \theta_E \text{ sgn } (\dot{X}_{MG}-\dot{X}_{BUFF})-C_{BUFF}(\dot{X}_{MG}-\dot{X}_{BUFF}) \quad \text{(A-80)}$$

M_{TOTAL} = mass of buffer weights = .224 lb

M_{BUFF} = effective mass of buffer tube = .146 lb (Includes $\frac{1}{3}$ mass of drive spring)

μ_{BUFF} = .2

C_{BUFF} = .01 lb-sec/ft

Forces Acting on the Buffer

A.2.38 FBUFF(1) = -FBC(6) (See previous derivation)

A.2.39 FBUFF(2) = -FMG(9) (See previous derivation)

A.2.40 FBUFF(3) Force from drive spring acting on buffer

$$FBUFF(3) = -K(X_{BUFF} - Y_1) - C(\dot{X}_{BUFF} - \dot{Y}_1) + \beta K_5 \quad \text{(A-81)}$$

See derivation of FMG(5)

A.2.41 FBUFF(4) Gravity force on buffer tube

This force acts continuously.

$$FBUFF(4) = -M_{BUFF} g \sin \theta_E \quad \text{(A-82)}$$

θ_E is the angle of elevation of the rifle

M_{BUFF} is the mass of the buffer tube.

A.2.42 FBUFF(5) Total buffer tube interaction with buffer weights.

This force arises from friction and from impact between the buffer tube and the end weights. Two methods of treating this force are presented. The first assumes that all the buffer weights are lumped together as a single mass. The second provides each mass with a separate degree of freedom.

LUMPED MASSES:

Consider the following idealization in Figure 41.

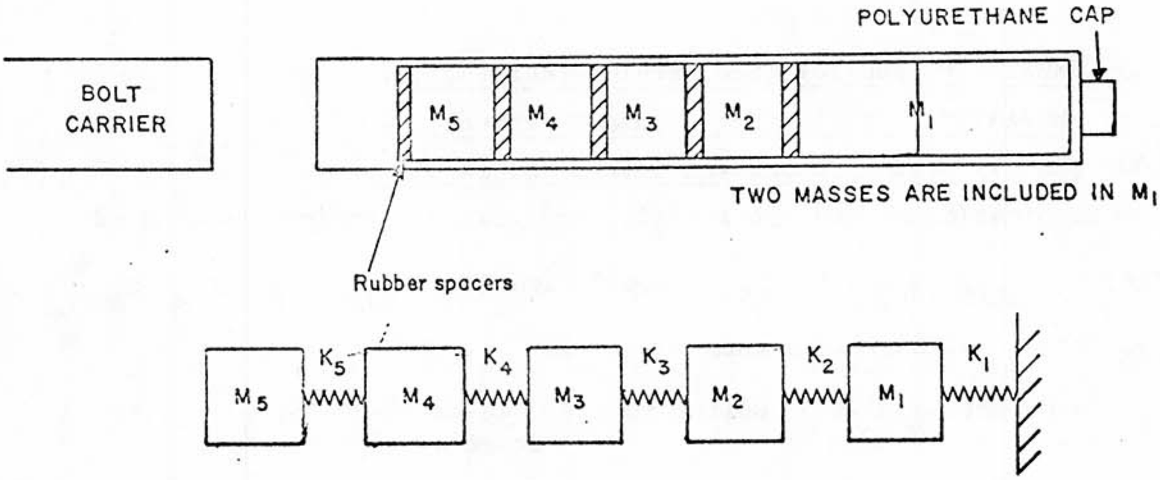
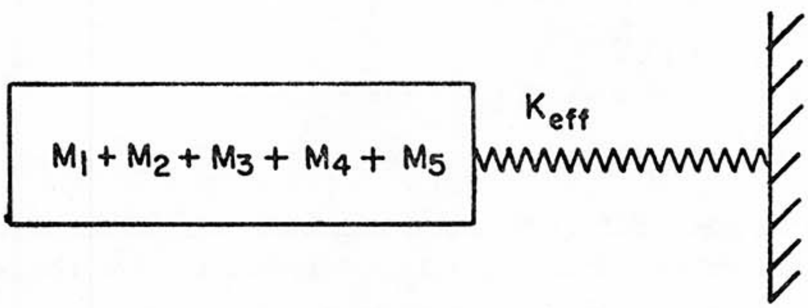


Figure 41 Schematic of Buffer Assembly

The equations of motion are:

$$M_1\ddot{X}_1 = -K_1X_1 + K_2(X_2-X_1)$$
$$M_2\ddot{X}_2 = -K_2(X_2-X_1) + K_3(X_3-X_2)$$
$$M_3\ddot{X}_3 = -K_3(X_3-X_2) + K_4(X_4-X_3)$$
$$M_4\ddot{X}_4 = -K_4(X_4-X_3) + K_5(X_5-X_4)$$
$$M_5\ddot{X}_5 = -K_5(X_5-X_4)$$

What is now sought is an effective spring constant for the following lumped system:



This effective spring constant will be chosen first by computation of lowest natural frequency of the five-mass system, and then by calculation of $K_{eff} = M_{TOTAL}\,\omega^2_{MIN}$. M_{TOTAL} is the total mass of the weights and spacers. ω_{MIN} is the lowest natural frequency, and K_{eff} is the effective spring constant of the system.

Now compute the lowest natural frequency. The equations of motion given above can be rewritten in matrix form as: $M\ddot{X} + KX = 0.$ (A-83)

$$M = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 \\ 0 & 0 & 0 & M_4 & 0 \\ 0 & 0 & 0 & 0 & M_5 \end{bmatrix} \quad \text{(A-84)}$$

$$K = \begin{bmatrix} K_1+K_2 & -K_2 & 0 & 0 & 0 \\ -K_2 & K_2+K_3 & -K_3 & 0 & 0 \\ 0 & -K_3 & K_3+K_4 & -K_4 & 0 \\ 0 & 0 & -K_4 & K_4+K_5 & -K_5 \\ 0 & 0 & 0 & -K_4 & K_5 \end{bmatrix} \quad \text{(A-85)}$$

$$X = (X_1, X_2, X_3, X_4, X_5)^T \quad \text{(A-86)}$$

Assume the following solution to the matrix equations: $X = X_o \sin \omega t$
Substitution into the equation then yields $\omega^2 MX_o = KX_o$

Let $\lambda \equiv \frac{1}{\omega^2}$. Then $\lambda X_o = K^{-1} M X_o$. The solution of this matrix equation is an eigenvalue problem. The largest eigenvalue λ_{MAX}, which corresponds to ω_{MIN}, is obtained by a standard numerical technique known as the power method. Then $K_{eff} = \frac{M_{TOTAL}}{\lambda_{MAX}}$. For purposes of calculation, all springs are assumed to act simultaneously. When the masses strike the forward end of the buffer body, $m_1 = m_2 = m_3 = m_4 = .0406$ lb, $m_5 = .0615$ lb, $K_1 = K_2 = K_3 = K_4 = K_5 = K_6 = 60{,}000$ lb/ft

Then

$\omega_{MIN} = 1801$ cps, and $K_{eff}^{(F)} = 22{,}567$ lb/ft.

When the masses strike the rearward end of the buffer tube, $m_1 = .0615$ lb, $m_2 = m_3 = m_4 = m_5 = .0406$ lb, $K_1 = K_2 = K_3 = K_4 = K_5 = 60{,}000$ lb/ft.

Then

$\omega_{MIN} = 1944$ cps, and $K_{eff}^{(R)} = 26{,}405$ lb/ft

Let X_{WTS} denote the position of the center of mass of the buffer masses. X_{WTS} is zero when the weights are fully forward. The following friction force exists between the buffer masses and the buffer tube:

$$F_{FRICTION} = -\mu_{WTS} M_{TOTAL} g \cos \theta_E \text{ sgn } (\dot{X}_{WTS} - \dot{X}_{BUFF})$$

μ_{WTS} is the coefficient of friction and θ_E is the angle of elevation of the rifle.

Let superscripts (R) and (F) denote rearward and forward impact, respectively.

Let X_s denote the total amount of play between the buffer weights and the ends of the buffer tube.

Then

$$FBUFF(5) = F_{FRICTION} + K_{eff}^{(F)} (X_{WTS} - X_{BUFF}) + C_{eff}^{(F)} (\dot{X}_{WTS} - \dot{X}_{BUFF}) \quad (A-87)$$

If $X_{WTS} - X_{BUFF} \geq 0$

$$FBUFF(5) = F_{FRICTION} + K_{eff}^{(R)} (X_{WTS} - X_{BUFF} + X_s) + C_{eff}^{(R)} (\dot{X}_{WTS} - \dot{X}_{BUFF}) \quad (A-88)$$

If $X_{WTS} - X_{BUFF} \leq -X_s$

$$X_s = .13 \text{ in}$$

Nominal values are: $\mu_{WTS} = .2$

$C_{eff}^{(F)} = 6.44$ lb-sec/ft = 33% critical

$C_{eff}^{(R)} = 7.56$ lb-sec/ft = 33% critical

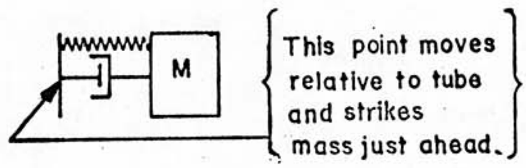
In the solution of the equations of motion involving X_{WTS}, the force acting on the lumped mass is then

$$F_{WTS} = -FBUFF(5) - M_{TOTAL} g \sin \theta_E$$

This lumped mass approach appears as an option in the computer program.

Individual Masses

In this approach, the motion of each mass must be considered to predict the motions of the end masses, by which the force input to the buffer body is controlled. Five systems, each consisting of a mass and a rubber spacer, are assumed to be free to move within the buffer body. These systems are not joined to each other as in the case of the lumped-mass analysis. Schematically, each system is idealized as:



and appears in the buffer body as shown in Figure 42.

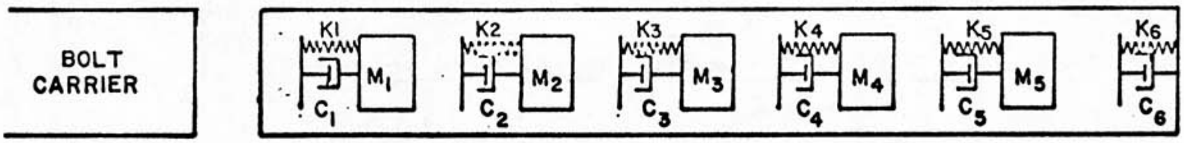


Figure 42 Schematic of Buffer Model

In this idealization, the metal spacer in the body is assumed to be joined to one of the buffer weights. Six masses could be used if a very stiff spring were assumed to act between this metal spacer and the adjacent buffer weight. However, the stiff spring would require the use of a very small integration interval, and the slight increase in realism would not be worth the increase in computation time.

The buffer weights also exert force on the buffer tube through friction. The total force on the tube from the weights is then:

$$FBUFF(5) = +\sum_{i=1}^{5} \mu_i M_i\, g\, \mathrm{sgn}(\dot{X}_i - \dot{X}_{BUFF}) \cos\theta_E + K_1 (X_1 - X_{BUFF}) + C_1 (\dot{X}_1 - \dot{X}_{BUFF}) \quad \text{(A-89)}$$

If $X_1 - X_{BUFF} \geq 0$

$$FBUFF(5) = -\sum_{i=1}^{5} \mu_i M_i\, g\, \mathrm{sgn}(\dot{X}_i - \dot{X}_{BUFF}) \cos\theta_E + K_6 (X_5 - X_{BUFF} + X_s) + C_6 (\dot{X}_5 - \dot{X}_{BUFF}) \quad \text{(A-90)}$$

If $X_5 - X_{BUFF} + X_s \leq 0$

X_{BUFF} is the position of the center of mass of the buffer tube,

μ_i is the coefficient of friction between the ith mass and the buffer tube,

θ_E is the angle of elevation of the rifle, and

g is the acceleration of gravity.

Nominal values for spring constants, masses, and damping coefficients are:

$M_1 = M_2 = M_3 = M_4 = .0406$ lb, $M_5 = .0615$ lb,

$C_1 = C_2 = C_3 = C_4 = C_5 = 12$ lb-sec/ft

$K_1 = K_2 = K_3 = K_4 = K_5 = K_6 = 60{,}000$ lb/ft

This individual-mass formulation is the one normally used in the computer program.

A.2.43 FBUFF(6) = -FMG(14) (See previous derivation)

Forces Acting on the Buffer Weights

A.2.44 F_{WTS} force acting on the lumped mass

This force is not used in the individual mass approach. Therefore, it is used only as an option. Part of this force is caused by friction, and part is due to impact of the lumped mass with the buffer body. The remainder is caused by gravity. $F_{WTS} = -FBUFF(5)' - M_{TOTAL}\, g \sin \theta_E$

A.2.45 $FM_{i,1}$ (i = 1,---5) force acting on individual buffer mass due to spring just forward of a mass

These forces do not occur when the lumped-mass approach is used. These spring forces act only when adjacent masses are in contact. In writing the equations, use of the following asymmetrical unit-step function is helpful:

$$U_+(X) = \begin{cases} 0 \text{ for } X \le 0 \\ 1 \text{ for } X > 0 \end{cases}$$

Then define:

$$F_i \equiv [K_{i+1}(X_{i+1} - X_i) + C_{i+1}(\dot{X}_{i+1} - \dot{X}_i)]\, U_+(X_{i+1} - X_i) \qquad \text{(A-91)}$$

$$i = 1,2,3,4$$

$$F_o \equiv [K_1(X_1 - X_{BUFF}) + C_1(\dot{X}_1 - \dot{X}_{BUFF})]\, U_+(X_1 - X_{BUFF}) \qquad \text{(A-92)}$$

$$F_5 \equiv [K_6(X_{BUFF} - X_5 - X_s) + C_6(\dot{X}_{BUFF} - \dot{X}_5)]\, U_+(X_{BUFF} - X_5 - X_s) \qquad \text{(A-93)}$$

$FM_{i,1} = -F_{i-1}$ i, 1,2,...5

X_s = "play" between buffer weights and tube in longitudinal direction

X_{BUFF} = position of C.G. of buffer tube.

A.2.46 $FM_{i,2}$ Force acting on individual buffer mass due to spring just rearward of mass

This force does not occur in the lumped mass approach.

$FM_{i,2} = F_i \quad i = 1,2,\ldots 5$ See A.2.45 for definition of F_i. (A-94)

(The subscript "i" refers to the ith mass.)

A.2.47 $FM_{i,3}$ Friction between buffer tube and buffer weight

This force does not occur in the lumped-mass approach.

$$FM_{i,3} = -\mu_i M_i g \cos\theta_E \operatorname{sgn}(\dot{X}_i - \dot{X}_{BUFF}) \qquad \text{(A-95)}$$

$$i = 1,2,3,4,5$$

μ_i is the coefficient of friction between the ith mass and the buffer tube.

θ_E is the angle of elevation of the rifle.

A.2.48 $FM_{i,4}$ Gravity force

This force does not occur in the lumped-mass approach.

$$FM_{i,4} = -M_i g \sin\theta_E \qquad \text{(A-96)}$$

$$i = 1,2,3,4,5$$

Torques That Affect Rifle Rotation

A.2.49 $T_{SHOULDER}$, T_{BODY}: Shooter's body applies torques that both increase and decrease rifle rotation

The model developed to account for rotational motion allows only one-way coupling between axial and rotational motion. That is, the forces acting parallel to the bore axis are allowed to affect the rotation of the rifle, but rotational motion is not allowed to affect axial motion. When the present M16 Rifle math model is joined with the man model under development, complete coupling will occur between axial and rotational motions.

In this model, the effect of the shooter's body is described by torsional and linear springs, and dashpots as shown in Figure 43.

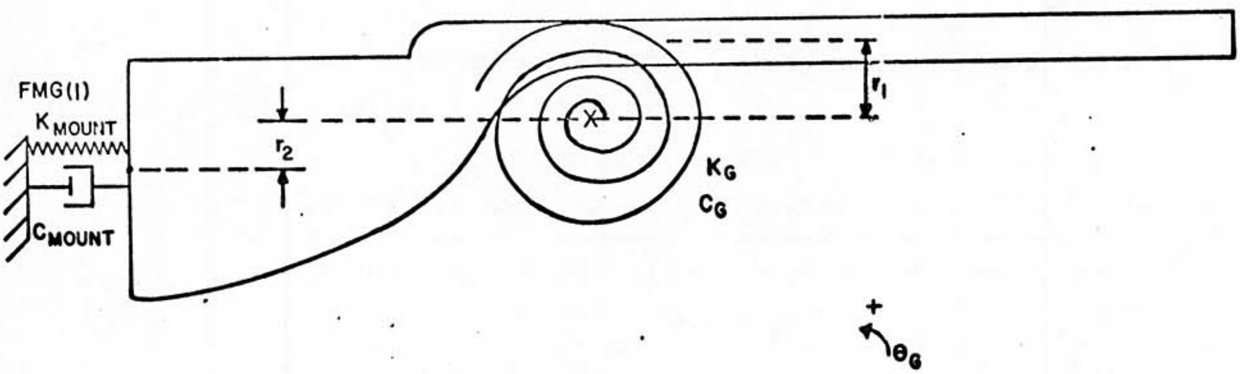


Figure 43 Schematic of Simple Man-Weapon Interaction Model

r_1 = distance from centerline of barrel to center

r_2 = distance from butt plate center to center of gravity of rifle

K_θ = effective torsional spring constant

C_θ = effective torsional damping constant

I_G = moment of inertia of rifle about a transverse axis through the center of gravity

θ_G = angle of elevation of barrel

The spring and dashpot at the butt are designed to approximate resistance from the shooter's shoulder. This resistance tends to cause rotation. The force acting through moment arm r_2 is identical to FMG(1). $T_{SHOULDER} = -r_2$ FMG(1).

The torsional spring and dashpot approximate the restoring moment exerted by the shooter's body in response to pitch motion. It includes the effects of his hands, arms, and upper torso. The values for K_G and C_G were selected as the same used for the XM19 Rifle model. These values lead to dispersion in bullet impact that accurately reflects what is found by experimental firings. That these values work well for two such diverse weapons is an indication that they constitute a valid approximation. They are not necessarily the only values that will give the proper dispersion, but they are physically reasonable.

$$T_{BODY} = K_G \theta_G - C_G \dot{\theta}_G \quad \text{(A-97)}$$

Nominal values are: r_2 = .34 in

K_G = 50 ft-lb/rad

C_G = 2.83 ft-lb-sec/rad

I_G = .135 slug-ft^2

K_{MOUNT} = 300 lb/ft

C_{MOUNT} = 9.53 lb-sec/ft

A.2.50 $T_{INTERNAL\ GUN\ MOTION}$: Motion of internal operating parts tends to increase rifle rotation

These forces are assumed to act along the centerline of the barrel. They consist of all forces on the main gun except the mount force FMG(1). The force names are: FMG(2), FMG(3), FMG(4), FMG(5), FMG(6), FMG(7), FMG(8), FMG(9), FMG(10), FMG(11), FMG(12), FMG(13), FMG(14). The moment arm through which they act is r_1 = .78 inch. From Newton's Second Law, the sum of these forces is $\ddot{X}_{MG}$ (M_{MG}) -FMG(1).

$$T_{INTERNAL\ GUN\ MOTION} = r_1 \left[M_{MG} \ddot{X}_{MG} - FMG(1)\right] \quad \text{(A-98)}$$

M_{MG} = mass of main gun = 6.2 -.0252 (N -19) lb

N = number of rounds of ammo in rifle at any given time

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APPENDIX B

SOLUTION OF EQUATIONS OF MOTION

In solving the equations of motion by the Runge-Kutta technique, one must put the equations in the form $\ddot{X} = f(y,x,\dot{x})$. Thus, second derivatives must be found explicitly. When constraint forces act, the equations for these second derivatives are coupled, and algebraic simultaneous equations must be solved. Otherwise, the second derivatives can be found explicitly by the simple rearrangement of terms in the equations. Cases where simultaneous equations must be solved to find second derivatives are described below. Cases in which no coupling of second derivatives exists, and in which second derivatives can be found by equation rearrangement will not be discussed.

B.1 Coupling between $\ddot{X}_{MG}$ and $\ddot{\theta}$ due to constraint force FMG(8) between hammer and gun

This force acts at all times.

$$M_{MG}\ddot{X}_{MG} = \sum_{\substack{i=1\\(i\neq 8)}}^{15} FMG(i) + \left\{-M_H\left[r_0\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right] - FMG(7) - FBC(8)\right\} \quad \text{(B-1)}$$

$$I_H\ddot{\theta} = T_{SPRING} - r_2\cos(\alpha_2+\theta)\,FMG(7) - M_H\ddot{X}_{MG}(\cos\theta)\,r_0 - r\cos(\theta+\alpha)\,FBC(8) \quad \text{(B-2)}$$

$$\ddot{X}_{MG} = \left[M_{MG} + M_H - \frac{M_H^2\,r_0^2\cos^2\theta}{I_H}\right]^{-1}\left\{\sum_{\substack{i=1\\(i\neq 7\,\mathrm{or}\,8)}}^{15} FMG(i) - M_H\left[r_0\left(\left(T_{SPRING} - r_2\cos(\theta+\alpha_2)\,FMG(7) - r\cos(\alpha+\theta)\,FBC(8)\right)\cos\theta\,I_H^{-1} - \dot{\theta}^2\sin\theta\right)\right] - FBC(8)\right\} \quad \text{(B-3)}$$

$\ddot{\theta}$ can be found by substituting $\ddot{X}_{MG}$ into (B-2) and rearranging terms.

B.2 Interaction among $\ddot{X}_B$, $\ddot{X}_{BC}$, and $\ddot{X}_{MG}$ while bolt and main gun are locked together after locking

$$\begin{cases} \ddot{X}_B = \ddot{X}_{MG} \\ M_{MG}\,\ddot{X}_B = \sum_{i=1}^{15} FMG(i) + F_{con} \\ M_{B(E)}\,\ddot{X}_B = \sum_{i=1}^{12} FB(i) - F_{con} \end{cases}$$

$$\ddot{X}_B = \ddot{X}_{MG} = \left\{ \sum_{i=1}^{12} FB(i) + \sum_{\substack{i=1 \\ (i\neq 7 \text{ or } 8)}}^{15} FMG(i) - FBC(8) - M_H \left[r_o (\theta_T \cos\theta - \dot{\theta}^2 \sin\theta) \right] \right\}$$

$$\times \left[M_H + M_{MG} - \frac{M_H^{\;2}\, r_o^{\;2} \cos^2\theta}{I_H} \right]^{-1} \qquad \text{(B-4)}$$

where $\theta_T \equiv [T_{SPRING} - r_2 \cos(\alpha_2 + \theta)\ FMG(7) - r \cos(\alpha + \theta)\ FBC(8)]\, I_H^{-1}$ (B-5)

B.3 Coupling between $\ddot{X}_B$ and $\ddot{X}_{BC}$ during the process of locking

$$\begin{cases} M_{B(E)}\ddot{X}_B = \sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i) + \left[\dfrac{d\theta_B}{d(X_B - X_{BC})} \right]^2 I_B (\ddot{X}_{BC} - \ddot{X}_B) & \text{(B-6)} \\ M_{BC}\ddot{X}_{BC} = \sum_{\substack{i=1 \\ i\neq 5}}^{12} FBC(i) - \left[\dfrac{d\theta_B}{d(X_B - X_{BC})} \right]^2 I_B (\ddot{X}_{BC} - \ddot{X}_B) & \text{(B-7)} \end{cases}$$

$$\ddot{X}_B = \left\{ M_{BC} \sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i) + \left[\frac{d\theta_B}{d(X_B - X_{BC})} \right]^2 I_B \left(\sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i) + \sum_{\substack{i=1 \\ i\neq 5}}^{12} FBC(i) \right) \right\} \left\{ M_{B(E)} M_{BC} + \left[\frac{d\theta_B}{d(X_B - X_{BC})} \right]^2 I_B (M_{B(E)} + M_{BC}) \right\}^{-1} \qquad \text{(B-8)}$$

$$\ddot{X}_{BC} = \left\{ M_{B(E)} \sum_{\substack{i=1 \\ i\neq 5}}^{12} FBC(i) + \left[\frac{d\theta_B}{d(X_B - X_{BC})} \right]^2 I_B \left(\sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i) + \sum_{\substack{i=1 \\ i\neq 5}}^{12} FBC(i) \right) \right\} \left\{ M_{B(E)} M_{BC} + \left[\frac{d\theta_B}{d(X_B - X_{BC})} \right]^2 I_B (M_{B(E)} + M_{BC}) \right\}^{-1} \qquad \text{(B-9)}$$

B.4 Coupling between $\ddot{X}_B$ and $\ddot{X}_{MG}$ while the bolt is locked into the main gun, but before the locking process is completed

$$\ddot{X}_B = \ddot{X}_{MG} \quad \text{(B-10)}$$

$$\left[M_{MG} + M_H - \frac{M_H{}^2 r_o{}^2 \cos^2\theta}{I_H}\right] \ddot{X}_{MG} = \sum_{\substack{i=1 \\ i\neq 7 or 8}}^{15} FMG(i) - M_H r_o(\theta_T \cos\theta - \dot{\theta}^2 \sin\theta) - FBC(8) + F_{con} \quad \text{(B-11)}$$

$$M_{B(E)}\ddot{X}_B = \sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i) + \left[\frac{d\theta_B}{d(X_B - X_{BC})}\right]^2 I_B(\ddot{X}_{BC} - \ddot{X}_B) - F_{con} \quad \text{(B-12)}$$

$$M_{BC}\ddot{X}_{BC} = \sum_{i=1}^{12} FBC(i) - \left[\frac{d\theta_B}{d(X_B - X_{BC})}\right]^2 I_B(\ddot{X}_{BC} - \ddot{X}_B) \quad \text{(B-13)}$$

$$\ddot{X}_{MG} = \ddot{X}_B \quad \text{(B-14)}$$

The equation resulting from the addition of (B-11), (B-12) and (B-14) have the same form as (B-6) and (B-7) if M_B is replaced by

$$\left(M_{MG} + M_{B(E)} + M_H - \frac{M_H{}^2\ r_o{}^2\ \cos^2\ \theta}{I_H}\right) \text{ and } \sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i) \text{ is replaced by}$$

$$\sum_{\substack{i=1 \\ (i\neq 7 or 8)}}^{15} FMG(i) - M_H\ r_o\ (\theta_T \cos\theta - \dot{\theta}^2 \sin\theta) - FBC(8) + \sum_{\substack{i=1 \\ i\neq 7}}^{12} FB(i)$$

Hence, $\ddot{X}_B$ and $\ddot{X}_{BC}$ can be found.

B.5 Interaction among $\ddot{\theta}$, $\ddot{X}_{BC}$, $\ddot{X}_{MG}$ during cocking while the hammer and bolt-carrier are constrained to move together

$$M_{BC}\ \ddot{X}_{BC} = \sum_{\substack{i=1 \\ i\neq 8}}^{12} FBC(i) + FBC(8) \quad \text{(B-15)}$$

$$M_{MG}\ \ddot{X}_{MG} = \sum_{\substack{i=1 \\ (i\neq 7 or 8)}}^{15} FMG(i) - M_H\ [\ r_o\ (\ddot{\theta} \cos\theta - \dot{\theta}^2 \sin\theta) + \ddot{X}_{MG}] - FBC(8) \quad \text{(B-16)}$$

$$I_H \ddot{\theta} = T_{SPRING} - r\ FBC(8) \cos(\theta + \alpha) - M_H \ddot{X}_{MG} (\cos\theta)\ r_o \tag{B-17}$$

The constraint equation and its derivatives are:

$$\theta = f(X_{MG} - X_{BC}) = -\alpha_1 + \tan^{-1}\left[\frac{r_1 \sin\alpha_1 + (X_{BC} - X_{MG})}{r_1 \cos\alpha_1}\right] \tag{B-18}$$

$$\dot{\theta} = f'(X_{MG} - X_{BC})(\dot{X}_{MG} - \dot{X}_{BC}) \tag{B-19}$$

$$\ddot{\theta} = f''(X_{MG} - X_{BC})(\dot{X}_{MG} - \dot{X}_{BC})^2 + f'(X_{MG} - X_{BC})(\ddot{X}_{MG} - \ddot{X}_{BC}) \tag{B-20}$$

where $f'(X_{MG} - X_{BC}) = -\left\{\left[1 + \left(\frac{r_1 \sin\alpha_1 + X_{BC} - X_{MG}}{r_1 \cos\alpha_1}\right)^2\right] r_1 \cos\alpha_1\right\}^{-1}$

$$f''(X_{MG} - X_{BC}) = -2(r_1 \sin\alpha_1 + X_{BC} - X_{MG})\left\{\left[1 + \left(\frac{r_1 \sin\alpha_1 + X_{BC} - X_{MG}}{r_1 \cos\alpha_1}\right)^2\right]^2 r_1^3 \cos\alpha_1^3\right\}^{-1}$$

From (12) $FBC(8) = [T_{SPRING} - I_H \ddot{\theta} - M_H r_o (\cos\theta) \ddot{X}_{MG}][r \cos(\theta + \alpha)]^{-1}$ (B-21)

Substitution of (B-20) into (B-21) and the result into (B-15) yields:

$$\left[M_{BC} - \frac{I_H f'(X_{MG} - X_{BC})}{r\cos(\alpha+\theta)}\right]\ddot{X}_{BC} + \left[\frac{I_H f'(X_{MG} - X_{BC}) + M_H r_o \cos\theta}{r\cos(\alpha+\theta)}\right]\ddot{X}_{MG} \tag{B-22}$$

$$= \sum_{\substack{i=1 \\ i\neq 8}}^{12} FBC(i) + \frac{T_{SPRING}}{r\cos(\theta+\alpha)} - \frac{I_H f''(X_{MG} - X_{BC})(\dot{X}_{MG} - \dot{X}_{BC})^2}{r\cos(\alpha+\theta)}$$

Substitution of (B-21) into (B-16) yields:

$$\left[\frac{I_H f'(X_{MG} - X_{BC})}{r\cos(\alpha+\theta)} - M_H f'(X_{MG} - X_{BC}) r_o \cos\theta\right]\ddot{X}_{BC} \tag{B-23}$$

$$+\left[-\frac{M_H^2 r_o^2 \cos^2\theta}{I_H} + M_{MG} + f'(X_{MG} - X_{BC}) M_H r_o \cos\theta + M_H - \frac{I_H f'(X_{MG} - X_{BC}) - M_H r_o \cos\theta}{r\cos(\alpha+\theta)}\right]\ddot{X}_{MG}$$

$$= \sum_{\substack{i=1 \\ i\neq 7 or 8}}^{15} FMG(i) + M_H r_o \dot{\theta}^2 \sin\theta - \frac{T_{SPRING}}{r\cos(\alpha+\theta)} + \frac{I_H f''(X_{MG} - X_{BC})(\dot{X}_{MG} - \dot{X}_{BC})^2}{r\cos(\alpha+\theta)}$$

$$- M_H r_o \cos\theta\ f''(X_{MG} - X_{BC})(\dot{X}_{MG} - \dot{X}_{BC})^2$$

Thus, there are two equations (B-22) and (B-23), and two unknowns $\ddot{X}_{BC}$ and $\ddot{X}_{MG}$.

The equations are linear and of the form:

$$\begin{cases} T_1 \ddot{X}_{BC} + T_2 \ddot{X}_{MG} = T_3 & \text{(B-24)} \\ U_1 \ddot{X}_{BC} + U_2 \ddot{X}_{MG} = U_3 & \text{(B-25)} \end{cases}$$

$$\therefore \quad \ddot{X}_{BC} = \frac{T_3 U_2 - T_2 U_3}{T_1 U_2 - U_1 T_2} \tag{B-26}$$

$$\ddot{X}_{MG} = \frac{T_1 U_3 - U_1 T_3}{T_1 U_2 - U_1 T_2} \tag{B-27}$$

B.6 <u>Interaction among $\ddot{\theta}$, $\ddot{X}_{BC}$, $\ddot{X}_{MG}$, $\ddot{X}_B$ during cocking while the bolt is locked into the main gun and the hammer is constrained to move with the bolt-carrier</u>

$$\begin{cases} M_{B(E)} \ddot{X}_B = \sum_{i=1}^{12} FB(i) + F_{CON} & \text{(B-28)} \\ \ddot{X}_B = \ddot{X}_{MG} & \text{(B-29)} \\ T_1 \ddot{X}_{BC} + T_2 \ddot{X}_{MG} = T_3 & \text{(B-30)} \\ U_1 \ddot{X}_{BC} + U_2 \ddot{X}_{MG} = U_3 - F_{CON} & \text{(B-31)} \end{cases}$$

F_{CON} is a constraint force that acts between the bolt and main gun to prevent relative motion between them.

Add (B-30) and (B-24) to eliminate F_{CON}, then the equations to be solved are:

$$\begin{cases} U_1 \ddot{X}_{BC} + (U_2 + M_{B(E)}) \ddot{X}_{MG} = U_3 + \sum_{i=1}^{12} FB(i) & \text{(B-32)} \\ T_1 \ddot{X}_{BC} + T_2 \ddot{X}_{MG} = T_3 & \text{(B-33)} \\ \ddot{X}_B = \ddot{X}_{MG} & \text{(B-34)} \end{cases}$$

The first two equations are of the form (B-24) and thus have similar solutions. The third equation establishes $\ddot{X}_B$.

B.7 Interaction among $\ddot{X}_{MG}$, $\ddot{X}_{BC}$, $\ddot{X}_B$, $\ddot{\theta}$ while the hammer is cocking; the hammer and bolt-carrier are constrained to move together; unlocking is taking place, but the bolt is still locked into the main gun

This case is a combination of cases B.5 and B.6.

The equation for the bolt is:

$$M_B \ddot{X}_B = \sum_{\substack{i=1 \\ i\neq 6}}^{12} FB(i) + \left[\frac{d\theta_B}{d(X_B-X_{BC})}\right]^2 I_B(\ddot{X}_{BC}-\ddot{X}_B) + \frac{d\theta_B}{d(X_B-X_{BC})} \frac{2}{3} \mu_c \frac{r_2^{\,3}-r_1^{\,3}}{r_2^{\,2}-r_1^{\,2}} F_N + F_N \quad \text{(B-35)}$$

where M_B is the mass of the bolt alone.

The negative of FB(6) must now appear in the equation for the bolt-carrier

$$T_1\ddot{X}_{BC} + T_2\ddot{X}_{MG} = T_3 - \left[\frac{d\theta_B}{d(X_B-X_{BC})}\right]^2 I_B(\ddot{X}_{BC}-\ddot{X}_B) - \frac{d\theta_B}{d(X_B-X_{BC})} \frac{2}{3} \mu_c \frac{r_2^{\,3}-r_1^{\,3}}{r_2^{\,2}-r_1^{\,2}} F_N \quad \text{(B-36)}$$

NOTE: $\ddot{\theta}$ has already been eliminated in this equation.

For convenience in eliminating F_N, one can consider that the cartridge case is now part of the main gun rather than the bolt. The equation for the main gun becomes:

$$U_1 \ddot{X}_{BC} + (U_2 + M_c)\ddot{X}_{MG} = U_3 - F_N. \quad \text{(B-37)}$$

NOTE: $\ddot{\theta}$ has already been eliminated in this equation.

The equations can be summarized as follows:

$$\left\{ M_B + \left[\frac{d\theta_B}{d(X_B-X_{BC})}\right]^2 I_B \right\}\ddot{X}_B - \left\{\left[\frac{d\theta_B}{d(X_B-X_{BC})}\right]^2 I_B\right\}\ddot{X}_{BC} = \sum_{\substack{i=1 \\ i\neq 6}}^{12} + \frac{d\theta_B}{d(X_B-X_{BC})}\frac{2}{3}\mu_c \frac{r_2^{\,3}-r_1^{\,3}}{r_2^{\,2}-r_1^{\,2}} F_N + F_N$$

$$-\left\{\left[\frac{d\theta_B}{d(X_B-X_{BC})}\right]^2 I_B\right\}\ddot{X}_B + \left\{T_1 + \left[\frac{d\theta_B}{d(X_B-X_{BC})}\right]^2 I_B\right\}\ddot{X}_{BC} + T_2\ddot{X}_{MG} = T_3 - \frac{d\theta_B}{d(X_B-X_{BC})} \frac{2}{3} \mu_c \frac{r_2^{\,3}-r_1^{\,3}}{r_2^{\,2}-r_1^{\,2}} F_N \quad \text{(B-38)}$$

$$U_1 \ddot{X}_{BC} + (U_2 + M_c)\ddot{X}_{MG} = U_3 - F_N \quad \text{(B-39)}$$

$$\ddot{X}_{MG} = \ddot{X}_B \quad \text{(B-40)}$$

To condense the equations, define the following:

$$K_1 \equiv \left[\frac{d\theta_B}{d(X_B - X_{BC})} \right] I_B$$

$$K_2 \equiv \frac{d\theta_B}{d(X_B - X_{BC})} \; \frac{2}{3} \mu_c \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$$

The equations become:

$$[M_B + K_1] \ddot{X}_B - K_1 \ddot{X}_{BC} = \sum_{\substack{i=1 \\ i \neq 6}}^{12} FB(i) + K_2 F_N + F_N \qquad \text{(B-41)}$$

$$- K_1 \ddot{X}_B + [T_1 + K_1] \ddot{X}_{BC} + T_2 \ddot{X}_{MG} = T_3 - K_2 F_N \qquad \text{(B-42)}$$

$$U_1 \ddot{X}_{BC} + (U_2 + M_c) \ddot{X}_{MG} = U_3 - F_N \qquad \text{(B-43)}$$

$$\ddot{X}_{MG} = \ddot{X}_B \qquad \text{(B-44)}$$

Now eliminate F_N:

Multiply (B-43) by $(K_2 + 1)$ and add the result to (B-41).

$$[-K_1 + U_1 (K_2 + 1)] \ddot{X}_{BC} + [M_B + K_1 + (K_2 + 1)(U_2 + M_c)] \ddot{X}_{MG} \qquad \text{(B-45)}$$

$$= \sum_{\substack{i=1 \\ i \neq 6}}^{12} FB(i) + [1 + K_2] U_3$$

Next, multiply (B-43) by (K_2) and subtract the result from (B-42).

$$[T_1 + K_1 - U_1 K_2] \ddot{X}_{BC} + [T_2 - K_1 - K_2 (U_2 + M_c)] \ddot{X}_{MG} = T_3 - U_3 K_2 \qquad \text{(B-46)}$$

$\ddot{X}_{BC}$ and $\ddot{X}_{MG}$ can be found by solving (B-45) and (B-46) simultaneously.

$\ddot{X}_B = \ddot{X}_{MG}$ determines $\ddot{X}_B$.

$\ddot{\theta}$ can then be found from (B-17).

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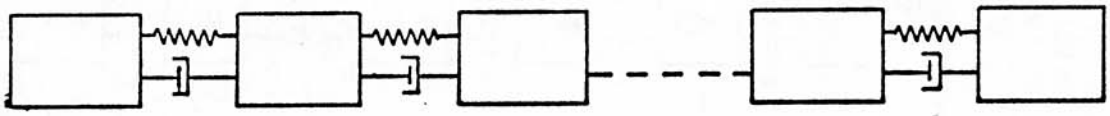
APPENDIX C

SPRING-SURGING AND COIL-CLASHING ANALYSES

Many phenomena occur in weapon operation that can be modeled as a series of masses connected to ideal springs and dashpots in the following manner:



For example, instead of making the common assumption that one mass upon impact with another encounters an ideal stiff spring, one can account for the wave-like propagation of stresses by assuming that the body encounters the spring system shown above. The body being impacted is thus assumed to be composed of many masses separated by springs with stiffness based on Young's modulus. With proper selection of masses, springs, and dampers, one could better approximate impact and include higher modes of oscillation.

As an alternative to using linear springs for physical simulation of helicopters and other vehicles as firing platforms, one could include higher modes of oscillation and possibly more closely duplicate the response of the actual system to firing loads. For a helicopter simulation, one would need one set of springs for each mode of vehicle flight such as hovering, diving, etc.

One can also use this spring-mass system to represent the action of rapidly compressed springs in the philosophy of the finite-element or lumped-mass concept. Each point mass could be considered to be one coil mass, although several coils could be combined and considered, in a rough approximation, as one point mass. Spring constants could be set so as to force the overall system to have the same spring rate under static conditions as that of the actual spring. Damping coefficients could be set so as to make the overall damping equivalent to the best estimate for the actual spring.

For all the above-listed applications, including the spring, the governing equation for the ith mass, where "i" denotes neither the first or last mass is:

$$M\ddot{X}_i + 2C\,\dot{X}_i + 2K\,X_i = K(X_{i-1} + X_{i+1}) + C(\dot{X}_{i-1} + X_{i+1})\,. \qquad \text{(C-1)}$$

In this formulation, mass, spring, and damping values do not change along the spring.

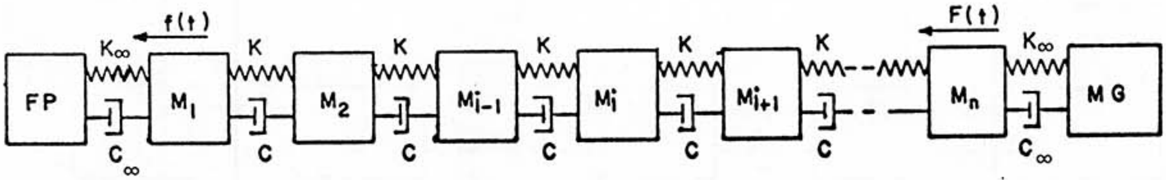


Figure 44 Drive Spring Model

Consider the following drive spring. The main gun is on one end, and the firing pin is on the other. The equations are:

$$M\ddot{X}_1 = -[X_1 - X_2 - (\Delta-\delta)]\,K - (\dot{X}_1 - \dot{X}_2)\,C + f(t) \qquad \text{(C-2)}$$

$$M\ddot{X}_i = -(X_i - X_{i+1})\,K - (\dot{X}_i - \dot{X}_{i+1})\,C - (X_i - X_{i-1})\,K - C\,(\dot{X}_i - \dot{X}_{i-1}) \qquad \text{(C-3)}$$

$$M\ddot{X}_n = [X_{n-1} - X_n - (\Delta-\delta)]\,K + C\,(\dot{X}_{n-1} - \dot{X}_n) + F(t) \qquad \text{(C-4)}$$

X's are measured with respect to ground at $t = 0$ for a preloaded spring.

The free length of each spring is Δ = (Free length of total spring)/(Number of coils + 1).

The length of each spring at $t = 0$ is δ = (Initial length of total spring)/(Number of coils + 1).

The length at any t is $\delta + X_j - X_{j+1}$ for a spring located between mass j and mass j+1.

The force exerted on masses j and j+1 by the spring located between them is $[\Delta - \delta - X_i + X_{i+1}]\,K_{TOT}(n+1)$ where K_{TOT} is constant for the entire spring and n is the number of coils.

Now

$$f(t) = K_\infty\,[X_{FP} - X_{FP_o} - X_1 - (\Delta_\infty - \delta_\infty)] + C_\infty\,(\dot{X}_{FP} - \dot{X}_1).$$

Set $(\Delta_\infty - \delta_\infty)$ such that the net force on M_1 is zero at $t = 0$.

Then

$$K_\infty\ [-\Delta_\infty + \delta_\infty] = (\Delta-\delta)\ K$$

$$-\ \Delta_\infty + \delta_\infty = \frac{K}{K_\infty}\ (\Delta-\delta)$$

$$f(t) = K_\infty\ [X_{FP} - X_{FP_o} - X_1 - \frac{K}{K_\infty}\ (\Delta-\delta)] + C_\infty\ [\dot{X}_{FP} - \dot{X}_1] \qquad \text{(C-5)}$$

Now $F(t) = K_\infty\ [X_{MG}-X_{MG_o}-X_n + (\Delta'_\infty\ \delta'_\infty)] + C_\infty\ [\dot{X}_{MG}-\dot{X}_n]$

Set $[\Delta'_\infty - \delta'_\infty]$ such that the net force on M_n is zero at t=o.

$$K(\Delta-\delta) = K_\infty\ [\Delta'_\infty - \delta'_\infty]$$

$$\Delta'_\infty - \delta'_\infty = \frac{K}{K_\infty}\ [\Delta-\delta]$$

$$F(t) = K_\infty\ [X_{MG}-X_{MG_o}-X_n + \frac{K}{K_\infty}\ (\Delta-\delta)] + C_\infty\ [\dot{X}_{MG}-\dot{X}_n] \qquad \text{(C-6)}$$

NOTE: f(t) is the force exerted by FP on M_1, and F(t) is the force exerted by MG on M_n.

In many instances, one is not interested in the details of the motions for each mass, but rather in how the first and last masses respond to input signals. Thus, for many applications, it would be very useful and economical to use a transfer function for the system, if such a ratio of output to input could be found. Laplace transforms were applied to the equations, but no inverse was found. Because this approach still seems potentially profitable, the preliminary work is now presented.

First substitute (C-5) and (C-6) into (C-2) and (C-4), respectively. The term $K(\Delta - \delta)$ is cancelled, and the result is:

$$M\ddot{X}_1 + (K+K_\infty)\ X_1 + (C+C_\infty)\ \dot{X}_1 - KX_2 - C\dot{X}_2 = K_\infty\ [X_{FP}-X_{FP_o}] + C_\infty\dot{X}_{FP} \qquad \text{(C-7)}$$

.
.
.

$$M\ddot{X}_i + 2KX_i + 2C\dot{X}_i - KX_{i+1} - C\dot{X}_{i+1} - KX_{i-1} - C\dot{X}_{i-1} = 0 \qquad \text{(C-8)}$$

.
.
.

$$M\ddot{X}_n + (K+K_\infty)\ X_n + (C+C_\infty)\ \dot{X}_n - KX_{n-1} - C\dot{X}_{n-1} = K_\infty\ (X_{MG}-X_{MG_o}) + C_\infty\dot{X}_{MG} \qquad \text{(C-9)}$$

Now define the following linear operations:

$$L_o = -C \frac{d()}{dt} - K$$

$$L_1 = m \frac{d^2()}{d+^2} + (C+C_\infty) \frac{d()}{dt} + (K+K_\infty)$$

$$L_2 = m \frac{d^2()}{d+^2} + 2C \frac{d()}{dt} + 2K$$

Then equations (C-7), (C-8), and (C-9) can be rewritten as:

$$\begin{aligned}
&L_1X_1 + L_oX_2 + 0X_3 + 0X_4 + \cdots + 0X_n = K_\infty (X_{FP}-X_{FP_o}) + C_\infty\dot{X}_{FP} \\
&L_oX_1 + L_2X_2 + L_oX_3 + 0X_4 + \cdots + 0X_n = 0 \\
&0X_1 + L_oX_2 + L_2X_3 + L_oX_4 + \cdots + 0X_n = 0 \\
&\vdots \\
&0X_1 + 0X_2 + 0X_3 + 0X_4 + L_oX_{n-1} + L_1X_n = K_\infty (X_{MG}-X_{MG_o}) + C_\infty\dot{X}_{MG}
\end{aligned} \qquad \text{(C-10)}$$

Now take Laplace transforms of $L_oX(t)$, $L_1X(t)$, $L_2X(t)$

$$\mathcal{L}\{L_o[X(t)]\} = -C\ [S\overline{X}(S) - X(0) - K\overline{X}(S)]$$

$$\mathcal{L}\{L_1[X(t)]\} = m\ [S^2\overline{X}(S) - SX(0) - \frac{dx}{dt}(0)] + (C+C_\infty)[S\overline{X}(S) - X(0)] + (K+K_\infty)\ \overline{X}(S)$$

$$\mathcal{L}\{L_2[X(t)]\} = m\ [S^2\overline{X}(S) - SX(0) - \frac{dx(0)}{dt}] + 2C\ [S\overline{X}(S) - X(0)] + 2K\overline{X}(S)$$

Assume $X(0) = \dot{X}(0) = 0$

$$\mathcal{L}\{L_o[X(t)]\} = -C\ [S + \frac{K}{C}]\ \overline{X}(S) \equiv \alpha_o(S)\overline{X}(S)$$

$$\mathcal{L}\{L_1[X(t)]\} = [mS^2 + (C+C_\infty)\ S + (K+K_\infty)]\overline{X}(S) \equiv \alpha_1(S)\overline{X}(S)$$

$$\mathcal{L}\{L_2[X(t)]\} = [mS^2 + 2CS + 2K]\ \overline{X}(S) \equiv \alpha_2(S)\overline{X}(S)$$

Also define

$$\mathcal{L}\{K_\infty[X_{FP}-X_{FP_o}] + C_\infty[\dot{X}_{FP}]\} \equiv g_1(S)$$

$$\mathcal{L}\{K_\infty[X_{MG}-X_{MG_o}] + C_\infty[\dot{X}_{MG}]\} \equiv g_n(S)$$

Now take the Laplace transforms of Equation (C-10)

$$\alpha_1(S)\,\overline{X}_1(S) + \alpha_o(S)\,\overline{X}_2(S) + 0\overline{X}_3(S) + 0\overline{X}_4(S) + \cdots + 0\overline{X}_n(S) = g_1(S)$$

$$\alpha_o(S)\,\overline{X}_1(S) + \alpha_2(S)\,\overline{X}_2(S) + \alpha_o(S)\overline{X}_3(S) + 0\overline{X}_4(S) + \cdots + 0\overline{X}_n(S) = 0$$

$$0\,\overline{X}_1(S) + \alpha_o(S)\,\overline{X}_2(S) + \alpha_2(S)\overline{X}_3(S) + \alpha_o(S)\overline{X}_4(S) + \cdots + 0\overline{X}_n(S) = 0$$

$$\vdots$$

$$0\,\overline{X}_1(S) + 0\,\overline{X}_2(S) + 0\,\overline{X}_3(S) + 0\,\overline{X}_4(S) + \alpha_o(S)\,\overline{X}_{n-1}(S) + \alpha_1(S)\overline{X}_n(S) = g_2(S)$$

Consider $g_1(S)$ and $g_2(S)$ as known. Such an assumption implies that X_{FP} and X_{MG} are given.

$$\begin{bmatrix} \alpha_1 & \alpha_o & 0 & 0 & \cdots & 0 \\ \alpha_o & \alpha_2 & \alpha_o & 0 & \cdots & 0 \\ 0 & \alpha_o & \alpha_2 & \alpha_o & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \cdots & \alpha_o\alpha_1 \end{bmatrix} \begin{bmatrix} \overline{X}_1(S) \\ \overline{X}_2(S) \\ \overline{X}_3(S) \\ \vdots \\ \overline{X}_n(S) \end{bmatrix} = \begin{bmatrix} g_1(S) \\ 0 \\ 0 \\ \vdots \\ g_2(S) \end{bmatrix}$$

Solve for $\overline{X}_1(S)$ using Cramer's rule. The denominator of the resulting expression for $\overline{X}_1(S)$ is:

$$\det[\alpha] = \alpha_1\left[\alpha_1 \frac{U^{n-1}-V^{n-1}}{U-V} - \alpha_o^2 \frac{U^{n-2}-V^{n-2}}{U-V}\right] - \alpha_o^2\left[\alpha_1 \frac{U^{n-2}-V^{n-2}}{U-V} - L_o^2 \frac{U^{n-3}-V^{n-3}}{U-V}\right]$$

$$\det[\alpha] = \frac{U^{n-3}}{U-V}\,[U\alpha_1 - \alpha_o^2]^2 - \frac{V^{n-3}}{U-V}\,[\alpha_1 V - \alpha_o^2]$$

$$\text{where } U \equiv \frac{\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_o^2}}{2} \qquad V \equiv \frac{\alpha_2 - \sqrt{\alpha_2^2 - 4\alpha_o^2}}{2}$$

In Reference [11], it is shown that the determinate of a MXM triple-banded matrix is:

$$\begin{vmatrix} A_2 & A_o & 0 & 0 & 0 & \cdots & 0 & 0 \\ A_o & A_2 & A_o & 0 & 0 & \cdots & 0 & 0 \\ 0 & A_o & A_2 & A_o & 0 & \cdots & 0 & 0 \\ \vdots \\ 0 & 0 & 0 & 0 & \cdots & A_o & A_2 & A_o & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_o & A_2 & A_o \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_o & A_2 \end{vmatrix} = \frac{U^{m+1} - V^{m+1}}{U-V}$$

where U and V are the roots of $X^2 - A_2 X - A_o^2 = 0$

By this fact, the surprisingly simple result for det [α] was made possible.

The numerator of the Cramer's rule solution for $\overline{X}_1(S)$ is:

$$g_1 [\alpha_1 K_{n-2} - \alpha_o^2 K_{n-3}] + (-1)^{n+1} g_2 \alpha_o^{n-1}$$

where $K_i \equiv \dfrac{U^{i+1} - V^{i+1}}{U-V}$

thus

$$\overline{X}_1(S) = \frac{(-1)^{n+1} g_2 \alpha_o^{n-1} + g_1 \left[\alpha_1 \dfrac{U^{n-1} - V^{n-1}}{U-V} - \alpha_o^2 \dfrac{U^{n-2} - V^{n-2}}{U-V} \right]}{\dfrac{U^{n-3}}{U-V} \left[U \alpha_1 - \alpha_o^2 \right]^2 - \dfrac{V^{n-3}}{U-V} \left[\alpha_1 V - \alpha_o^2 \right]^2}$$

$$\overline{X}_1(S) = \frac{(-1)^{n+1} g_2 \alpha_o^{n-1} (U-V) + g_1 U^{n-2} (\alpha_1 U - \alpha_o^2) - V^{n-2} g_1 (\alpha_1 V - \alpha_o^2)}{U^{n-3} [\alpha_1 U - \alpha_o^2]^2 - V^{n-3} [\alpha_1 V - \alpha_o^2]^2}$$

[11] Muir, Thomas, A Treatise on the Theory of Determinants, Dover Publication, pp 564.

One finds a similar expression for the last coil. If these are rewritten in the K_i-notation, the result is:

$$\bar{X}_1(S) = \frac{(-1)^{n+1} g_2 \alpha_o^{n-1} + g_1 (\alpha_1 K_{n-2} - \alpha_o^2 K_{n-3})}{\alpha_1^2 K_{n-2} - 2\alpha_1 \alpha_o^2 K_{n-3} + \alpha_o^4 K_{n-4}}$$

$$\bar{X}_n(S) = \frac{(-1)^{n+1} g_1 \alpha_o^{n-1} + g_2 (\alpha_1 K_{n-2} - \alpha_o^2 K_{n-3})}{\alpha_1^2 K_{n-2} - 2\alpha_1 \alpha_o^2 K_{n-3} + \alpha_o^4 K_{n-4}}$$

The great difficulty is in taking the inverse transforms. If one can find the inverse transform of $\bar{X}_1$ (S), one can then easily evaluate the spring force acting on the firing pin if the motion of the firing pin is given. Many methods for finding inverse transforms were used. The following expression is shown in Reference[12].

$$f(t) = \mathcal{L}^{-1} [\bar{f}(s)] = e^{-t/2} \sum_{k=o}^{\infty} \frac{C_k L_k(t)}{k!}$$

where $C_k = \sum_{j=o}^{k} \binom{k}{j} \frac{1}{j!} \bar{f}^{\,j} \left(\frac{1}{2}\right) \quad k=o, 1, 2, \cdots$

$$L_k(t) = e^t \frac{d^k}{dt^k} (t^k e^{-t}) = (-1)^k [X^k - k^2 X^{k-1} + \frac{k^2(k-1)^2}{2!} X^{k-2} \mp \cdots]$$

The $K_K(t)$ are Laguerre polynominals.

The above method did not appear to converge. Because of time consideration, the transfer function approach was temporarily abandoned, and a direct solution of the equation was sought. However, the finding of a transfer function for such a mechanical system is of such importance to weapon modeling that efforts should be resumed; perhaps Fourier transforms could also be tried.

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[12] Korn, G.A. and Korn, T.M., Mathematical Handbook for Scientists and Engineers; definitions, theorems, and formulas for reference and review, McGraw-Hill, New York, N.Y. (1961), pg 226.

In solving the equations directly, the approach was to assume that the right side of the equation for the ith coil,

$$M\ddot{X}_i + 2C\dot{X}_i + 2KX_i = K(X_{i-1} + X_{i+1}) + C(\dot{X}_{i-1} + \dot{X}_{i+1}),$$

is constant within any given time increment of integration. It can, of course, vary from increment-to-increment. With the right side thus held constant temporarily, homogeneous and particular solutions are easily obtained for the ith coil, and one can evaluate its position and velocity at the end of each time interval so that these become initial conditions for the next time increment. The method was found to give more accurate results than the fourth-order Runge-Kutta algorithm in less than one-half the computation time. Comparisons were made between these two methods, and the exact solutions for two and three masses.

A derivation of the method now will be presented. Assume, for simplicity, that $c_\infty = c$ and $K_\infty = K$, and rewrite the equations as follows:

$$M\ddot{X}_1 + 2KX_1 + 2C\dot{X}_1 = KX_2 + C\dot{X}_2 + K(X_{FP} - X_{FP_o}) + C\dot{X}_{FP} = h_1 \quad \text{(C-11)}$$

$$\vdots$$

$$M\ddot{X}_i + 2KX_i + 2C\dot{X}_i = K(X_{i+1} + X_{i-1}) + C(\dot{X}_{i+1} + \dot{X}_{i-1}) = h_i \quad \text{(C-12)}$$

$$\vdots$$

$$M\ddot{X}_n + 2KX_n + 2C\dot{X}_n = K\,X_{n-1} + K(X_{MG} - X_{MG_o}) + C(\dot{X}_{MG} + \dot{X}_{n-1}) = h_n \quad \text{(C-13)}$$

Integrate these equations from $t=t_1$ to $t=t_1 + \Delta t$. During this interval, assume that the right side of each equation does not change. The homogeneous equations are of the form

$$M\ddot{X}_h + \alpha\,\dot{X}_h + \beta\,X_h = 0,$$

where

$$\alpha = 2C,\ \beta = 2K.$$

Assume homogeneous solutions of the form $X = e^{rt}$.

Substitution of the assumed solution into the homogeneous equation yields the following:

$$mr^2 + \alpha r + \beta = 0$$

$$r = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\, m\beta}}{2m}$$

Three cases exist that are analogous to the underdamped, overdamped, and critically-damped cases for a single mass-spring-dashpot system.

CASE I

$\alpha^2 - 4\, m\beta < 0$ or $\frac{2K}{m} - (\frac{C}{m})^2 > 0$ This case corresponds to underdamping.

$$r = -\frac{\alpha}{2m} \pm i\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}$$

$$X_h = e^{-\frac{\alpha}{2m}t}\left[Ae^{i\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\,t} + Be^{-i\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\,t}\right]$$

$$X_h = e^{-\frac{\alpha}{2m}t}\left[(A+B)\cos\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\;t + i\,(A-B)\sin\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\;t\right]$$

$$X_h = e^{-\frac{\alpha}{2m}t}\left[C_1\cos\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\;t + C_2\sin\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\;t\right]$$

Now, find a particular solution to:

$$M\ddot{X}_j + \alpha\,\dot{X}_j + \beta\,X_j = h_j = \text{constant} = h_j\ (t=0).$$

$$X_{P_j} = \frac{h_j}{\beta}$$

Thus, the complete solution to equations (C-11), (C-12), (C-13) is:

$$X_j = e^{-\frac{\alpha}{2m}t}\left[C_{1j}\cos\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\;t + C_{2j}\sin\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\;t\right] + \frac{h_j(t=0)}{\beta} \quad \text{(C-14)}$$

C_{1j} and C_{2j} are determined from the initial conditions

$$X_j(t_1) = P_j \quad \text{(C-15)}$$

and

$$\dot{X}_j(t_1) = V_j. \quad \text{(C-16)}$$

For simplicity, equation (C-14) will be applied individually over each time increment. Thus, t_1 will be equated to zero; and P_j and V_j will be equated to the respective values at the end of the previous time increment. A separate accounting is made of the true value of t as the solution progresses.

Application of the initial condition (C-15) yields:

$$P_j = C_{1j} + \frac{h_j(t=0)}{\beta} \qquad \text{(C-17)}$$

$$C_{1j} = P_j - \frac{h_j(t=0)}{\beta} \qquad \text{(C-18)}$$

Now,

$$\dot{x}_j = -\frac{\alpha}{2m} e^{-\frac{\alpha}{2m}t} \left[C_{1j} \cos \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\, t \right.$$

$$\left. + C_{2j} \sin \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\, t \right]$$

$$+ e^{-\frac{\alpha}{2m}t} \left[-C_{1j} \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2} \sin \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\, t \right.$$

$$\left. + C_{2j} \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2} \cos \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}\, t \right]$$

Application of (C-16) yields:

$$V_j = -\frac{\alpha}{2m} C_{1j} + C_{2j} \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}$$

$$V_j = -\frac{\alpha}{2m} \left[P_j - \frac{h_j(t=0)}{\beta} \right] + C_{2j} \sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}$$

$$C_{2j} = \frac{1}{\sqrt{\frac{\beta}{m} - (\frac{\alpha}{2m})^2}} \left[V_j + \frac{\alpha}{2m} \left(P_j - \frac{h_j(t=0)}{\beta} \right) \right] \qquad \text{(C-19)}$$

From (C-14), (C-18), and (C-19)

$$X_j = e^{-\frac{\alpha}{2m}t}\left\{\left(P_j - \frac{h_j(t=0)}{\beta}\right)\cos\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\,t \right. \tag{C-20}$$

$$\left. + \frac{1}{\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}}\left[V_j + \frac{\alpha}{2m}\left(P_j - \frac{h_j(t=0)}{\beta}\right)\right]\sin\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\,t\right\}$$

$$+ \frac{h_j(t=0)}{\beta}$$

$$X_j\,(t = t_1 + \Delta t) = e^{-\frac{C\Delta t}{m}}\left\{\left(P_j - \frac{h_j(t=t_1)}{2K}\right)\cos\left(\sqrt{\frac{2K}{m} - \left(\frac{C}{m}\right)^2}\,\Delta t\right)\right. \tag{C-21}$$

$$\left. + \frac{1}{\sqrt{\frac{2K}{m} - \left(\frac{C}{m}\right)^2}}\left[V_j + \frac{C}{m}\left(P_j - \frac{h_j(t=t_1)}{2K}\right)\right]\sin\left(\sqrt{\frac{2K}{m} - \left(\frac{C}{m}\right)^2}\,\Delta t\right)\right\}$$

$$+ \frac{h_j(t=t_1)}{2K}$$

P_j and V_j are position and velocity of the jth mass as calculated at the end of the previous time interval.

Since the value of $\dot{X}_j$ at the end of each time interval is needed as an initial condition for the next increment, $\dot{X}_j$ will now be calculated:

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left\{\left[-\frac{\alpha}{2m}C_{1j} + C_{2j}\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\right]\cos\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\,t\right. \tag{C-22}$$

$$\left. - \left[\frac{\alpha}{2m}C_{2j} + C_{1j}\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\right]\sin\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\,t\right\}$$

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left\{\left[-\frac{\alpha}{2m}\left(P_j - \frac{h_j}{\beta}\right) + \frac{1}{\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}}\left(V_j + \frac{\alpha}{2m}\left(P_j - \frac{h_j}{\beta}\right)\right)\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\right]\right.$$

$$\times\cos\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\,t + \left[-\frac{\alpha}{2m}\,\frac{1}{\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}}\left(V_j + \frac{\alpha}{2m}\left(P_j - \frac{h_j}{\beta}\right)\right)\right. \tag{C-23}$$

$$\left.\left. - \left(P_j - \frac{h_j}{\beta}\right)\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\right]\sin\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\,t\right\}$$

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left\{V_j \cos\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}\, t - \left[\frac{\alpha V_j + 2\beta\left(P_j - \frac{h_j}{\beta}\right)}{2m\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2}}\right] \sin\sqrt{\frac{\beta}{m} - \left(\frac{\alpha}{2m}\right)^2 t}\right\} \quad \text{(C-24)}$$

$$\dot{X}_j(t=t_1 + \Delta t) = e^{-\frac{C}{m}\Delta t}\left\{V_j \cos\left(\sqrt{\frac{2K}{m} - \left(\frac{C}{m}\right)^2}\,\Delta t\right)\right. \quad \text{(C-25)}$$

$$\left. - \left[\frac{CV_j + 2K\left(P_j - \frac{h_j}{2K}\right)}{m\sqrt{\frac{2K}{m} - \left(\frac{C}{m}\right)^2}}\right] \sin\left(\sqrt{\frac{2K}{m} - \left(\frac{C}{m}\right)^2}\,\Delta t\right)\right\}$$

P_j and V_j are position and velocity calculated at the end of the previous time interval.

CASE II

$\alpha^2 - 4m\beta > 0$

or

$\frac{2K}{m} - \left(\frac{C}{m}\right)^2 < 0$ This case corresponds to overdamping.

$$r = -\frac{\alpha}{2m} \pm \sqrt{\left(\frac{\alpha}{2m}\right)^2 - \frac{\beta}{m}}$$

$$X_h = e^{-\frac{\alpha}{2m}t}\left[Ae^{\sqrt{\left(\frac{\alpha}{2m}\right)^2 - \frac{\beta}{m}}\, t} + Be^{-\sqrt{\left(\frac{\alpha}{2m}\right)^2 - \frac{\beta}{m}}\, t}\right]$$

$$X_{P_j} = \frac{h_j}{\beta}$$

$$X_j = e^{-\frac{\alpha}{2m}t}\left[C_{1j}\, e^{\sqrt{\left(\frac{\alpha}{2m}\right)^2 - \frac{\beta}{m}}\, t} + C_{2j}\, e^{-\sqrt{\left(\frac{\alpha}{2m}\right)^2 \;\; \frac{\beta}{m}}\, t}\right] + \frac{h_j}{\beta} \quad \text{(C-26)}$$

at

$t = 0,\; X_j = P_j$ (C-27)

and

$\dot{X}_j = V_j$ (C-28)

Let

$$\left(\frac{\alpha}{2m}\right)^2 - \frac{\beta}{m} \equiv D$$

Then

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left[C_{1j}\sqrt{D}\,e^{\sqrt{D}\,t} - C_{2j}\sqrt{D}\,e^{-\sqrt{D}\,t}\right]$$

$$-\frac{\alpha}{2m}\left[C_{1j}e^{\sqrt{D}\,t} + C_{2j}e^{-\sqrt{D}\,t}\right]$$

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left[e^{\sqrt{D}\,t}\left(C_{1j}\sqrt{D} - C_{1j}\frac{\alpha}{2m}\right) + e^{-\sqrt{D}\,t}\left(-C_{2j}\sqrt{D} - \frac{\alpha}{2m}C_{2j}\right)\right]$$

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left\{e^{\sqrt{D}\,t}\,C_{1j}\left[\sqrt{D} - \frac{\alpha}{2m}\right] + e^{-\sqrt{D}\,t}(-C_{2j})\left[\sqrt{D} + \frac{\alpha}{2m}\right]\right\}$$

Application of (C-27) and (C-28) yields

$$\begin{cases} V_j = C_{1j}\left(\sqrt{D} - \frac{\alpha}{2m}\right) - C_{2j}\left(\sqrt{D} + \frac{\alpha}{2m}\right) \\ P_j = C_{1j} + C_{2j} + \frac{h_j(t=0)}{\beta} \end{cases}$$

$$\begin{cases} C_{1j}\left[\sqrt{D} - \frac{\alpha}{2m}\right] + C_{2j}\left[-\sqrt{D} - \frac{\alpha}{2m}\right] = V_j \\ C_{1j}\,[1] + C_{2j}\,[1] = P_j - \frac{h_j}{\beta} \end{cases}$$

Now apply Cramer's Rule.

$$C_{1j} = \frac{\begin{vmatrix} V_j & -\sqrt{D} - \frac{\alpha}{2m} \\ P_j - \frac{h_j}{\beta} & 1 \end{vmatrix}}{\begin{vmatrix} \sqrt{D} - \frac{\alpha}{2m} & -\sqrt{D} - \frac{\alpha}{2m} \\ 1 & 1 \end{vmatrix}}$$

$$C_{1j} = \frac{V_j + (P_j - \frac{h_j}{\beta})(\sqrt{D} + \frac{\alpha}{2m})}{\sqrt{D} - \frac{\alpha}{2m} + \sqrt{D} + \frac{\alpha}{2m}}$$

$$C_{1j} = \frac{V_j + (P_j - \frac{h_j}{\beta})(\sqrt{D} + \frac{\alpha}{2m})}{2\sqrt{D}} \qquad \text{(C-29)}$$

$$C_{2j} = \frac{\begin{vmatrix} \sqrt{D} - \frac{\alpha}{2m} & V_j \\ 1 & P_j - \frac{h_j}{\beta} \end{vmatrix}}{2\sqrt{D}} = \frac{(\sqrt{D} - \frac{\alpha}{2m})(P_j - \frac{h_j}{\beta}) - V_j}{2\sqrt{D}} \qquad \text{(C-30)}$$

Substitute (C-29) and (C-30) into (C-26)

$$X_j = e^{-\frac{\alpha}{2m}t} \left[\frac{V_j + \left(P_j - \frac{h_j}{\beta}\right)\left(\sqrt{D} + \frac{\alpha}{2m}\right)}{2\sqrt{D}} e^{\sqrt{D}\,t} \right. \qquad \text{(C-31)}$$

$$\left. + \frac{\left(\sqrt{D} - \frac{\alpha}{2m}\right)\left(P_j - \frac{h_j}{\beta}\right) - V_j}{2\sqrt{D}} e^{-\sqrt{D}\,t} \right] + \frac{h_j}{\beta}$$

$$X_j(t=t_1+\Delta t) = e^{-\frac{c}{m}\Delta t}\left[\frac{V_j + \left(P_j - \frac{h_j(t=t_1)}{2K}\right)\left(\sqrt{(\frac{c}{m})^2 - \frac{2K}{m}} + \frac{c}{m}\right)}{2\sqrt{(\frac{c}{m})^2 - \frac{2K}{m}}} e^{\sqrt{(\frac{c}{m})^2 - \frac{2K}{m}}\,\Delta t}\right.$$

(C-32)

$$\left. + \frac{\sqrt{(\frac{c}{m})^2 - \frac{2K}{m}}\left(P_j - \frac{h_j}{2K}\right) - V_j}{2\sqrt{(\frac{c}{m})^2 - \frac{2K}{m}}} e^{-\sqrt{(\frac{c}{m})^2 - \frac{2K}{m}}\,\Delta t}\right] + \frac{h_j(t=t_1)}{2K}$$

P_j and V_j are position and velocity calculated at the end of the previous time interval.

Now, calculate $\dot{X}_j$.

$$\dot{X}_j = e^{-\frac{\alpha t}{2m}}\left[C_{1j}\sqrt{D}\,e^{\sqrt{D}\,t} - C_{2j}\sqrt{D}\,e^{-\sqrt{D}\,t}\right]$$

$$-\frac{\alpha}{2m} e^{-\frac{\alpha t}{2m}}\left[C_{1j}\,e^{\sqrt{D}\,t} + C_{2j}\,e^{-\sqrt{D}\,t}\right]$$

$$\dot{X}_j = e^{-\frac{\alpha t}{2m}}\left[e^{\sqrt{D}\,t}\left(C_{1j}\sqrt{D} - \frac{\alpha}{2m}C_{1j}\right) + e^{-\sqrt{D}\,t}\left(-C_{2j}\sqrt{D} - \frac{\alpha}{2m}C_{2j}\right)\right]$$

$$\dot{X}_j = e^{-\frac{\alpha t}{2m}}\left[C_{1j}\left(\sqrt{D} - \frac{\alpha}{2m}\right)e^{\sqrt{D}\,t} + C_{2j}\left(-\sqrt{D} - \frac{\alpha}{2m}\right)e^{-\sqrt{D}\,t}\right]$$

$$\dot{X}_j(t=t_1+\Delta t) = e^{-\frac{c}{m}t}\left\{\left[\frac{V_j+\left(P_j-\frac{h_j(t=t_1)}{2K}\right)\left(\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}+\frac{c}{m}\right)}{2\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}}\right]\right. \tag{C-33}$$

$$\times\left[\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}-\frac{c}{m}\right]e^{\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}\,\Delta t}$$

$$+\left[\frac{\left(\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}-\frac{c}{m}\right)\left(P_j-\frac{h_j(t=t_1)}{2K}\right)-V_j}{2\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}}\right]\left[-\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}-\frac{c}{m}\right]$$

$$\left.\times\ e^{-\sqrt{(\frac{c}{m})^2-\frac{2K}{m}}\,\Delta t}\right\}$$

CASE III

$\alpha^2 = 4m\beta$

or

$\frac{2K}{m} = (\frac{c}{m})^2$ This case corresponds to critical damping.

$r = -\frac{\alpha}{2m}$ Repeated roots.

$$X_h = A_e^{-\frac{\alpha}{2m}t} + B\,t\,e^{-\frac{\alpha}{2m}t}$$

$$X_P = \frac{h_j}{\beta}$$

$$X_j = A_j\,e^{-\frac{\alpha}{2m}t} + B_j\,t\,e^{-\frac{\alpha}{2m}t} + \frac{h_j}{\beta}$$

$$\dot{X}_j = -A_j\frac{\alpha}{2m}e^{-\frac{\alpha}{2m}t} + B_j\left[e^{-\frac{\alpha}{2m}t} - \frac{\alpha}{2m}\,t\,e^{-\frac{\alpha}{2m}t}\right]$$

$$\dot{X}_j = e^{-\frac{\alpha}{2m}t}\left[-A_j\frac{\alpha}{2m} + B_j - B_j\frac{\alpha}{2m}t\right]$$

At $t = 0$, $X_j = P_j$, $\dot{X}_j = V_j$

$$\therefore \quad P_j = A_j + \frac{h_j}{\beta}$$

$$V_j = -A_j \frac{\alpha}{2m} + B_j$$

$$A_j = P_j - \frac{h_j}{\beta}$$

$$B_j = V_j + A_j \frac{\alpha}{2m} = V_j + \frac{\alpha}{2m} (P_j - \frac{h_j}{\beta}$$

$$X_j (t=t_1 + \Delta t) = e^{-\frac{c}{m}\Delta t} \left[P_j - \frac{h_j(t=t_1)}{2K} + \Delta t \left(V_j + \frac{c}{m} \left(P_j - \frac{h_j(t=t_1)}{2K} \right)\right)\right] + \frac{h_j(t=t_1)}{2K} \qquad \text{(C-34)}$$

$$\dot{X}_j (t=t_1 + \Delta t) = e^{-\frac{c}{m}\Delta t} \left\{ - \left(P_j - \frac{h_j(t=t_1)}{2K} \right) \frac{c}{m} + \left[V_j + \frac{c}{m} \left(P_j - \frac{h_j(t=t_1)}{2K} \right)\right] \left[1 - \frac{c}{m}\Delta t \right] \right\} \qquad \text{(C-35)}$$

P_j and V_j are positions and velocities calculated at the end of the previous time interval.

The choice of which pair of equations ((C-21) (C-25) or (C-32) (C-33) or (C-34) (C-35)) should be used for calculating the motions of the masses is determined by the value of $\frac{2K}{m} - (\frac{c}{m})^2$.

Using the latest available calculated parameter values, one evaluates h_j at each time interval. All equations for X_j (j = 1,2,3...) are evaluated sequentially for one time interval before the next time interval is examined.

NOTE: The above solutions can be generalized to a system with all different masses, spring constants, and dampers if the following substitutions are made:

$\beta = K_i + K_{i+1}$, $\alpha = c_i + c_{i+1}$, $m = m_i$

Spring surging and coil clashing can also be analyzed from a continuum rather than a finite element. A derivation of the appropriate equations appears in Reference[13]. This work was done under contract to the Army Weapons Command, and a computer program is available. This approach appears to be more economical than the finite-element approach unless a transfer function can be found. The search for such a function is continuing.

A simpler treatment of inertial effects is to assume that an ideal spring is acting on a mass equal to $\frac{1}{3}$ the mass of the spring. Because the underlying assumptions are not immediately apparent, a derivation is shown below:

Assume that the velocity of any point on the spring varies linearly with spring mass between that point and the fixed end; that is,

$$V = \frac{m}{M_{SPRING}} V_{end} .$$

Thus, the velocity is zero at the fixed end (m=0) and is equal to V_{end} at the forced end ($m = M_{SPRING}$). Now, calculate the total kinetic energy (KE) of all mass points along the spring.

$$KE = \frac{1}{2} \int_o^{M_{SPRING}} V^2 dm = \frac{1}{2} \int_o^{M_{SPRING}} \left(\frac{m}{M_{SPRING}}\right)^2 V_{end}^2 \, dm$$

$$= \frac{1}{2} \left(\frac{V_{end}}{M_{SPRING}}\right)^2 \frac{1}{3} m^3 \Big|_o^{M_{SPRING}} = \frac{1}{2} \left[\frac{1}{3} M_{SPRING}\right] V_{end}^2$$

Thus, a mass equal to $\frac{1}{3} M_{SPRING}$ that is forced against a massless spring with velocity V_{end} will have the same kinetic energy as the spring with the distributed mass that is forced with the same velocity. The key assumption is linearity of velocity along the spring. As the spring is forced with greater velocity, this assumption becomes less accurate.

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[13] Phillips, J.W. and Costello, G.A., Large Deflections of Impacted Helical Springs, Journal of the Acoustical Society of America, VOL 51, No. 3 (Part 2), pg 967 (1972)

APPENDIX D

EXTENSION OF DETERMINISTIC MODELS TO PROBABILISTIC REGIME

The M16A1 Rifle model described in this report is deterministic. That is, one is given certain single-valued input data, and the model provides single-valued output information. Such a model describes only one particular weapon under one particular set of firing conditions, and with one particular round of ammunition. However, input data in reality have a range of values. Not all weapon parts will have the same weights and dimensions because of certain manufacturing tolerances. Not all rounds of ammunition will produce the same pressure-time curve. Not all mounts will have the same stiffness. Not all crosswinds will have the same velocity. Not all shooters will hold the weapon the same way. Not all weapons will have the same amount of lubricant and contaminant. Not all weapons are fired at the same ambient temperature. Such a list is virtually endless; however, the problem is not as hopeless as it may seem. Most of these quantities have reasonably well-defined bounds, that is, wind velocity does not vary from plus infinity to minus infinity. A reasonable estimate of mean value and standard deviation is possible. One can then construct a reasonable probability distribution function. The more experimental data that are available, the more accurate this curve is likely to be. Similarly for the other input data, one can approximate probability distribution functions; of course, some will be more difficult than others. The next problem is the determination of how to process this information. The computer should accept this stochastic input information, operate with it, and provide stochastic output.

Two possible methods are discussed by which a model could be designed to carry out this process. The first is the Monte Carlo technique and the second is a generalization of the method of partial derivatives. An alternative to statistics is also presented. This is a perturbation technique.

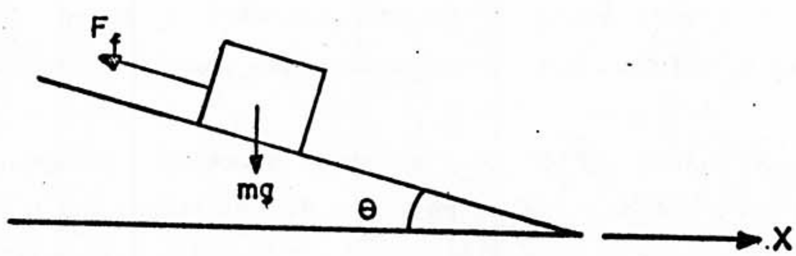
The Monte Carlo approach requires that one run the deterministic model for many input values. These values are chosen to reflect the probability distribution of the appropriate quantity. The method of partial derivatives attempts to approximate the output means and standard deviation given the input function. The perturbation technique is simply a way to examine the derivative of various quantities, with respect to the others, to determine how sensitive the first variable is to changes in the second. One can then use this knowledge of the range of values for the second variable to determine the significance of the derivative. The Monte Carlo method has a serious disadvantage in that it requires a large number of computer runs. For a complex model, this shortcoming causes the computer costs to become excessive.

The method of partial derivatives requires only one computer run and would save computer costs, but this method is more difficult to set up. The technique does not provide the probability density function; it provides a measure of the variability in output that could be expected. The perturbation technique provides even less information directly.

The three techniques will be discussed in some technical detail in the following paragraphs.

D.1 Introductory Example

First, an introductory problem will be presented to illustrate some main ideas of this discussion about probabilistic models. The problem is atypical in its lack of complexity. Let "m" be a mass sliding on an inclined plane, as shown below:



F_f is a friction force equal to $\mu mg \cos\theta$, where μ is the coefficient of sliding friction. The equation of motion for the mass is

$$m\ddot{x} = mg \sin\theta - \mu mg \cos\theta, \; \theta \neq 0$$

$$\text{I.C. } x(0) = \dot{x}(0) = 0$$

The solution is

$$x(t) = [g \sin\theta - \mu g \cos\theta]\frac{t^2}{2}, \; \dot{x}(t) = [g \sin\theta - \mu g \cos\theta]t$$

or

$$x(t) = A + B\,\mu, \; \dot{x}(t) = A_1 + B_1 \mu$$

where

$$A = [g \sin\theta]\frac{t^2}{2}, \quad B = -[g \cos\theta]\frac{t^2}{2}$$

$$A_1 = [g \sin\theta]t, \quad B_1 = -[g \cos\theta]t$$

Suppose that one selects the inclined plane and the sliding mass from a stockpile of these objects where the coefficient of friction has a normal distribution. Certainly, the surfaces will have some differences in finish and lubrication, and not all the coefficients can be expected to be the same. Suppose that the distribution has a mean of .3 and a standard deviation of .05. Then, from probability theory, $x(t)$ and $\dot{x}(t)$ have a normal distribution with means of $\overline{x} = A + B\,\overline{\mu} = A + .3B$,

$\overline{\dot{x}} = A_1 + B_1 \overline{\mu} = A_1 + .3B_1$

and with standard deviations of

$\sigma_x = [B^2\,(.05)^2\,]^{1/2}$

and

$\sigma_{\dot{x}} = [B_1^2\,(.05)^2\,]^{1/2}$, respectively.

At any given time, t_s, one can determine the probability that $x(t_s)$ or $\dot{x}(t_s)$ will lie within certain limits.

For example, let us find the values of x(.5) that lie within two standard deviations of the mean of x(.5).

Let

$$\theta = 45°, \quad g = 32.2 \text{ ft/sec}^2, \quad t_s = .5 \text{ sec}$$

Then

$$A = 2.8461 \text{ ft}, \quad B = -2.8461 \text{ ft}$$

and

$$\overline{x}\,(.5) = 2.8461 - (.3)(2.8461) = 1.9923$$

$$\sigma_x = [(2.8461)^2\ (.05)^2]^{\frac{1}{2}} = .1423$$

$$P(1.9923 - 2\,\sigma_x \leq x(.5) \leq 1.9923 + 2\,\sigma_x) = .95$$

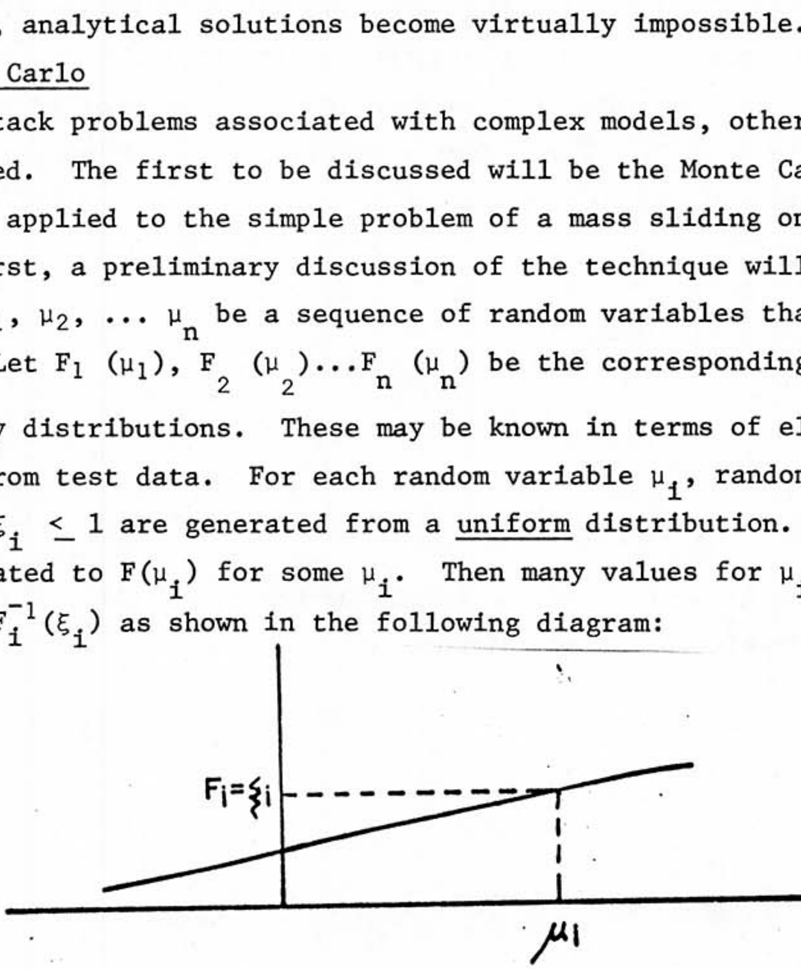
$$P(1.7077 \leq x(.5) \leq 2.2769) = .95$$

The previous results are dependent upon the fact that the differential equation can be solved analytically, and that the distribution is normal and completely determined. If μ had a log-normal distribution, it would be difficult to find the probability distribution for x(t). With a complex model, analytical solutions become virtually impossible.

D.2 Monte Carlo

To attack problems associated with complex models, other techniques are required. The first to be discussed will be the Monte Carlo technique. It will be applied to the simple problem of a mass sliding on an inclined plane. First, a preliminary discussion of the technique will be made.

Let $\mu_1, \mu_2, \ldots \mu_n$ be a sequence of random variables that are constant in time. Let $F_1\ (\mu_1), F_2\ (\mu_2) \ldots F_n\ (\mu_n)$ be the corresponding cumulative probability distributions. These may be known in terms of elementary functions or from test data. For each random variable μ_i, random numbers ξ_i where $0 \leq \xi_i \leq 1$ are generated from a uniform distribution. The numbers ξ_i are equated to $F(\mu_i)$ for some μ_i. Then many values for μ_i are computed from $\mu_i = F_i^{-1}(\xi_i)$ as shown in the following diagram:



The differential equations are then solved many times with the use of the μ_i values. The output values are then distributed in some fashion not necessarily similar to the input values.

Recall that the inclined plane problem is described by:

$\ddot{x} = g \sin \theta - \mu g \cos \theta$, $x(0) = \dot{x}(0) = 0$, $\theta = 45°$, and $g = 32.2$ ft/sec^2 where μ is normally distributed with a mean of .3 and a standard deviation of .05.

Let $Z \equiv \frac{\mu - .3}{.05}$. Then "Z" has a standard normal distribution. Let F(x) be the cumulative probability distribution function of the standard normal.

$$F(Z) = \int_{-\infty}^{Z} f(s)\, ds.$$ Generate random ξ_i and compute $Z = F^{-1}(\xi_i)$.

Then $\mu = .05\,Z + .3$. Table 11 contains the values obtained.

By taking a sufficiently large sample, one can find approximate probabilities that x(t) at a given time will lie within certain limits. For example,

$$P(29 \leq x(2) \leq 31) \simeq \frac{\#\text{trials } (29 \leq x(2) \leq 31)}{\text{total } \#\text{ trials}} \simeq \frac{12}{15} = .80$$

or

$$P(1.7077 \leq x(.5) \leq 2.2769) \simeq \frac{\#\text{trials } (1.7077 \leq x(.5) \leq 2.2769)}{\text{total } \#\text{ trials}} \simeq \frac{14}{15} = .93$$

From the previously derived exact solution, $P(1.7077 \leq x(.5) \leq 2.2769) = .95$

It is possible to place confidence limits on the estimate that the probability is .93 that x(.5) lies between 1.7077 and 2.2769. Such information is tabulated. From pg. 147 in Reference[14], a 95% confidence interval about .93 is (.67, 1). This interval could be reduced if a larger sample were used.

An estimate of the sample size required for a $1 - \alpha$ confidence interval $(P - E, P + E)$ for the probability "P" is $$n = \frac{P(1-P)\left[Z_{1-\frac{\alpha}{2}}\right]^2}{E^2}.$$ ($Z_{1-\frac{\alpha}{2}}$ indicates that "Z" is a function of $1 - \frac{\alpha}{2}$.)

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[14]Haugen, E., Probabilistic Approaches to Design, John Wiley & Sons, Inc., New York (1968)

TABLE 11 SAMPLE MONTE CARLO CALCULATIONS

	ξ	Z	μ	x(.5)	x(2)
1.	.816	.90	.345	1.864	29.82
2.	.763	.72	.336	1.889	30.22
3.	.061	-1.54	.323	1.927	30.83
4.	.988	2.28	.4125	1.672	26.75
5.	.174	-.94	.253	2.126	30.02
6.	.709	.55	.3275	1.914	30.62
7.	.889	1.22	.361	1.819	29.10
8.	.772	.75	.3375	1.885	30.16
9.	.893	1.24	.362	1.816	29.05
10.	.232	.73	.3365	1.888	30.21
11.	.091	1.33	.3665	1.803	28.85
12.	.133	1.11	.3555	1.834	29.34
13.	.197	.85	.3425	1.871	29.94
14.	.469	.10	.305	1.978	31.65
15.	.061	-1.55	.2225	2.213	35.10

For the case $(1.7077 \leq x(.5) \leq 2.2769)$, E = .05, α = .05, P = .93 and

$$n = \frac{.93(1 - .93)(1.96)^2}{(.05)^2} = 100$$

Thus, a sample size of 100 would be needed to get a 95% confidence interval of (.88, .98).

D.3 Partial Derivatives

Now, consider the method of partial derivatives. This method provides estimates for the means and standard deviations of the output given the means and standard deviations of the input data. It does not provide a probability density function for the output. The technique is applied to algebraic equations in Haugen's book.[14] Here, the technique is extended to ordinary differential equations.

Consider the differential equation $\dot{x}(t) = F(x, a, t)$ (D-1)

where $x(0) = x_o$. The variable "a" is random and has a mean "$\bar{a}$" and a standard deviation "σ".

[14] Haugen, E., Probabilistic Approaches to Design, John Wiley & Sons, Inc., New York (1968)

The solution of the equation is $x(t) = \int_o^t F(x, a, \tau)\, d\tau + x_o$. (D-2)

Thus, the variable x(t) is also random.

Given that $x_1, x_2 \ldots x_n$ are random variables with means $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n$ and standard deviations $S_{x_1}, S_{x_2}, \ldots S_{x_n}$, define $z = \Psi(x_1, x_2, \ldots x_n)$.

Approximations to the mean and standard deviation of "z" are:

$$\bar{Z} \simeq \Psi(\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n) \tag{D-3}$$

$$S_Z \simeq \left[\sum_{j=1}^{n} \left(\frac{\partial Z}{\partial x_j}\right)^2 S^2_{x_j} \right]^{1/2} \tag{D-4}$$

NOTE: In the approximation of the mean, higher-order terms in a Taylor series expansion of "z" about $\bar{x}_i$ are neglected. Some could be included for greater accuracy. The partial derivatives in the approximation to the standard deviation are evaluated at the mean values of the x_i's. These approximations are quite good if the variances are not too large. The remaining development of the method of partial derivatives will be based on these approximations.

Now apply (D-3) to (D-2)

$$\bar{x}(t, a) = \int_o^t F(x, \bar{a}, \tau)\, d\tau + x_o$$

This equation means that the mean of x(t, a) can be approximated by the solution of the differential equation where "$\bar{a}$" has been substituted for "a".

Now

$$S_{x(t,\, a)} = \left[\left(\frac{\partial\, x(t, a)}{\partial a}\right)^2 S^2_a \right]^{1/2}$$

From (D-2)

$$\frac{\partial\, x(t, a)}{\partial a}\bigg|_{a = \bar{a}} = \int_o^t \frac{\partial\, F(x, a, \tau)}{\partial a}\bigg|_{a = \bar{a}} d\tau$$

$$S_a = \sigma$$

$$\therefore \quad S_{x(t,\, a)} = \int_0^t \frac{\partial F(x, a, \tau)}{\partial a} \Big|_{a = \bar{a}} d\tau\, \sigma$$

Consider the example

$$\dot{x}(t) = a\, e^{-t}$$

$$x(0) = 0$$

The variable "a" has a mean "$\bar{a}$" and a standard deviation σ.

$$\bar{x}(t, a) \simeq \int_0^t F(x, \bar{a}, \tau)\, d\tau + x_o = \int_0^t \bar{a}\, e^{-\tau}\, d\tau + 0 = \bar{a}\,(1 - e^{-t})$$

$$\frac{\partial F(x, a, t)}{\partial a} \Big|_{a = \bar{a}} = e^{-t}$$

$$\frac{\partial x(t, a)}{\partial a} \Big|_{a = \bar{a}} = \int_0^t \frac{\partial F(x, a, \tau)}{\partial a} \Big|_{a = \bar{a}} d\tau = \int_0^t e^{-\tau}\, d\tau = 1 - e^{-t}$$

$$S_{x(t,\, a)} \simeq \left| \frac{\partial x(t, a)}{\partial a}\, \sigma \right| = \sigma\,(1 - e^{-t})$$

For this example, the exact result can be obtained.

$$x(t, a) = a\,(1 - e^{-t})$$

$$\bar{x}(t, a) = \bar{a}\,(1 - e^{-t})$$

$$S_x(t, a) = \sigma\,(1 - e^{-t})$$

Thus, the approximate and exact results are equal.

This scheme will now be generalized to a system of first-order differential equations. Since one or more higher-order differential equations can be transformed to a system of first-order equations, this is a powerful generalization.

The system of equations is described by:

$$\dot{x}(t) = F(x, a, t)$$

$$x(0) = g(a)$$

where

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} \qquad F(x, a, t) = \begin{bmatrix} F_1(x, a, t) \\ F_2(x, a, t) \\ \vdots \\ F_n(x, a, t) \end{bmatrix}$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \qquad g(a) = \begin{bmatrix} g_1(a) \\ g_2(a) \\ \vdots \\ g_n(a) \end{bmatrix} \qquad \bar{a} = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \vdots \\ \bar{a}_m \end{bmatrix}$$

then

$$x(t, a) = \int_0^t F(x, a, \tau)\, d\tau + g(a)$$

$$\bar{x}(t, a) \simeq \int_0^t F(x, \bar{a}, \tau)\, d\tau + g(\bar{a})$$

$$\left[\begin{pmatrix} \left(\dfrac{\partial x_1(t, a)}{\partial a_1}\right)^2 & - - - & \left(\dfrac{\partial x_1(t, a)}{\partial a_m}\right)^2 \\ \vdots & & \\ \left(\dfrac{\partial x_n(t, a)}{\partial a_1}\right)^2 & - - - & \left(\dfrac{\partial x_n(t, a)}{\partial a_m}\right)^2 \end{pmatrix} \begin{pmatrix} \sigma_{a_1}^2 \\ \vdots \\ \sigma_{a_m}^2 \end{pmatrix} \right]^{1/2} \tag{D-5}$$

The square root of the column vector is the column vector formed from the square root of each term. Partial derivatives are evaluated at $\bar{a}_1, \bar{a}_2, \ldots \bar{a}_n$.

$$\frac{\partial x(t, a)}{\partial a_j}\bigg|_{a=\bar{a}} = \int_0^t \frac{\partial F(x, a, \tau)}{\partial a_j}\bigg|_{a=\bar{a}} d\tau + \frac{\partial g(a)}{\partial a_j}\bigg|_{a=\bar{a}} . \tag{D-6}$$

The following is a simple example for which exact results can also be obtained. Consider a free-falling mass. $\ddot{x} = -g$

$$x(0) = a_1$$
$$\dot{x}(0) = a_2$$

Assume a_1 has a mean of zero and a standard deviation of σ_{a_1}.

Assume a_2 has a mean of zero and a standard deviation of σ_{a_2}.

Make the following transformation:
$$\begin{cases} x_1 \equiv x \\ x_2 \equiv \dot{x} \end{cases}$$

$$\therefore \quad x_1(0) = a_1$$
$$x_2(0) = a_2$$
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -g \end{pmatrix}$$

$$\begin{pmatrix} \dfrac{\partial x_1}{\partial a_1} \\ \dfrac{\partial x_2}{\partial a_1} \end{pmatrix} = \begin{pmatrix} \displaystyle\int_o^t \frac{\partial x_2}{\partial a_1} \, d\,t \\ \displaystyle -\int_o^t \frac{\partial g}{\partial a_1} \, d\,t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dfrac{\partial x_1}{\partial a_2} \\ \dfrac{\partial x_2}{\partial a_2} \end{pmatrix} = \begin{pmatrix} \displaystyle\int_o^t \frac{\partial x_2}{\partial a_2} \, d\,t \\ \displaystyle -\int_o^t \frac{\partial g}{\partial a_2} \, d\,t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} t \\ 1 \end{pmatrix}$$

Insert these expressions into (D-5).

$$S_x(t, a) \equiv \left[\begin{pmatrix} 1 & t^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_{a_1}^2 \\ \sigma_{a_2}^2 \end{pmatrix} \right]^{1/2} = \left[\begin{pmatrix} \sigma_{a_1}^2 & + & \sigma_{a_2}^2 & t^2 \end{pmatrix}^{1/2} \\ \sigma_{a_2} \right]$$

Now, find the exact results for $S_{x(t,\ a)}$ and compare with the approximate expression above.

$$\ddot{x} = -g$$

$$x(0) = a_1 \qquad \dot{x}(0) = a_2$$

$$x(t) = -\frac{1}{2} g\ t^2 + a_2 t + a_1$$

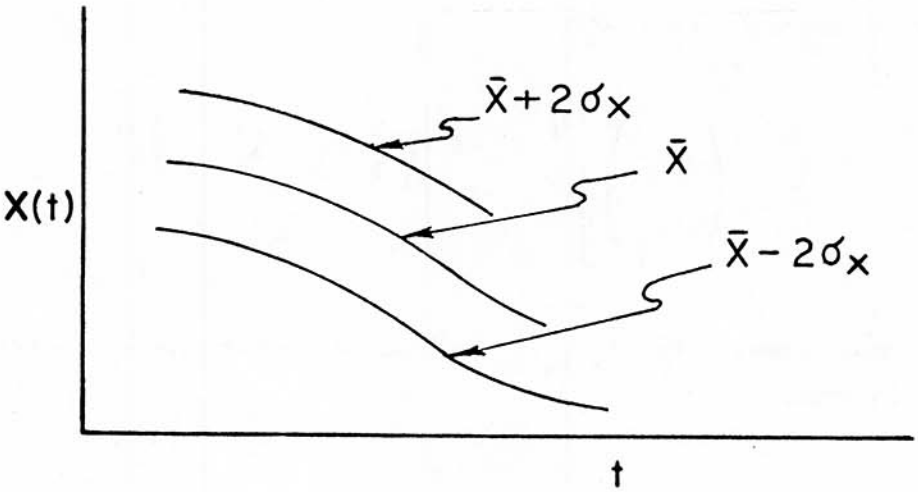
$$S_x = \left[\sigma_{a_2}^2\ t^2 + \sigma_{a_1}^2 \right]^{1/2}$$

$$\dot{x}(t) = -g\ t + a_2$$

$$S_{\dot{x}} = \sigma_{a_2}$$

Thus, the approximate and exact results for the standard deviations are equal. It can also be shown that the approximate and exact mean values are equal.

Using this approximate method, one could get a good idea of the nature of the output without resorting to the time-consuming Monte Carlo technique. By calculating approximate output means and standard deviations, one can "bracket" the solution by graphing the results as follows:



Using the method of partial derivatives, one can avoid transformation to a system of first-order equations and can find partial derivatives directly. For example, consider a projectile that leaves a barrel with a random muzzle velocity having a mean v_o and a standard deviation σ_{v_o}. Assume that the angle of elevation θ is a random variable with mean $\overline{\theta}$ and standard deviation σ_θ. Assume that the only force on the projectile is that of gravity.

Then

$$m\ddot{y} = -g \qquad y(0) = 0, \quad \dot{y}(0) = v_o \sin\theta$$

$$m\ddot{x} = 0 \qquad x(0) = 0, \quad \dot{x}(0) = v_o \cos\theta$$

Do not transform the system to first-order, but directly take partial derivatives of the equations with respect to the random variables.

Note that

$$y_\alpha \equiv \frac{\partial y}{\partial \alpha}, \quad x_\alpha \equiv \frac{\partial x}{\partial \alpha}$$

$$m\ddot{y}_{v_o} = 0$$

$$y_{v_o}(0) = 0$$

$$\dot{y}_{v_o}(0) = \sin\theta$$

$$\therefore\ \dot{y}_{v_o} = \alpha$$

$$y_{v_o} = \alpha t + \beta$$

$$\begin{cases} y_{v_o} = (\sin\theta)\, t \\ \dot{y}_{v_o} = \sin\theta \end{cases}$$

$$m\ddot{x}_{v_o} = 0$$

$$x_{v_o}(0) = 0$$

$$\dot{x}_{v_o}(0) = \cos\theta$$

$$\therefore\ \dot{x}_{v_o} = \alpha$$

$$x_{v_o} = \alpha t + \beta$$

$$\begin{cases} x_{v_o} = (\cos\theta)\, t \\ \dot{x}_{v_o} = \cos\theta \end{cases}$$

$$m\ddot{y}_\theta = 0$$

$$y_\theta(0) = 0$$

$$\dot{y}_\theta(0) = v_o \cos\theta$$

$$\therefore\ \dot{y}_\theta = \alpha$$

$$y_\theta = \alpha t + \beta$$

$$\begin{cases} y_o = v_o (\cos\theta)\, t \\ \dot{y}_\theta = v_o \cos\theta \end{cases}$$

$$m\ddot{x}_\theta = 0$$

$$x_\theta(0) = 0$$

$$\dot{x}_\theta(0) = -v_o \sin\theta$$

$$\therefore\ \dot{x}_\theta = \alpha$$

$$x_\theta = \alpha t + \beta$$

$$\begin{cases} x_\theta = -(v_o \sin\theta)\, t \\ \dot{x}_\theta = -v_o \sin\theta \end{cases}$$

Equation (D-4) is

$$S_Z \approx \left[\sum_{j=1}^{n} \left(\frac{\partial Z}{\partial x_j} \right)^2 S_{x_j}^{\ 2} \right]^{1/2}$$

$$\therefore \quad S_{x(t)} \equiv [\sigma_{v_o}^{\ 2} \cos^2 \bar{\theta} + \sigma_\theta^{\ 2} (\bar{v}_o \sin \bar{\theta})^{\ 2}]^{1/2} \ t$$

$$S_{\dot{x}(t)} \approx [\sigma_{v_o}^{\ 2} \cos^2 \bar{\theta} + \sigma_\theta^{\ 2} (\bar{v}_o \sin \bar{\theta})^{\ 2}]^{1/2}$$

$$S_{y(t)} \approx [\sigma_{v_o}^{\ 2} \sin^2 \bar{\theta} + \sigma_\theta^{\ 2} (\bar{v}_o \cos \bar{\theta})^{\ 2}]^{1/2} \ t$$

$$S_{\dot{y}(t)} \approx [\sigma_{v_o}^{\ 2} \sin^2 \bar{\theta} + \sigma_\theta^{\ 2} (\bar{v}_o \cos \bar{\theta})^{\ 2}]^{1/2}$$

The following is an illustration of the types of conclusions that can be drawn from the results of this method.

Suppose

$$\bar{\theta} = 0$$

Then

$$S_{x(t)} \equiv \sigma_{v_o} t$$

$$S_{\dot{x}(t)} \equiv \sigma_{v_o}$$

$$S_{y(t)} \equiv \sigma_\theta \bar{v}_o t$$

$$S_{\dot{y}(t)} \equiv \sigma_\theta \bar{v}_o$$

Thus, when the average elevation is zero, σ_θ has virtually no effect on $S_{x(t)}$ or $S_{\dot{x}(t)}$ if σ_θ is small.

One can, by maximizing $S_{x(t)}$, find the average elevation at which x(t) has a maximum variation. Thus, one must find $\bar{\theta}$ that maximizes

$$S_{x(t)} = [\sigma_{v_o}^{\ 2} \cos^2 \bar{\theta} + \sigma_\theta^{\ 2} v_o^{\ 2} \sin^2 \theta]^{1/2} \ t$$

Substitute $\quad \sin^2 \bar{\theta} = 1 - \cos^2 \bar{\theta}$

$$S_{x(t)} = [(\sigma_{v_o}^{\ 2} - \sigma_\theta^{\ 2} v_o^{\ 2}) \cos^2 \bar{\theta} + \sigma_\theta^{\ 2} v_o^{\ 2}]^{1/2} t$$

If

$\theta = 0,$

then

$S_{x(t)}$ is a maximum.

D.4 Perturbation

The following perturbation method is an alternative to the statistical approach. This method is not as powerful, but it can yield much useful information.

Consider the equation $\dot{x} = f(x, t, a)$, where "a" is a random variable, and $x(0) = x_o$.

The solution of this equation is

$$x(t) = \int_o^t f(x, t, a)\, dt + x_o$$

The fundamental objective of this method is to evaluate the effect on x(t) caused by a small change in "a".

The change in x(t) can be written $\Delta x(t) = \frac{\partial x(t)}{\partial a} \Delta a$, if only the first two terms in a Taylor series expansion of "x" about "a" are used.

From (D-1)

$$\frac{\partial x(t)}{\partial a} = \int_o^t \frac{\partial f(x, t, a)}{\partial a}\, dt$$

As a simple illustration, consider the following:

$$\dot{x}(t) = e^{-at}$$

$$x(0) = 0$$

$$\frac{\partial f(x, t, a)}{\partial a} = -t\, e^{-at}$$

$$\frac{\partial x(t)}{\partial a} = \int_o^t -t\, e^{-at}\, dt = \frac{t\, e^{-at}}{a} + \frac{1}{a^2}(e^{-at} - 1)$$

$$\Delta x(t) = \left[\frac{t\, e^{-at}}{a} + \frac{1}{a^2}(e^{-at} - 1) \right] \Delta a$$

NOTE: If the equation cannot be solved analytically, $\frac{\partial x}{\partial a}$ can still be found numerically by integrating $\frac{\partial f(x, t, a)}{\partial t}$ over time with zero initial conditions. The basic approach can be extended to systems of differential equations.

Now, consider the case of a free-falling mass and find the effects of slight variations in initial velocity and displacement on velocity and displacement at some time "t".

$$m\ddot{x} = -g$$

$$x(0) = x_o$$

$$\dot{x}(0) = \dot{x}_o$$

Take partial derivatives of each equation with respect to x_o and $\dot{x}_o$ and solve the resulting equations.

$$m\ddot{x}_{x_o} = 0$$

$$x_{x_o}(0) = 1$$

$$\dot{x}_{x_o}(0) = 0$$

$$x_{x_o} = \alpha t + \beta$$

$$\begin{cases} x_{x_o}(t) = 1 \\ \dot{x}_{x_o}(t) = 0 \end{cases}$$

$$\Delta x(t) = x_{x_o}(t)\,\Delta x_o + x_{\dot{x}_o}(t)\,\Delta \dot{x}_o$$

$$\therefore \quad \Delta x(t) = \Delta x_o + t\,\Delta \dot{x}_o$$

$$m\ddot{x}_{\dot{x}_o} = 0$$

$$x_{\dot{x}_o}(0) = 0$$

$$\dot{x}_{\dot{x}_o}(0) = 1$$

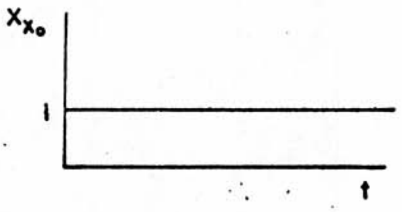
$$x_{\dot{x}_o} = \alpha t + \beta$$

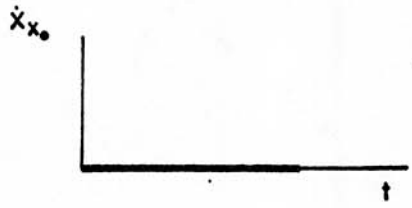
$$\begin{cases} x_{\dot{x}_o} = t \\ \dot{x}_{\dot{x}_o} = 1 \end{cases}$$

$$\Delta \dot{x}(t) = \dot{x}_{x_o}(t)\,\Delta x_o + \dot{x}_{\dot{x}_o}(t)\,\Delta \dot{x}_o$$

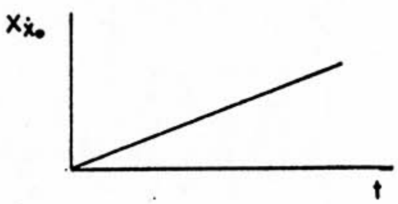
$$\Delta \dot{x}(t) = \Delta \dot{x}_o$$

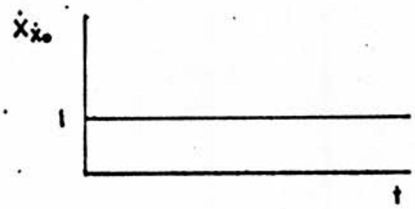
A graph can often be helpful in determining which variables should be altered to achieve a desired change in displacement or velocity.





Thus, one can see that a small change in initial displacement changes the displacement at each point in time by an equal amount. Velocity can be seen to be unaffected by a small change in displacement.





Thus, a change in initial velocity affects the displacement by an amount that increases with time. A change in initial velocity is simply added to the velocity at any other time.

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APPENDIX E

"BROAD SPECTRUM OF AMMUNITION" STUDY

The "Broad Spectrum of Ammunition" Study initiated under the Weapons Laboratory (WECOM) is designed to investigate an important, but relatively unexplored area of weapon-ammunition interaction. Emphasis is on the determination of how various ammunition parameters affect weapon operation and how these parameters are distributed in normal ammunition production. The term "broad spectrum" refers to the wide range in parameter values that occurs in production. Potentially, many useful results exist from such an investigation. For example, by understanding how gas behavior is affected by nonuniformity of propellant grain shape and, in turn, how gas behavior affects mechanism performance and reliability, one could then either stiffen or relax manufacturing tolerances on uniformity of grain shape. Also, having a more complete understanding of the weapon-ammunition interface, one could better design weapons so that they are relatively insensitive to ammunition tolerances difficult to maintain. Reference [15] provides additional background.

The general approach of this study is to answer the following three basic questions: (1) What measurable weapon characteristics should be used as indicators of weapon performance? (2) How do ammunition tolerances affect these performance indicators? (3) In normal production, what are the distributions of ammunition tolerance that are important to weapon performance? Mathematical models of weapons can be highly useful in the study of the first two questions. Statistical analyses of acceptance test data on breech and port pressures have been performed in support of the third question. See Reference [16].

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[15] "Ammunition Selection for Verification Testing of the 5.56mm, XM207, Belt-Fed Machine Gun", Technical Report 70-103, Rock Island, Illinois (Aug 1969)

[16] "Investigation of the Interaction of Weapon-Ammunition Subsystems", Technical Report SWERR-TR-72-30, AD 742723, Rock Island, Illinois (May 1972)

This task is very difficult, but it is also important. The problem it addresses cannot be solved in a straight-forward manner. Although a long-term effort is required, interim results should prove highly useful. This work interfaces with the probabilistic modeling described in Appendix D.

APPENDIX F

METHODS OF TREATING IMPACT

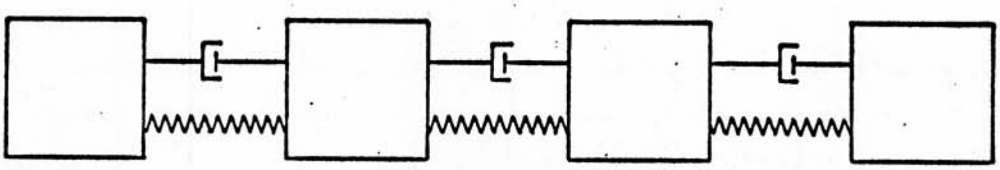
The impact between moving bodies in a weapon mechanism is very difficult to model accurately. Knowledge in this field is important for stress, wear, and fatigue analysis. In reality, time-dependent elastoplastic effects are involved. The nature of the impact depends on the motions of the masses just prior to impact, the mechanical properties of the material, the area of contact, the angle of contact, the boundary conditions on the colliding bodies, the physical dimensions of the masses, etc. However, in the interest of reasonable computation time for an already complex weapon model and in view of the lack of current theory to handle accurately impact situations, large approximations are made.

Often, a coefficient of restitution is used. This coefficient is an indication of the kinetic energy that is lost by the colliding bodies. However, with this method, one knows neither the duration of contact nor the forces involved. A better approximation would be to assume that, at the time of impact, a stiff spring and dashpot hypothetically appear between the two colliding masses. Coulomb, rather than viscous damping, could be assumed, but the question of which should be used is somewhat academic since the collision process is far too complex to be accurately modeled by either one.

Two major difficulties exist in the spring approach. The first difficulty is that it is difficult to properly choose the most appropriate spring and damping values. To some extent, the spring constant can be based on Young's modulus if one assumes that the constant is approximately AE/L. However, then a question arises as to the choice of length L. Possibly, one could let L vary and be equal to the instantaneous distance that the stress wave has traveled in the body. Another way to arrive at spring and damping constants is to correlate them with a coefficient of restitution. These coefficients are easy to measure and are often tabulated. Mr. Robert Coberly, Research Directorate, Weapons Laboratory, WECOM, has derived a relationship that will permit one to choose spring and dashpot values if the coefficient of restitution is known. The spring will then account for the force history of impact; this feat cannot be accomplished with the use of a coefficient of restitution without the spring.

The second difficulty with the spring concept is that, when a digital computer is used to solve the equations, an extremely small time increment is needed when a stiff spring is present. Otherwise, the momenta of the colliding bodies will, in the space of a single time-increment, carry these bodies so deeply into each other that the displacements represent extremely high force levels when multiplied by a large spring constant.

Another method of treating impact is to consider that the colliding bodies are hypothetically broken into a series of point masses, springs, and dashpots as follows:



This mechanical system is studied in Appendix C. However, the analysis is still too time-consuming on a computer to be practical in a large weapon model. What is needed is a transfer function that will give a direct relation between input displacement and output force. Such a model could account for another facet of impact not considered by the previously described methods. That facet is the propagation and reflection of stress waves. Parts of a colliding body, not close to the impact point, do not receive information that contact has been made until a finite length of time has elapsed. How the stress wave is reflected depends on the boundary conditions. This approach by which the body is divided into a number of segments is called a lumped-mass or finite-element approach. Simplification of this approach may be possible by replacement of the many masses and springs with a single mass-spring system where the spring constant is a function of time. This constant would be $K = M\ddot{X}/X$, where the position X of the single mass would be equal to the position history of the first mass in the large finite-element model. This ratio has been found to remain constant even if the initial conditions are changed. Thus, for a particular problem, one might establish K(t) and expect it to hold regardless of the impact velocities.

A continuum approach might also be considered. This can also be used to account for the propagation and reflection of stress waves. An example of how it might be used follows:

Navier's equation is:

$$G \nabla^2 U_i + (\lambda + G) \frac{\partial \Delta}{\partial x_i} = \rho \ddot{U}_i$$

Assume

$$\omega = v = 0$$

$$U = U(x, t)$$

Substitute into Navier's equation to obtain

$$\frac{\partial^2 U}{\partial x^2} = \frac{\rho}{\lambda + 2G} \frac{\partial^2 U}{\partial t^2}$$

$$U_{tt} = \frac{\lambda + 2G}{\rho} U_{xx}$$

$$U_{tt} = c^2 U_{xx}$$

$$c = \sqrt{\frac{\lambda + 2G}{\rho}}$$

Δ = dilitation

λ = Lamé's constant

G = Shear modulus

$$U_{tt} = c^2 U_{xx}$$

$$U(0, t) = f(t)$$

$$U(L, t) = 0$$

$$U(x, 0) = 0$$

$$U_t(x, 0) = h(x)$$

Let

$$U = v + g(x) f(t)$$

$$v(0,t) + g(0) f(t) = f(t)$$

$$v(L, t) + g(L) f(t) = 0$$

Require

$$v(0, t) = v(L, t) = 0$$

$$g(0) = 1$$

$$g(L) = 0$$

$$v_{tt} + g(x)\ f''(t) = c^2\ v_{xx} + g''(x)\ f(t)$$

$$v_{tt} - c^2\ v_{xx} = g''(x)\ f(t) - g(x)\ f''(t)$$

Let

$$g(x) = 1 - \frac{x}{L}$$

$$v_{tt} - c^2\ v_{xx} = (\frac{x}{L} - 1)\ f''(t)$$

$$v(0,\ t) = 0$$

$$v(L,\ t) = 0$$

$$v(x,\ 0) = -(1 - \frac{x}{L})\ f(0)$$

$$= 0,$$

Since $f(0) = 0$

$$v_t(x,\ 0) = h(x) - g(x)\ f'(0)$$

Let

$$v = \Sigma\ \omega_n(t)\ \sin\frac{n\pi x}{L}$$

$$\text{I.C.} \Rightarrow \Sigma\ \omega_n(0)\ \sin\frac{n\pi x}{L} = 0$$

$$\therefore\ \omega_n(0) = 0$$

Write equation as

$$v_{tt} - c^2\ v_{xx} = \Sigma\ Q_n(t)\ \sin\frac{n\pi x}{L},$$

where

$$Q_n(t) = \frac{2}{L}\int_0^L (\frac{x}{L} - 1)\ f''(t)\ \sin\frac{n\pi x}{L}\ dx.$$

By substitution,

$$\Sigma\ \omega_n''\ \sin\frac{n\pi x}{L} + \Sigma\ c^2\ \frac{n^2\pi^2}{L^2}\ \omega_n\ \sin\frac{n\pi x}{L} = \Sigma\ Q_n\ \sin\frac{n\pi x}{L}$$

$$\omega_n''(t) + c^2\ \frac{n^2\pi^2}{L^2}\ \omega_n(t) = Q_n(t)$$

$$U_t(x, 0) = h(x)$$

$$v_t(x, 0) + g(x)\, f'(0) = h(x)$$

$$v_t(x, 0) = h(x) - f'(0)(1 - \frac{x}{L})$$

$$v_t(x, 0) = \Sigma\, \beta_n \sin \frac{n\pi x}{L}$$

where

$$\beta_n = \frac{2}{L} \int_o^L [h(x) - f'(0)(1 - \frac{x}{L}) \sin \frac{n\pi x}{L}]\, dx$$

$$v_t(x, 0) = \Sigma\, \omega_n' (0) \sin \frac{n\pi x}{L}$$

$$\therefore \omega_n' (0) = \beta_n$$

$$\omega_n''(t) + \frac{c^2 n^2 \pi^2}{L^2} \omega_n(t) = G_n(t)$$

$$\omega_n(0) = 0$$

$$\omega_n'(0) = \beta_n$$

One can solve for ω_n.

Then,

$$v = \Sigma\, \omega_n(t) \sin \frac{n\pi x}{L}$$

$$U = v + (1 - \frac{x}{L})\, f(t)$$

$$U = \Sigma\, \omega_n(t) \sin \frac{n\pi x}{L} + (1 - \frac{x}{L})\, f(t)$$

Thus,

for a given input displacement $f(t)$ at $x=0$, one knows $U(x, t)$

Thus,

one can find $\varepsilon = \frac{\partial U}{\partial x}$.

The generalized Hook's Law reduces to

$$\sigma = 2\,G\,\varepsilon + \lambda\,\varepsilon = (2\,G + \lambda)\,\varepsilon$$

$$\sigma = (2\,G + \lambda)\,\frac{\partial U}{\partial x}$$

What is needed is

$$-\sigma A = K\,U + c\,\dot{U} \doteq K\,f(t)\text{ , neglecting damping}$$

$$K = \frac{-\sigma A}{f(t)}$$

$$K = \frac{-(2\,G + \lambda)\,\frac{\partial U}{\partial x}\,A}{f(t)}$$

$$K = -\frac{2\,G + \lambda}{f(t)}\left[\frac{\pi}{L}\sum_{n=1}^{\infty} n\,\omega_n(t) - \frac{1}{L}\,f(t)\right] A \qquad (X=L)$$

where

$$\omega_n''(t) + \frac{c^2 n^2 \pi^2}{L^2}\,\omega_n(t) = Q_n(t)$$

$$\omega_n(0) = 0$$

$$\omega_n'(0) = \beta_n$$

$$Q_n(t) = \frac{2}{L}\int_o^L \left(\frac{x}{L} - 1\right) f''(t)\,\sin\frac{n\pi x}{L}\,dx$$

$$\beta_n = \frac{2}{L}\int_o^L \left(h(x) - f'(0)\left(1 - \frac{x}{L}\right)\right)\sin\frac{n\pi x}{L}\,dx$$

λ = Lamé's constant

G = Shear modulus

$f(t)$ = displacement history at $x=0$

$h(x) = U_t(x, 0)$ = rate of change of U along body at $t=0$.

Probably $U_t(0, 0) = f'(0)$

$U_t(x > 0, 0) = 0$

Let

$$f(t) = a\,t$$

$$Q_n(t) = \frac{2}{L}\int_o^L \left(\frac{x}{L} - 1\right)(0)\,\sin\left(\frac{n\pi x}{L}\right) dx = 0$$

$$h(x) = \begin{cases} f'(0) \text{ at } x=0 \\ 0 \text{ at } x > 0 \end{cases}$$

$$\beta_n = \frac{2}{L}\int_o^L \left(-f'(0)\,\left(1 - \frac{x}{L}\right)\sin\frac{n\pi x}{L}\right) dx$$

$$\beta_n = -\frac{2\,a}{L}\int_o^L \left(1 - \frac{x}{L}\right)\sin\frac{n\pi x}{L}\,dx$$

$$= -\frac{2\,a}{L}\left[\int_o^L \sin\frac{n\pi x}{L}\,dx - \int_o^L \frac{x}{L}\sin\frac{n\pi x}{L}\,dx\right]$$

$$= -\frac{2\,a}{L}\left[-\frac{L}{n\pi}\cos\frac{n\pi x}{L}\Big|_o^L - \frac{1}{L}\left(\frac{L^2}{n^2\pi^2}\sin\frac{n\pi x}{L} - \frac{L}{n\pi}\,x\cos\frac{n\pi x}{L}\right)_o^L\right]$$

$$= -\frac{2\,a}{L}\left[-\frac{L}{n\pi}(-1)^n + \frac{L}{n\pi} - \frac{1}{L}\left(-\frac{L^2}{n\pi}\,(-1)^n\right)\right]$$

$$= -\frac{2\,a}{L}\left[\frac{L}{n\pi}\right] = \frac{-2\,a}{n\pi}$$

$$\beta_n = \frac{-2\,a}{n\pi}$$

$$\omega_n'' + \gamma^2\,\omega_n = 0$$

$$\omega_n(0) = 0$$

$$\omega_n'(0) = -\frac{2\,a}{n\pi}$$

$$\omega_n = A_n\cos\gamma_n t + B_n\sin\gamma_n t$$

$$\omega_n(0) = 0 \Rightarrow A_n = 0$$

$$\omega_n' = B_n\,\gamma\cos\gamma_n\,t$$

$$\omega_n'(0) = B_n\,\gamma_n = -\frac{2\,a}{n\pi}$$

$$B_n = -\frac{2\,a}{\gamma_n n\pi}$$

$$B_n = -\frac{2\,a}{n\pi}\ \frac{L}{cn\pi} = -\frac{2\,a\,L}{cn^2\pi^2}$$

$$\omega_n = -\frac{2\,a\,L}{cn^2\pi^2}\ \sin\frac{cn\pi}{L}\,t$$

$$\therefore\quad K = -A\,\frac{2G+\lambda}{f(t)}\left[\sum_{n=1}^{\infty}\left(-\frac{2\,a}{cn\pi}\ \sin\frac{cn\pi}{L}\,t\right)-\frac{f(t)}{L}\right]$$

$$K = -A\,\frac{2G+\lambda}{f(t)}\left[\sum_{n=1}^{\infty}\left(-\frac{2\,a}{cn}\ \sin\frac{cn\pi}{L}\,t\right)-\frac{f(t)}{L}\right]$$

$$f(t) = a\,t$$

$$K = A(2G+\lambda)\left[+\frac{1}{L}+\frac{1}{t}\sum_{n=1}^{\infty}\frac{2}{cn}\ \sin\frac{cn\pi}{L}\,t\right]$$

$$K = +A(2G+\lambda)\left[\frac{1}{L}+\frac{1}{t}\sum_{n=1}^{\infty}\frac{2}{cn}\ \sin\frac{cn\pi}{L}\,t\right]$$

$$K = +AE\left[\frac{1}{L}+\frac{1}{t}\sum_{n=1}^{\infty}\frac{2}{cn}\ \sin\frac{cn\pi}{L}\,t\right]$$ Spring Constant

This K can be shown to be $\quad c = \sqrt{\frac{E}{\rho}}$

$$K = AE\left[\frac{1}{L}+\frac{\pi}{ct}\ \left(1-\frac{c}{L}\,t\right)\right]\ ,\ 0 < t < \frac{2L}{c}\ ,\ \text{periodically repeated.}$$

APPENDIX G

COUPLING OF COMPLEX MODELS

Many occasions arise when it would be desirable to couple complex mathematical models. Often a very large effort is needed to successfully combine the computer programs. Sometimes it is better "to start from scratch" than to attempt to make the programs compatible that have been written by two different people. This problem arises in weapon dynamic studies where an important consideration is the interaction between a mount-vehicle system and a weapon system. For example, if someone has a model of a certain weapon mechanism, much information could be gained by coupling it with someone else's model of a certain helicopter. The basic problem is an extremely important one and has applications far beyond weapons. However, the problem is also very difficult. The coupled system behaves as a completely new system and often bears little relation to the independent behavior of either system. A review of the literature reveals very little information concerning this problem.

One approach is to simplify one or both of the complex models. In effect, one models the model. It is possible to simplify the model to the point where it is a single analytic expression relating the force exerted by the mount to the position, velocity, and acceleration of the weapon. (See Reference[3]) However, considerable deterioration in accuracy can easily occur. Another approach is outlined below. It is much more successful in its ability to preserve the true natures of the interacting systems. However, it is more difficult to apply; but for many problems, the application of the method is simpler than simultaneous solution of all equations in both models.

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[3]Ehle, P.E., "Mathematical Model of the Stoner 5.56mm Medium Machine Gun, XM207," WECOM Technical Report 70-114, AD 862081L, Research and Engineering Directorate, Rock Island, Ill. (Oct 1969)

Let $\vec{x}_V$ locate the position of the point on the vehicle that is connected to the gun. Let $\vec{x}_G$ locate the position of the point on the gun that is connected to the vehicle. Assume that some unknown interaction force exists at this common point that causes the vehicle and gun to move together there. Assume that this force can be represented by a Fourier sine or cosine series in $(0, t_o)$.

That is,

$$\vec{F}(t) = \sum_{n=1}^{K} \vec{A}_n \sin \frac{n\pi t}{t_o}$$

where

$$\vec{A}_n = \frac{2}{t_o} \int_o^{t_o} \vec{F}(t) \sin \frac{n\pi t}{t_o} \, d\, t$$

and t_o is the maximum time for which the solution is desired. The assumption that the masses move together at the coupling point is equivalent to the constraint equation $\vec{x}_V - \vec{x}_G = \vec{\alpha}$, where $\vec{\alpha}$ accounts for any difference in the origins of the two coordinate systems. The procedure is as follows:

1. Make an initial guess for the $\vec{A}_n$ values. Denote these by $\vec{A}_{n_{(0)}}$.
2. Run the vehicle and gun programs independently with a forcing function equal to the Fourier series established with $\vec{A}_{n_{(0)}}$ values. The result will be $\vec{x}_{V_{(0)}}(t)$ and $\vec{x}_{G_{(0)}}(t)$. However, these functions will almost certainly not satisfy the constraint equation.
3. Expand $\vec{x}_V$ and $\vec{x}_G$ in a Taylor series about $\vec{A}_{n_{(0)}}$.

$$\vec{x}_V = \vec{x}_{V_{(0)}} + \sum_{n=1}^{K} \left[\frac{\partial \vec{x}_{V_{(0)}}}{\partial \vec{A}_{n_{(0)}}} \right] \Delta \vec{A}_n + \sum_{n=1}^{K} \left[\frac{\partial^2 \vec{x}_{V_{(0)}}}{\partial \vec{A}_{n_{(0)}}^2} \right] \left(\Delta \vec{A}_n \right)^2 + \cdots$$

$$\vec{x}_G = \vec{x}_{G_{(0)}} + \sum_{n=1}^{K} \left[\frac{\partial \vec{x}_{G_{(0)}}}{\partial \vec{A}_{n_{(0)}}} \right] \Delta \vec{A}_n + \sum_{n=1}^{K} \left[\frac{\partial^2 \vec{x}_{G_{(0)}}}{\partial \vec{A}_{n_{(0)}}^2} \right] \left(\Delta \vec{A}_n \right)^2 + \cdots$$

4. Substitute the results of (3.) into the constraint equation. Neglect second and higher order terms.

$$\vec{x}_{v_{(0)}}(t) - \vec{x}_{G_{(0)}}(t) - \vec{\alpha} = \sum_{n=1}^{K} \left[\frac{\partial \vec{x}_{G_{(0)}}}{\partial \vec{A}_{n_{(0)}}} - \frac{\partial \vec{x}_{v_{(0)}}}{\partial \vec{A}_{n_{(0)}}} \right] \Delta \vec{A}_n$$

5. From the gun model determine $\frac{\partial \vec{x}_{G_{(0)}}}{\partial \vec{A}_{n_{(0)}}}$; and from the vehicle model determine $\frac{\partial \vec{x}_{v_{(0)}}}{\partial \vec{A}_{n_{(0)}}}$. If the governing equations are linear, the partial derivatives can be found directly. Otherwise, they can be estimated by the running of each model and the noting of changes in $\vec{x}_{G_{(0)}}$ and $\vec{x}_{v_{(0)}}$ as the $\vec{A}_n$ are varied.

6. Write the equation in (4.) for each of k points in time scattered throughout $(0, t_o)$. One knows the values of $\vec{x}_{v_{(0)}}$, $\vec{x}_{G_{(0)}}$, $\vec{\alpha}$, $\frac{\partial \vec{x}_{G_{(0)}}}{\partial \vec{A}_{n_{(0)}}}$ and $\frac{\partial \vec{x}_{v_{(0)}}}{\partial \vec{A}_{n_{(0)}}}$. Thus, one has k algebraic equations and k unknowns $\Delta \vec{A}_n$.

7. Solve for $\Delta \vec{A}_n$ and write $\vec{A}_{n_{(1)}} = \vec{A}_{n_{(0)}} + \Delta \vec{A}_n$.

8. Repeat the above process, and use $\vec{A}_{n_{(1)}}$ as the initial guess in the Fourier series.

9. Repeat the entire process until $\vec{x}_{v_{(i)}} - \vec{x}_{G_{(i)}}$ is as close as possible or as close as desired to $\vec{\alpha}$.

There are two basic problems that may be encountered when one applies this method. First, for a very irregular interaction force over a long time interval, a large k value may be needed. Also, for some problems, a large number of iterations may be necessary.

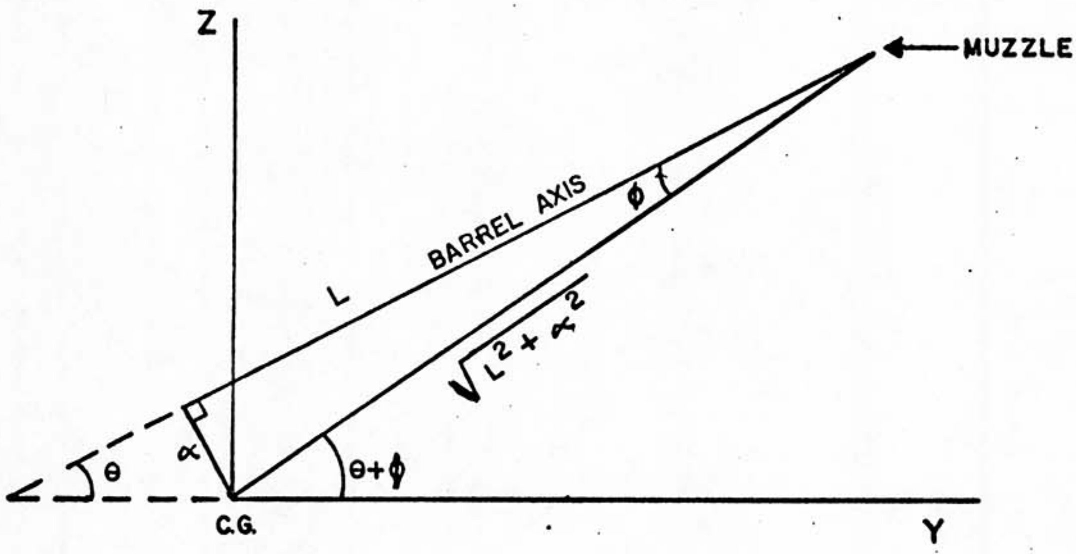
The method was applied to simple spring mass systems with nonlinear spring constants. It was also applied to a complex weapon model (XM207) in conjunction with a simple mount model. In all cases, convergence to the known interaction force was very rapid. Four iterations gave very good results. The method will not work equally well for all problems.

The approach described represents a "first cut" at the problem. Much work remains to satisfactorily solve it.

APPENDIX H

SIMPLIFIED EXTERIOR BALLISTICS ANALYSIS

A simplified exterior ballistics analysis was made for the determination of accuracy trends as various weapon parameters were varied. A sophisticated analysis is considered unnecessary for this purpose. The projectile is given only two degrees of freedom, and the drag force is assumed to be zero. Gravity is the only force that acts on the projectile. The axis of the barrel does not pass through the pivot point, which is assumed to be the center of gravity.



NOTE: α = Minimum distance from pivot point to bore axis

Figure 45 Geometry for Exterior Ballistics Analysis

Coordinates of Muzzle:

$$Z = \sqrt{L^2 + \alpha^2}\ \sin(\theta + \phi)$$

$$Y = \sqrt{L^2 + \alpha^2}\ \cos(\theta + \phi)$$

Components of Muzzle Velocity:

$$V_y = V \cos \Theta \qquad V = \text{muzzle velocity}$$

$$V_z = V \sin \Theta$$

Total velocity components of projectile at muzzle:

$$\dot{y} = \dot{Y} + V \cos \Theta = - \dot{\Theta} \sqrt{L^2 + \alpha^2} \sin (\Theta + \phi) + V \cos \Theta$$

$$\dot{z} = \dot{Z} + V \sin \Theta = \dot{\Theta} \sqrt{L^2 + \alpha^2} \cos (\Theta + \phi) + V \sin \Theta$$

The equation governing vertical motion is:

$$\ddot{z} = -g$$

$$\dot{z} = -gt + c_1$$

$$z = - \tfrac{1}{2} gt^2 + c_1 t + c_2$$

$$\text{at } t = t_i: z = z_i \quad \dot{z} = \dot{z}_i$$

$$\therefore \quad z_i = - \tfrac{1}{2} gt_i^2 + c_1 t_i + c_2$$

$$\dot{z}_i = -gt_i + c_1$$

$$c_1 = \dot{z}_i + gt_i$$

$$c_2 = z_i + \tfrac{1}{2} gt_i - t_i (\dot{z}_i + gt_i)$$

$$\therefore \quad z = - \tfrac{1}{2} gt^2 + [\dot{z}_i + gt_i] t + [z_i + \tfrac{1}{2} gt_i - t_i (\dot{z}_i + gt_i)] \qquad \text{(H-1)}$$

The equation governing horizontal motion is:

$$\ddot{y} = 0$$

$$y = k_1 t + k_2$$

$$\text{at } t = t_i: y = y_i \quad \dot{y} = \dot{y}_i$$

$$\therefore \quad y_i = k_1 t_i + k_2$$

$$\dot{y}_i = k_1$$

$$k_2 = y_i - \dot{y}_i t_i$$

$$y = \dot{y}_i t + [y_i - \dot{y}_i t_i] \quad \text{(H-2)}$$

Now determine t when y = R = range to target

$$R = \dot{y}_i t_R + [y_i - \dot{y}_i \, t_i]$$

$$t_R = \frac{1}{\dot{y}_i} [R - y_i + \dot{y}_i t_i] \quad \text{(H-3)}$$

Next determine z when $t = t_R$ by substitution of (H-3) into (H-1)

$$z_R = -\tfrac{1}{2} g \frac{1}{\dot{y}_i} [R-y_i + \dot{y}_i t_i]^2 + [\dot{z}_i + yt_i] \frac{1}{\dot{y}_i} [R - y_i + \dot{y}_i \, t_i]$$
$$+ [z_i + \tfrac{1}{2} gt_i - t_i(\dot{z}_i + gt_i)] \quad \text{(H-4)}$$

Assume initial time $t_i = 0$.

Then

$$z_R = -\tfrac{1}{2} g \frac{1}{\dot{y}_i^2} [R - y_i]^2 + \dot{z}_i \frac{1}{\dot{y}_i} [R - \dot{y}_i] + z_i \quad \text{(H-5)}$$

where

$$z_i = \sqrt{L^2 + \alpha^2} \; \sin(\Theta + \phi)$$

$$\dot{z}_i = \dot{\Theta} \sqrt{L^2 + \alpha^2} \; \cos(\Theta + \phi) + V \sin \Theta$$

$$y_i = \sqrt{L^2 + \alpha^2} \; \cos(\Theta + \phi)$$

$$\dot{y}_i = -\dot{\Theta} \sqrt{L^2 + \alpha^2} \; \sin(\Theta + \phi) + V \cos \Theta$$

$$\phi = \sin^{-1} \frac{\alpha}{L}$$

z_R is the vertical coordinate of the impact point at range R.

Nominal Values are:

α = .065 ft

L = 1.645 ft

R = 450 ft

V = 3180 ft/sec

$\Theta = 0$

$\dot{\Theta}$ is obtained from the analysis of rifle rotation.

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APPENDIX I
GAS DYNAMICS

Accurate representations of gas forces are of great importance to mathematical models of automatic gas-operated weapons. At the present time, experimental pressure-time curves provide probably the most accurate simulation of these forces. However, to retain this accuracy, one must make new measurements each time changes are made in weapon parameters. Also, because of round-to-round variations, the use of any single curve is not realistic. Theoretical descriptions would allow one to account for the influence on gas force caused by changes in weapon parameters, but much work remains before really accurate predictions can be made.

In the M16A1 model, experimental pressure curves are used. However, there are many uncertainties associated with these measurements. Pressure gauges made by different manufacturers tend to disagree with one another. Also, it is very difficult to get measurements at exact points where they are needed. In addition, the presence of the gauge tends to perturb the normal fluid flow. Nevertheless, experimental measurements are probably more reliable than theoretical predictions.

There exist many computer programs that treat interior ballistics. Very few consider propellants with deterrent coatings. An unpublished one that does was constructed by T. Trafton at BRL. All of these analyses introduce gross approximations, but for many purposes, the results are satisfactory. One of the most successful analyses of flow in the M16A1 gas tube is described in Reference[9]. This analysis is fairly detailed and complex, and it provides an excellent tool for individual study of the gas tube. However, in the modeling of the dynamics of an entire weapon, a delicate balance must be struck between complexity and the limitations of computer storage and running time. This problem is compounded if the Monte Carlo technique, which requires

[9] Spurk, J.H., "The Gas Flow in Gas-Operated Weapons," Ballistic Research Laboratories Report No. 1475, Aberdeen Proving Ground, Md. (Feb 1970)

many computer runs, is used. What is needed in this instance is a relatively simple model that can accept wide modifications in flow geometry and thermal properties and provide approximate answers. With such a gas flow analysis included in the weapon model, one could, for example, look at trends in unlocking force as changes are made in the location of the gas port, the thermal properties of the tube, or changes in the tube geometry. Such an analysis has been developed in theory in Reference[7]. However, the present computer program exhibits an instability for high rates of change of pressure typical of a weapon, and this problem is currently under investigation.

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[7] Ehle, P.E. and Rahe, A.E., "Development of a Finite Element Approach for Approximate Analysis of Unsteady Compressible Fluid Flow," WECOM Technical Report SWERR-TR-72-36, AD 746234, Rock Island, Ill (Jun 72)

APPENDIX J

CONVERSION OF PHYSICAL UNITS

TABLE 12 CONVERSION OF PHYSICAL UNITS

Area: $1\ m^2 = 10.764\ ft^2 = 1{,}550\ in^2$

Force: $1\ newton = 1\ kg\text{-}m/sec^2 = .22482\ lb_{force}$

Length: $1\ m = 3.281\ ft = 39.37\ in$

Mass: $1\ kg = 2.205\ lb_{mass}$

Pressure: $1\ newton/m^2 = 1\ kg/(m\text{-}sec^2) = .14511 \times 10^{-3}\ psi$

Velocity: $1\ m/sec = 3.281\ ft/sec = 39.37\ in/sec$

Cyclic rate:

Time for One Shot (milliseconds)	Shots Per Minute
60	1,000
61	984
62	968
63	952
64	938
65	923
66	909
67	896
68	882
69	870
70	857
71	845
72	833
73	822
74	811
75	800
76	789
77	779
78	769
79	759
80	750
81	741
82	732
83	723
84	714
85	706

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APPENDIX K

COMPUTER PROGRAM FOR M16A1 RIFLE AND SAMPLE OUTPUT

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```
0001          IMPLICIT REAL*8(A-H,O-Z)
0002          EXTERNAL FCT,OUTP
0003          REAL*4 ITMIN,ITMAX,ITX,ITY
0004          COMMON/MAS/NMASS
0005          COMMON/LAIN/ITMIN,ITMAX,ITX,ITY
0006          COMMON/RUNKA/PRMT(4),X(21),XD(21),XDD(21)
0007          COMMON/MODES/IMODE,IFG(30),IPT,IHAM,IUNLO,ISEP,IBBC,INIT,IRND
             1,NPOUN,ISR,IFFEQ,IRX,IPICK,I4DG
0008          COMMON/ASTHT/PBRCX     ,PBRCY(50),TFIRE,CCL,XLAMCC,XMUCC,ACC,BOREA,
             1 XKHYP,XCHYP,CF4,XMU4,XMB,XLD,PBRL,PCAL,CARTF(50),CARTX,PCAR
0009          COMMON/ASIT/XPBC,XKBBC,XCBBC,DERIV,XMBUL,XMARC  ,PUC,AREBOL,
             1 TDEL,XIB,G,TPORT,XCHCMG,XMUBCG,EJCAR,ELEV           ,XKLBC,XCLBC
0010          COMMON/NUM/NBRCH,IFLAG,NCAV,IFLG,IFIRE
0011          COMMON/ASTHEE/PCAVT      ,PCAVY(50),FBORY(50),PBORE,FBORX
0012          COMMON/ER/XKBUFF,XCBUFF,XMBC,XMBV(3),XIM,RH,ALPHA,TSPRIN,
             1 RHH0,ALH0,RHH1,ALH1,HK,HC,
             1   DCOCK  ,XKMCON,XCMOUN,XUHBC,XDHA,EXTC,EXTF,FDIS,FSTIC
             1,R1B,R2B,XMUB,FCONX
0013          COMMON/S/XK9,XC9,XK5,XC5,BETA,ALP1,THETO,XKHG,XCHG,XMHAM,XMG,
             1     XMBUFF,CON1,XPUC,XPUC1,RO,CON2,XMUFF,XCUFF
0014          COMMON/BUFF/XBFORW,XCFORW,XBREAR,XCREAR,SLACK,XUBUF,XMWT,WTI
0015          COMMON/GRAP/TGR1,TGR2
0016          FCONX=-1.D-10
0017          DO 1 I=1,21
0018          X(I)=0.0D0
0019          XD(I)=0.0D0
0020        1 XDD(I)=0.0D0
         C      I4DG       USE ONE IF THE BUFFER MASSES ARE TREATED SEPERATELY AND 0
         C      IF USING A LUMPED MASS MODEL
0021          IPT=-1
0022          DO 2000 I=1,30
0023     2000 IFG(I)=0
         C      IRND       IRND DENOTES WHICH ROUND IS BEING FIRED
         C      IMODE        IF IMODE IS ONE THEN PRINT FORCES
         C                   IF IMODE IS ZERO THEN DON'T PRINT FORCES
         C      ISEP       IS ZERO WHEN WEAPON FIRES
         C      ISEP       IS ONE WHEN CAVITY FORCE GOES ON
         C      IUNLO        IF 1 THEN BARREL AND THE BOLT CAN SEPERATE
         C                   IF ZERO THEN THEY REMAIN LOCKED TOGETHOR
         C      IHAM       ONCE LOCKING OF THE WEAPON HAS OCCURRED, THE
         C      HAMMER BEGINS TO ROTATE FORWARD.  IT STRIKES THE FIRING PIN,AND
         C      IT IS ASSUMED ALL ITS ENERGY IS ABSORBED IN FIRING THE WEAPON,
         C      AND WHAT EVER GETS TRANSFERRED TO THE MAIN GUN DUE TO THE SPRING FORCE.
         C      WHEN IT JUST BEGINS TO REVERSE DIRECTION, THEN ITS ROTATIONAL VELOCITY
         C      AND THE DERIVATIVE OF THE ROTATIONAL VELOCITY ARE SET TO ZERO.
         C      (IHAM TAKES ON THE VALUE ZERO AT THIS POINT.)
         C      NOTE: THERE IS NO ROTATIONAL MOTION UNTIL THE BOLT CARRIER BEGINS TO
         C      REVERSE . (IHAM IS RESET TO ONE WHEN THE HAMMER IS COCKED AGAIN.)
         C      IFLAG        TAKES THE VALUE ONE WHEN XMG - BLTC IS LESS THAN FDIS
         C             AND TAKES THE VALUE ZERO WHEN BLT - BLTC IS GREATER THAN XLD
         C      IFLG         1  BOLT HAS NO CARTRIDGE
         C                   2  BOLT HAS JUST THE CASE
         C                   3  BOLT HAS CASE AND PROJECTILE
```

```
0024            TFIRE=-.00000100
          C       IRBC       TAKES ON THF VALUE ONE WHEN THE SPRING FORCF BETWEEN THE ROLT
          C       CARRIER AND THE BOLT TS ON, AND WHEN WEAPON IS UNLOCKED
          C       INIT        WHEN WEAPON JUST BEGINS TO COCK IT TAKES ON THE VALUE ONE,
          C       AND IS SET BACK TO ZERO WHEN WEAPON FIRES.
          C       IFIRE        WHEN THE HAMMER ANGLE (THETA) RECOMES GREATER THAN ZERO, THEN
          C        THE WEAPON FIRES.(IFIRE IS SET TO ONF AT THIS POINT)
          C       IFIRE IS SET BACK TO ZERO WHEN ROLT BREAKS AWAY FROM THE GUN
          C       NROUN        NUMBER OF ROUNDS DESIRED
          C       IFREQ        FREQUENCY THAT THE FORCES ARE PRINTED OUT.
          C       IF       IREAD IS ONE THEN READ IN X AND XD
          C       IPICK        TAKES ON THE VALUE ONE WHEN THE BOLT CARRIER GETS
          C       BACK FAR ENOUGH TO PICK UP ANOTHER ROUND , AND GOES BACK TO ZERO WHEN THE
          C       GUN FIRES
          C        NSMPR       SAMPLE NUMBER FOR DETERMINING SAMPLE RATE IN GRAPHING
0025            NAMELIST/MODER/IRND,IMODE,ISEP,IUNLO,IHAM,IFLAG,IFLG,IBBC,INIT,
               1IFIRE,IFREQ,NROUN,IREAD,IPICK,NSMPR,I4DG,NMASS
0026            READ (5,MODER)
0027            PRINT 1776
0028       1776 FORMAT(30X,'CONTROL VARIABLES',/)
0029            WRITE(6,MODER)
0030            READ 100, PBRCX
0031            READ 2,NBRCH,(PBRCY(I),I=1,NBRCH)
0032            PBRCY(NBRCH + 1) =PBRCY(NBRCH)
0033            READ 100, PCAVT
0034            READ 2,NCAV,(PCAVY(I),I=1,NCAV)
0035            PCAVY(NCAV  + 1)=PCAVY(NCAV)
0036          2 FORMAT(I10,/,(8F10.5))
0037            PRINT 102, PBRCX ,( PBRCY(I),I=1,NBRCH)
0038        102 FORMAT(30X,' BREECH PRESSURE (LB/FT**2)',/,
               1 30X,' TIME STEP=',F16.8,/,(6F15.2))
0039            PRINT 103, PCAVT  ,(PCAVY(I),I=1,NCAV)
0040        103 FORMAT(20X,' CAVITY PRESSURE IN BOLT CARRIER (LB/FT**2)',/
               1,30X,' TIME STEP=',F16.8,/,(6F15.2))
0041            READ 100,CARTX
0042            READ 2,NCAR,(CARTF(I),I=1,NCAR)
0043            PRINT 165 , CARTX,(CARTF(I),I=1,NCAR)
0044            CARTF(NCAR  + 1)=CARTF(NCAR)
0045        165 FORMAT(30X,' CARTRIDGE CASE FORCE (LBS) ',/,30X,
               1' TIME STEP=',F16.8,/,(6F15.2))
0046            XI=NBRCH - 1
0047            PBRL=XI*PBRCX
0048            XI=NCAV - 1
0049            PCAL=XI*PCAVT
0050            XI=NCAR - 1
0051            PCAR=XI*CARTX
0052            READ 100,FBORX
0053            READ 2,NBOR,(FBORY(I),I=1,NBOR)
0054            XI=NBOR - 1
0055            PBORE=XI*FBORX
0056            PRINT 7001,FBORX,(FBORY(I),I=1,NBOR)
0057       7001 FORMAT(30X,' BORE FRICTION (LBS)',/,30X,
               1' TIME STEP=',F16.8,/,(6F15.2))
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0058        NAMELIST/INPT1/                       BOREA,XKHYP,XCHYP,XLD,CF4,XMU4
           1 ,XMB, G
0059        NAMELIST/INPT2/XBBC,XKBBC,XCBBC,DERIV,XKLBC,XMBUL,XCLBC,
           1 AREBOL      ,TDEL,XIB ,XMARC
0060        READ(5,INPT1)
0061        READ(5,INPT2)
      C       CONVERTS FROM POUNDS TO SLUGS
0062        XMBUL=XMBUL/G
0063        XMARC=XMARC/G
0064        XMB=XMB/G
0065        NAMELIST/INPT3/TPORT,XKBUFF,XCBUFF,XCBCMG,XIM,RH,ALPHA,
           1 XKMOUN,XCMOUN,TSPRIN,XMUBCG,XMBC,R18,R28,CCL,XMUB
0066         NAMELIST/INPT4/XK9,XC9,XK5,XC5,BETA,ALP1,THETO,XKHG,XCHG,XMHAM,
           1   XMG,XMBUFF  ,EJCAR,ELEV,XPUC,XPUC1,RO
0067        NAMELIST/INPT5/XBFORW,XCFORW,XBREAR,XCREAR,SLACK,XUBUF,XMWT
           1,TGR1,TGR2,XUHBC,XDHA,FXTC,EXTF,FDIS,XMUFF,XCUFF,FSTIC
           2,RHH0,RHH1,ALH0,ALH1,HK,HC
      C       COMMENTS ABOUT INPUTS
      C       TGR1         ESTIMATED TIME FOR THE FIRST SHOT
      C       TGR2       ESTIMATED TIME FOR SUBSEQUENT SHOTS
0068        READ(5,INPT3)
0069        READ(5,INPT4)
0070        READ(5,INPT5)
0071        CALL READIN
      C       CONVERTS FROM POUNDS TO SLUGS
0072        XMHAM=XMHAM/G
0073        XMG=XMG/G
0074        XMBUFF=XMBUFF/G
0075        XMBC=XMBC/G
0076        XMWT=XMWT/G
0077        PRINT 9
0078      9 FORMAT(' NAME ',29X,'DESCRIPTION',39X,'VALUE',2X,'UNITS',/)
0079        PRINT 10                          ,BOREA,XKHYP
0080     10 FORMAT(
          5' BOREA',4X,'BORE AREA',51X,F20.8,2X,'FT**2',//,
          6 ' XKHYP',4X,'SPRING THAT PREVENTS MOTION BETWEEN THE BOLT AND GUN
          7',8X,F20.8,2X,'LBS/FT',/)
0081        PRINT 11,XCHYP,XLD,CF4,XMU4,XMB,G
0082     11 FORMAT(' XCHYP',4X,'DAMPING FOR THE ABOVE SPRING',32X,F20.8,
          2 2X,'LBS-SEC',//,' XLD',6X ,
          2'DISTANCE BOLT CARRIER TRAVELS BEFORE GUN IS UNLOCKED',8X,
          * F20.8,2X,'FT',// ,
          3 ' CF4',6X,'DAMPING COEFFICIENT BETWEEN BOLT AND BOLT CARRIER',11X
          4 ,F20.8,2X,'LBS-SEC/FT',//,' XMU4',5X,
          5'COEFFICIENT OF FRICTION BETWEEN BOLT AND BOLT CARRIER',7X,F20.8,
          6 2X,'NONE',//,' XMB',6X,'MASS OF THE BOLT',44X,F20.8,2X,'SLUGS',//
          7,' G',8X,'GRAVITATIONAL CONSTANT',38X,F20.8,2X,'FT/SEC**2',/)
0083        PRINT 12 ,XBBC,XKBBC,XCBBC,DERIV,XKLBC,XMBUL
0084     12 FORMAT(' XBBC',5X,'DISTANCE BOLT TRAVELS IN THE BOLT CARRIER',19X,
          1 F20.8,2X,'FT'      ,//,' XKBBC',4X,
          2'SPRING BETWEEN BOLT AND BOLT CARRIER',24X,F20.8,2X,'LBS/FT',//,
          3' XCBBC',4X,'DAMPING COEFFICIENT FOR ABOVE SPRING',24X,F20.8,2X,
          4'LBS-SEC/FT',//,' DERIV',4X,'DERIVATIVE OF THETA WITH RESPECT TO X
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              4B-XBC',18X,F20.8,2X,'RAD/FT',//,
              5' XKLBC',4X,'SPRING BETWEEN BOLT AND BOLT CARRIER DURING LOCKING',
              69X,F20.8,2X,'LBS/FT',//,' XMBUL',4X,'MASS OF THE BULLET AND THE PR
              7OPELLANT',23X,F20.8,2X,'SLUGS',/)
0085          PRINT 13,XCLBC     ,AREROL      ,TDEL,XIB,XMARC
0086       13 FORMAT(' XCLBC',4X,'DAMPING CORRESPONDING TO THE SPRING XKLBC ',
              118X,F20.8,2X,'LBS-SEC/FT',//,
              3' AREROL',3X,'AREA OF CAVITY THE GAS FORCE ACTS ON',24X,F20.8,2X,
              4'FT**2',//,
              4' TDEL',5X,'TIME DELAY BETWEEN PORT AND CAVITY',26X,F20.8,2X,
              5   'SEC',//,
              5' XIB',6X,'MOMENT OF INERTIA OF THE BOLT',31X,
              5F20.8,2X,'SLUGS-FT**2'
              6 ,//,' XMARC',4X,'MASS OF THE CARTRIDGE CASE ',33X,F20.8,2X,'SLUGS
              6',/)
0087          PRINT 14,TPORT,XKBUFF,XCBUFF,XCBCMG,XIM,RH
0088       14 FORMAT(' TPORT',4X,'TIME IT TAKES PROJECTILE TO REACH PORT',22X,
              1 F20.8,2X,'SEC',//,' XKBUFF',3X,
              2 'INTERACTION SPRING BETWEEN BOLT CARRIER AND BUFFER',
              310X,F20.8,2X,'LBS/FT'
              3 ,//,' XCBUFF',3X,'DAMPING COEFFICIENT FOR ABOVE SPRING',24X,
              4 F20.8,2X,'LBS-SEC/FT',//,' XCBCMG',3X,
              4 'DAMPING BETWEEN BOLT CARRIER AND MAIN GUN',19X,F20.8,2X,
              4'LBS-SEC/FT'
              5  ,//,' XIM',5X,'MOMENT OF INERTIA OF THE HAMMER',29X,F20.8,2X,'SL
              6UGS-FT**2 ',//,' RH',7X,'RADIAL DISTANCE OF THE HAMMER TO POINT OF
              7 CONTACT',11X,F20.8,2X,'FT',/)
0089          PRINT 15,ALPHA            ,XKMOUN,XCMOUN,TSPRIN
0090       15 FORMAT(' ALPHA',4X,'REFERENCE ANGLE ON THE HAMMER',31X,F20.8,2X,
              2'RAD',//,
              3' XKMOUN',3X,'MOUNT SPRING CONSTANT',39X,F20.8,2X,'LBS/FT',//,
              4' XCMOUN',3X,'DAMPING FOR ABOVE SPRING',36X,F20.8,2X,'LBS-SEC/FT',
              6//,' TSPRIN',3X,'CONSTANT TORQUE DUE TO THE HAMMER SPRING',20X,
              6F20.8,2X,'FT-LBS',/)
0091          PRINT 16,XMUBCG,XMBC,XK9,XC9,XK5,XC5,BETA
0092       16 FORMAT(' XMUBCG',3X,'COEFFICIENT OF FRICTION BETWEEN THE BOLT CARR
              1RIER AND GUN',3X,F20.8,2X,'NONE',//,
              2' XMBC',5X,'MASS OF THE BOLT CARRIER',36X,F20.8,2X,'SLUGS',//,
              3' XK9',6X,'BACK PLATE SPRING CONSTANT',34X,F20.8,2X,'LBS/FT',//,
              3' XC9',6X,'DAMPING FOR THE ABOVE SPRING',32X,F20.8,2X,'LBS-SEC/FT'
              4,//,' XK5',6X,'DRIVE SPRING CONSTANT',39X,F20.8,2X,'LBS/FT',//,
              5' XC5', 6X,'DAMPING FOR ABOVE SPRING',36X,F20.8,2X,'LBS-SEC/FT',//
              6 ,' BETA',5X,'PRELOAD DISTANCE FOR THE DRIVE SPRING',23X,F20.8,2X,
              7'FT',/)
0093          PRINT 17,ALP1,THETO,XKHG,XCHG,XMHAM,XMG
0094       17 FORMAT(' ALP1',5X,'DISTANCE BUFFER TRAVELS BEFORE BACK PLATE IS HI
              1T',12X     ,F20.8,2X,'FT',//,
              2' THETO',4X,'INITIAL ANGLE OF THE HAMMER',33X,F20.8,2X,'RAD',//,
              3' XKHG',5X,'SPRING BETWEEN HAMMER AND GUN',31X,F20.8,2X,'LBS/FT',/
              4/,' XCHG' ,5X,'DAMPING FOR THE ABOVE SPRING',
              532X,F20.8,2X,'LBS-SEC/FT'
              5,//,' XMHAM',4X,'MASS OF THE HAMMER',42X,F20.8,2X,'SLUGS',//,
              6' XMG',6X,'MASS OF THE MAIN GUN',40X,F20.8,2X,'SLUGS',/)
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0095          PRINT 18,XMBUFF,EJCAR,FLEV,XPUC,XPUC1,RO
0096       18 FORMAT(' XMBUFF',3X,'MASS OF THE BUFFER',42X,F20.8,2X,'SLUGS',
             1//,' EJCAR',4X,        'DISTANCE BOLT TRAVELS BEFORE CARTRIDGE EJE
             2CTION ',12X,F20.8,2X,'FT',//,' ELEV',5X,
             3'ANGLE OF ELEVATION OF THE GUN',31X,F20.8,2X,'RAD',//,
             3' XPUC',5X,'DISTANCE OF BOLT WHEN ROUND IS PICKED UP',20X
             4  ,F20.8,2X,'FT',//,' XPUC1',4X,        'DISTANCE OF BOLT WHEN PIC
             5K UP FORCE IS OFF',18X,F20.8,2X,'FT',//,' RO',7X,
             6'DISTANCE FROM THE PIVOT POINT TO THE C.G. OF THE HAMMER', 5X
             6,F20.8,2X,'FT',/)
0097          PRINT 19,XBFORW,XCFORW,XBREAR,XCREAR
0098       19 FORMAT(' XBFORW',3X,'EFFECTIVE INTERNAL BUFFER SPRING CONSTANT (FO
             1RWARD)',9X,F20.8,2X,'LBS/FT',//,
             2' XCFORW',3X,'DAMPING COEF. FOR THE ABOVE SPRING',26X,F20.8,2X,
             3'LBS-SEC/FT',//,' XBREAR',3X,'EFFECTIVE INTERNAL BUFFER SPRING CON
             4STANT (REARWARD)',8X,F20.8,2X,'LBS/FT',//,
             5' XCREAR',3X,'DAMPING COEF. FOR THE ABOVE SPRING',26X,F20.8,2X,
             5'LBS-SEC/FT',/)
0099          PRINT 20,SLACK,XUBUF,XMWT,XUHBC,XDHA
0100       20 FORMAT(' SLACK',4X,'DISTANCE WTS. ARE ALLOWED TO MOVE',27X,F20.8,
             1 2X,'FT',//,
             2' XUBUF',4X,'COEF. OF FRICTION BETWEEN WTS. AND BUFFER',19X,
             3 F20.8,2X,'NONE',//,
             4' XMWT',5X,'MASS OF THE WTS IN THE BUFFER',31X,F20.8,2X,'SLUGS',//
             5,' XUHBC',4X,'COEF. OF FRICTION BETWEEN HAMMER AND B.C.',19X,
             6F20.8,2X,'NONE',//,' XDHA',5X,
             7'DISTANCE B.C. MOVES BEFORE FRICT. BETWEEN THE HAM. AND B.C.',
             81X,F20.8,2X,'FT',/)
0101          PRINT 31,EXTC,EXTF,FDIS,XMUFF,XCUFF,FSTIC
0102       31 FORMAT(' EXTC',5X,'EXTRACTOR DISTANCE ', 41X,F20.8,2X,'FT',// ,
             1' EXTF',5X,'EXTRACTOR FORCE',45X,F20.8,2X,'LBS',//,
             1' FDIS',5X,'DISTANCE BOLT CARRIER IS FROM BATTERY, HAMMER ROTATES'
             2 ,7X,F20.8,2X,'FT',//,
             3' XMUFF',4X,'COEF. OF FRIC. BETWEEN GUN AND BUFFER',23X,
             4F20.8,2X,'NONE',//,
             4' XCUFF',4X,'DAMPING BETWEEN BUFFER AND GUN',30X,F20.8,2X,'NONE',/
             5/,' FSTIC',4X,'STICTION FORCE',46X,F20.8,2X,'LBS',/)
0103          PRINT 35,RHH0,RHH1,ALH0,ALH1,HK,HC
0104       35 FORMAT(' RHH0',5X,'DISTANCE FROM HAMMER PIVOT TO B.C.',26X,
             1F20.8,2X,'FT',//,' RHH1',5X,
             2'DISTANCE FROM PIVOT TO TOP OF HAMMER ',23X,F20.8,2X,'FT',//,
             3' ALH0',5X, 'ANGLE CORRESPONDING TO RHH0',33X,F20.8,
             3 2X,'RAD',//,' ALH1',5X,'ANGLE CORRESPONDING TO RHH1 ',32X,F20.8,
             12X,'RAD',//,' HK',7X,'HAMMER IMPACT SPRING',40X,F20.8,2X,
             1'LBS/RAD',//,' HC',7X,'ABOVE DAMPING CONSTANT',38X,F20.8,
             12X,'LBS-SEC/RAD',/)
0105          PRINT 39,R1B,R2B,CCL,XMUB
0106       39 FORMAT(' R1B',6X,'INNER RADIUS CARTRIDGE CASE ',
             1 32X,F20.8,2X,'FT',//,' R2B',6X,'OUTER RADIUS CARTRIDGE CASE ',
             232X,F20.8,2X,'FT',//,' CCL',6X,
             3'CARTRIDGE CASE LENGTH ',38X,F20.8,2X,'FT',//,' XMUB',5X,
             4'COEFFICIENT OF FRICTION AT THE BASE',25X,F20.8,2X,'NONE',/)
0107          XMBV(1)=XMB
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            XMBV(2)=XMB + XMARC
            XMBV(3)=XMB + XMBUL   + XMARC
            X(5)=THETO
            CON1= DSIN(ELEV)
            CON2=DCOS(ELEV)
            IF(IREAD.EQ.1) GO TO 1700
C            COMPUTE EQUILIBRIUM POINTS
            X(2)=(-CON1*(XMWT+ XMBC + XMBUFF)*G+BETA*XK5)/XKLBC
            X(4)=X(2) +(-CON1*(XMWT + XMBUFF)*G+BETA*XK5)/XKBUFF
            IF(ELEV.GT.0.0D0)X(6)=X(4) - SLACK
            IF(ELEV.LE.0.0D0)X(6)=X(4)
            DO 1915  I=1,4
       1915 X(6 + I)=X(6)
       1700 READ 100,PRMT
            IF(IREAD.NE.1) GO TO 170
C            NOTE: X(6) CAN TAKE ON VALUES BETWEEN ZERO AND SLACK
            READ 100, X
             READ 100,XD
        100 FORMAT(8F10.5)
C ****FOLLOWING DETERMINES SAMPLING RATE FOR PLOTTING PURPOSES
        170 XR=NROUN
            WTI=0.0D0
       1929 NP=    .075/PRMT(3)
            ISR=NP/NSMPR
            IF(ISR.EQ.0) ISR=1
            ITMAX=    TGR1   + PRMT(1)
            ITMIN=PRMT(1)
            IRX=IRND
            CALL READIT
            IF(I4DG.NE.1) GO TO 2900
            CALL READUM
            CALL INITAL
C          INTEGRATION ROUTINE BEGINS HERE
       2900 KNUM=7 + NMASS
            IF(I4DG.EQ.1)KNUM=11   + NMASS
            PRINT 1938,NROUN
       1938 FORMAT('1',15X,'COMPUTED RESULTS FOR A ',I2,' SHOT BURST',//)
            N11=7
            IF(I4DG.EQ.1) N11=11
            L11=KNUM - 1
            DO 1942 I=N11,L11
       1942 X(I)=X(4)
            CALL RUNGE(KNUM,FCT,OUTP)
            CALL EXIT
            END
```

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0001          SUBROUTINE FCT(T,X,XD,XDD,ITEST)
0002          IMPLICIT REAL*8(A-H,O-Z)
0003          DIMENSION X(21),XD(21),XDD(21)
0004          COMMON/MODES/IMODE,IFG(30),IPT,IHAM,IUNLO,ISEP,IBBC,INIT,IRND
             1,NROUN,ISR,IFREQ,IRX,IPICK,I4DG
0005          COMMON/ASTHT/PBBCX     ,PBBCY(50),TFIRE,CCL,XLAMCC,XMUCC,ACC,BOREA,
             1 XKHYP,XCHYP,CF4,XMU4,XMB,XLD,PBRL,PCAL,CARTF(50),CARTX,PCAR
0006          COMMON/NUM/NBRCH,IFLAG,NCAV,IFLG,IFIRE
0007          COMMON/ASTHEE/PCAVT     ,PCAVY(50),FBORY(50),PBORE,FBORX
0008          COMMON/ASIT/XBBC,XKBBC,XCBBC,DERIV,XMBUL,XMARC ,PUC,AREBOL,
             1 TDEL,XIB,G,TPORT,XCBCMG,XMUBCG,EJCAR,FLEV          ,XKLBC,XCLBC
0009          COMMON/ER/XKBUFF,XCBUFF,XMBC,XMBV(3),XIM,RH,ALPHA,TSPRIN,
             1 RHHO,ALHO,RHH1,ALH1,HK,HC,
             1   DCOCK ,XKMOUN,XCMOUN,XUBBC,XDHA,EXTC,EXTF,FDIS,FSTIC
             1,R1B,R2B,XMUB,FCONX
0010          COMMON/S/XK9,XC9,XK5,XC5,BETA,ALP1,THETO,XKHG,XCHG,XMHAM,XMG,
             1     XMBUFF,CON1,XPUC,XPUC1,RO,CON2,XMUFF,XCUFF
0011          COMMON/BUFF/XBFORW,XCFORW,XBREAR,XCREAR,SLACK,XUBUF,XMWT,WTI
0012          COMMON/CNTRL/DELT,SUM
0013          COMMON/PRT/FB(12),FBC(12),FMG(15),FBUF(6)
0014          COMMON/DIF/DIFF1,DIFF2,DIFF3,DIFF4,DIFF5,DIFF6
0015          COMMON/MSSS/XMS(5),XKS(6),XCS(6)
0016          DATA TOFF/-10.D0/,TCOMP/0.0D0/,IKPTRK/0/ ,IBOR/0/,IHD/0/,IDID/0/
0017          COMMON/MAS/NMASS
        C      COMMENTS
        C      X(1),XD(1)       CORRESPOND TO THE BOLT
        C      X(2),XD(2)       CORRESPOND TO THE BOLT CARRIER
        C      X(3),XD(3)       CORRESPOND TO THE MAIN GUN
        C      X(4),XD(4)       CORRESPOND TO THE BUFFER
        C      X(5),XD(5)       CORRESPOND TO THE HAMMER
        C      X(5),XD(6)       CORRESPOND TO THE WTS. IN THE BUFFER
        C      OR X(6),X(7),X(8),X(9),X(10) CORRESPOND TO BUFFER WEIGHTS
        C      X(7) ...... CORRESPOND TO DRIVE SPRING MASSES
        C      OR X(11).......... CORRESPOND TO DRIVE SPRING MASSES
        C      LAST DEFINED PARAMETER IS THE GUN ROTATION
0018          DIFF1=X(1) - X(3)
0019          DIFD1=XD(1) - XD(3)
0020          DIFF2=X(1) - X(2)
0021          DIFD2=XD(1) - XD(2)
0022          DIFF3=X(3) - X(2)
0023          DIFD3=XD(3) - XD(2)
0024          DIFF4=X(4) - X(2)
0025          DIFD4=XD(4) - XD(2)
0026          DIFF5=X(3) - X(4)
0027          DIFD5=XD(3) - XD(4)
0028          DIFF6=X(6) - X(4)
0029          DIFD6=XD(6) - XD(4)
0030          IF(IPICK.EQ.0.AND.DIFF3.GT.XPUC)  IPICK=1
0031          IF(DIFF2.GE.XBBC.AND.IBBC.EQ.0) IBBC=1
0032          IF(IBBC.EQ.1.AND.DIFF1.GE.0.0D0.AND.IFIRE.EQ.0.AND.DIFF2.LT.XBBC)
             1  IBBC=0
0033          IF(IUNLO.EQ.1.AND.DIFF1.GT.0.0D0.AND.DIFD1.LT.0.0D0.AND.IFLG.EQ.3
             1.AND.ITEST.EQ.1) GO TO 79
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0034           IF(IFLAG.EQ.0.AND.DIFF3.LT.FDIS.AND.IBRC.EQ.0) IFLAG=1
0035           GO TO 788
0036        79 IUNLO=0
0037           XD(1)=XD(3)
0038       788 IF(-DIFF1.GT.EJCAR.AND.IFLG.EQ.2) IFLG=1
0039           IF(IFLG.EQ.1.AND. IFG(21).NE.1) GO TO 7500
0040           GO TO 7501
0041      7500 PRINT 7502,T
0042      7502 FORMAT(' CARTRIDGE CASE IS EJECTED AT T= ',F15.8)
0043           IFG(21)=1
0044      7501 IF(X(5).GE.0.0D0.AND.IFIRE.EQ.0.AND.IFLG.EQ.3) GO TO 75
0045      6655 IF(DIFF3.GE.XLD.AND.IFLAG.EQ.1) IFLAG=0
0046           IF(IBBC.EQ.1.AND.IFIRE.EQ.1.AND.FBC(8).LT.1.D-20)    IFIRE=0
0047           GO TO 700
0048        75 IF(IDID.EQ.0) TNOW=T
0049           IF(INIT.NE.0)     INIT=0
0050           IF(IDID.NE.1)  IDID=1
0051           IF(T.LT.TNOW + TBFO) GO TO 6655
0052           IDID=0
0053           IFIRE=1
0054           TFIRE=T
0055           IFG(21)=0
0056           IFLG=2
0057           INIT=0
0058           ISEP=0
0059           IRND=IRND + 1
0060           IPICK=0
0061            IF(IRND.EQ.1) THETK=THETO
0062           IF(IRND.EQ.1)THETO=THETO - .122
0063           PRINT 150,IRND
0064       150 FORMAT(' **********',' ROUND NUMBER ',I1,' BEGINS ',
              1'**********')
0065           PRINT 100,TFIRE
0066       100 FORMAT(' WEAPON FIRES AT T= ',F15.8)
          C     COMPUTE FB(1)
0067       700 TF1=T - TFIRE
0068           IF(IHAM.EQ.1.AND.XD(5).LT.0.0D0)IHAM=0
0069           IF(IBBC.EQ.1.AND.IHAM.EQ.0.AND.IUNLO.EQ.1) IHAM=1
0070           IF(TF1.GT.PBRL.OR.TF1.LT.0.0D0.OR.IFLG.EQ.3)  GO TO 10
0071            FB(1)=-XINTP(TF1,PBRCX,PBRCY)*BOREA
0072           IF(IFG(1).EQ.1) GO TO 11
0073           IFG(1)=1
0074           PRINT 550,T
0075       550 FORMAT(' BREECH FORCE IS ON AT T=',F15.8)
0076           GO TO 11
0077        10 FB(1)=0.0D0
0078           IF(IFG(1).EQ.0) GO TO 11
0079           IFG(1)=0
0080            PRINT 551,T
0081       551 FORMAT(' BREECH FORCE IS OFF AT T=',F15.8)
          C     COMPUTE FB(2)
0082        11 IF(-DIFF1.LT.XPUC.AND.DIFF3.GT.XPROJ.AND.IFLG.NE.3.AND.
              1XD(2).LT.0.0D0) GO TO 8915
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      GO TO 8916
 8915 TEMP=IKPTRK
      FB(2)=(XME*G*CON2+ XKMAG*(DELSP*TEMP - XFL + YIHT))* 2. *
     1DSIGN(1.D0, DIFD1)*XUMAG1
      IF(IFG(30).EQ.1) GO TO 13
      IFG(30)=1
      PRINT 8917,T,FB(2)
 8917 FORMAT(' AMMO AND BOLT FRICTION IS ON AT T=',F15.8,'FRIC=',F15.8)
      GO TO 13
 8916 FB(2)=0.0D0
      IF(IFG(30).EQ.0) GO TO 13
      IFG(30)=0
      PRINT 8918,T
 8918 FORMAT(' BOLT AND AMMO FRICTION IS OFF AT T=',F15.8)
C       COMPUTE FB(3)
   13 IF(IFIRE.EQ.1.OR.IUNLO.EQ.0 .OR.DIFF1.LT.-EXTC.OR.IFLG.EQ.2)
     1 GO TO 14
      FB(3)=-EXTF
      IF(DIFF1.GT.0.0D0) GO TO 9916
      GO TO 9917
 9916 FB(3)=-XKHYP*DIFF1 - XCHYP*DIFD1 +FB(3)
 9917 IF(IFG(3).EQ.1) GO TO 15
      IFG(3)=1
      PRINT 554,T,FB(3)
  554 FORMAT(' SPRING EXTRACTOR FORCE IS ON AT T=',F15.8,' CONSTANT FORC
     1E',  F15.8)
      GO TO 15
   14 FB(3)=0.0D0
      IF(IFG(3).EQ.0) GO TO 15
      IFG(3)=0
      PRINT 556,T
  556 FORMAT(' SPRING BETWEEN BOLT AND BARREL IS OFF AT T=',F15.8)
C       COMPUTE FB(4)
   15 FB(4)= - CF4*DIFD2 + XMU4*XMBV(IFLG)*G*DSIGN(1.0D0,-DIFD2)*CON2
C       COMPUTE FB(5)
      IF(IBBC.EQ.0.OR.(DIFF2.LT.XBBC.AND.DIFF2.GT.XBBC - .52083D-2)
     1.OR.IFG(13).EQ.1) GO TO 17
      IF(DIFF2.LE.XBBC-.52083D-2) GO TO 9161
      FB(5)=-XKBBC*(DIFF2 - XBBC) - XCBBC*DIFD2
      IF(FB(5).GT.0.0D0) FB(5)=0.0D0
      GO TO 9162
 9161 FB(5)=-XKBBC*(DIFF2 - XBBC + .52083D-2) - XCBBC*DIFD2
      IF (FB(5).LT.0.0D0) FB(5)=0.0D0
 9162 IF(IFG(4).EQ.1) GO TO 80
      IFG(4)=1
      PRINT 557,T
  557 FORMAT(' SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T
     1=',F15.8)
      GO TO 80
   17 FB(5)=0.0D0
      IF(IFG(4).EQ.0) GO TO 80
      IFG(4)=0
      PRINT 558,T
```

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0128         558 FORMAT(' SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=',
                1 F15.8)
           C       COMPUTE FB(6) AND FB(7)       (ZERO UNLESS SPECIFIED OTHERWISE)
0129          80 FB(6)=0.0D0
0130             FB(7)=0.0D0
           C       COMPUTE FB(8)
0131          87 IF(DIFF2.GE.0.0D0) GO TO 20
0132             FB(8)=- XKLBC*(DIFF2) - XCLBC*DIFD2
0133             IF(FB(8).LT.0.0D0)FB(8)=0.0D0
0134             IF(IFG(5).EQ.1) GO TO 201
0135             IFG(5)=1
0136             PRINT 559,T
0137         559 FORMAT(' (FORWARD) BOLT PIN STRIKES END OF CAM PATH AT T=',F15.8)
0138             GO TO 201
0139          20 FB(8)=0.0D0
0140             IF(IFG(5).EQ.0) GO TO 201
0141             PRINT 560,T
0142         560 FORMAT(' (FORWARD) FORCE AT END OF CAM PATH IS OFF AT T=',F15.8)
0143             IFG(5)=0
           C       COMPUTE FB(9)
0144         201 IF(IPICK.EQ.0) GO TO 21
0145             IF(-DIFF1. LT. XPUC .AND.-DIFF1 .GT. XPUC1 . AND.
                1 XMBC*XD(2) + XMBV(IFLG)*XD(1).GT.0.0D0.AND.(ITEST.EQ.1.OR.
                2IFG(6).EQ.1).AND.T - TOFF.GT.TCOMP)GO TO 2111
0146             GO TO 21
0147        2111 TEMP=IKPTRK
0148             FB(9)=DIAC*(XME*G*CON2 + XKMAG*(DELSP*TEMP - XFL + YIHT))*
                1(XUMAG2 + XUMAG3)*DSIGN(1.D0,DIFD1)/DELSP
0149             IF(IFG(6).EQ.1) GO TO 22
0150             PRINT 561,T
0151         561 FORMAT(' FORCE TO PICK UP NEW ROUND IS ON AT T=',F15.8)
0152             IFG(6)=1
0153             IFLG=3
0154             PRINT 7553
0155        7553 FORMAT(/,' INELASTIC COLLISION OF THE BOLT AND ROUND')
0156             PRINT 7550,XMG,XD(1),XD(3)
0157        7550 FORMAT(/,' MASS OF THE GUN AND VELOCITIES OF THE BOLT AND ROUND ',
                1 /,' PRIOR TO ROUND BEING PICKED UP',/,' GUN MASS=',F15.8,
                2'  BOLT VELOCITY=',F15.8,' ROUND VELOCTTY (VELOCITY OF THE GUN)=',
                3F15.8)
0158             XMG=XMG-(XMARC+XMBUL)
0159             XD(1)=(XD(3)*(XMARC+ XMBUL) + XD(1)*XMBV(IFLG))/
                1(XMARC + XMBUL + XMBV(IFLG))
0160             PRINT 7589,XMG,XD(1),XD(1)
0161        7589 FORMAT(/,' MASS OF THE GUN AND VELOCITIES OF THE BOLT AND ROUND ',
                1 /,' AFTER ROUND IS PICKED UP'        ,/,' GUN MASS=',F15.8,
                2'  BOLT VELOCITY=',F15.8,' ROUND VELOCITY =',F15.8)
0162               PRINT 7599,FB(9)
0163        7599 FORMAT(' FRICTIONAL FORCE TO PICK UP NEW ROUND IS',F15.8)
0164             GO TO 22
0165          21 FB(9)=0.0D0
0166             IF(IFG(6).EQ.0) GO TO 22
0167             PRINT 562,T
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0168       562 FORMAT(' FORCE TO PICK UP NEW ROUND IS OFF AT T=',F15.8)
0169           IFG(6)=0
0170           TOFF=T
0171           IKPTRK=IKPTRK + 1
0172           TEMP=IKPTRK
0173           FMI=-XKMAG*(TEMP*DELSP + YIHT - XFL)
0174           FMA=-XKMAG*((TEMP - 1.)*DELSP + YIHT  - XFL)
0175           TCOMP=DSORT(AVE/(EPSOL*XKMAG))*DARCOS(FMI/FMA)
0176            PRINT 7600,TCOMP,FMI,FMA
0177      7600 FORMAT(/,' RESPONSE TIME OF THE MAGAZINE IS', F15.8,' FMI=',
              1F15.8,' FMA=',F15.8)
         C      COMPUTE FB(10)
0178        22 TGAS= T - TFIRE - TPORT - TDEL
0179           IF(TGAS.LT.0.0D0.OR.TGAS.GT.PCAL .OR.IFLG.EQ.3)  GO TO 23
0180           FB(10)=+XINTP(TGAS,PCAVT,PCAVY)*AREBOL
0181           IF(IFG(8).EQ.1) GO TO 24
0182           PRINT 563,T
0183       563 FORMAT(' CAVITY FORCE IS ON AT T=',F15.8)
0184           IFG(8)=1
0185           ISEP=1
0186           GO TO 24
0187        23 FB(10)=0.0D0
0188           IF(IFG(8).EQ.0) GO TO 24
0189           PRINT 564,T
0190       564 FORMAT(' CAVITY FORCE IS OFF AT T=',F15.8)
0191           IFG(8)=0
0192        24 IF(IUNLO.EQ.0.OR.IBBC.EQ.0) GO TO 5510
  .      C     TBBC1=T - TBBC
         C     IF(TBBC1.GT.PCAR) GO TO 5510
         C     FB(11)= XINTP(TBBC1,CARTX,CARTF)
0193           IF(-DIFF1.GT.CCL.OR.-XMBC*DIFD3.GT.0.0D0) GO TO 5510
0194           FB(11)=-CARTF(1)*(DIFF1 + CCL)/CCL
0195           IF(IFG(20).EQ.1) GO TO 5520
0196           IFG(20)=1
0197           PRINT 5540,T
0198      5540 FORMAT( ' EXPERIMENTAL CARTRIDGE CASE EJECTION FORCE IS ON AT T= '
              1 , F15.3)
0199           GO TO 5520
0200      5510 FB(11)=0.0D0
0201           IF(IFG(20).EQ.0) GO TO 5520
0202           IFG(20)=0
0203           PRINT 5530,T
0204      5530 FORMAT( ' EXPERIMENTAL CARTRIDGE CASE EJECTION FORCE IS OFF AT T=
              1',F15.8)
         C      COMPUTE FORCES ON THE BOLT CARRIER
0205      5520 FB(12)=-CON1*G*XMBV(IFLG)
0206           FBC(1)= - FB(4)
0207           FBC(2)= - FB(5)
0208           FBC(3)=-FB(6)
0209           FBC(4)=-FB(8)
0210           FBC(5)=-FB(7)
         C      COMPUTE FBC(6)
0211           FBC(6)=XKBUFF*DIFF4 + XCBUFF*DIFD4
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0212            IF(FBC(6).LT.0.0D0)FBC(6)=0.0D0
0213            GO TO 29
0214         25 FBC(6)=0.0D0
           C      COMPUTE FBC(7)
0215         29 FBC(7)=-XCBCMG*(-DIFD3) + XMUBCG*(XMBV(IFLG) + XMBC)*G*DSIGN(1.D0,
               1DIFD3)*CON2
           C      COMPUTE FBC(8)   (ENTERS INTO THE CONSTRAINT EQUATIONS)
0216         27 FBC(8)=0.0D0
           C      COMPUTE FBC(9)
0217         28 FBC(9)=-FB(10)
           C      FRICTIONAL FORCE ACTING ON THE B.C. DUE TO THE AMMO IS COMPUTED HERE
0218            IF(T - TOFF.GT.TCOMP.AND.DIFF3.LT.XPROJ.AND.IFG(6 ).EQ.0)
               1GO TO 9600
0219            GO TO 9601
0220       9600 TEMP=IKPTRK
0221            FBC(10)=(XME*G*CON2+XKMAG*(DELSP*TEMP - XFL + YIHT))*2.*
               1DSIGN(1.D0,-DIFD3)*XUMAG1
0222             IF(IFG(28).EQ.1) GO TO 9650
0223            PRINT 9607,T,FBC(10)
0224            IFG(28)=1
0225       9607 FORMAT(' FRICTIONAL FORCE BETWEEN CARTRIDGES AND B.C. IS ON AT T=
               1',F15.8,' FBC(10)=',F15.8)
0226            GO TO 9650
0227       9601 FBC(10)=0.0D0
0228            IF(IFG(28).EQ.0) GO TO 9650
0229             IFG(28)=0
0230            PRINT 9609,T
0231       9609 FORMAT(' FRICTIONAL FORCE BETWEEN CARTRIDGES AND B.C. IS OFF AT T=
               1 ',F15.8)
0232       9650 IF(DIFF3.LT.XOHA.OR.IFG(12).NE.2) GO TO 9705
0233            FBC(11)=-DSIGN(1.D0,DIFD3)*TSPRIN*XUHBC*2./(RHH1*CONF1*DSIN(X(5)))
0234            IF(IFG(29).EQ.1) GO TO 9706
0235             IFG(29)=1
0236            PRINT 9707,T,FBC(11)
0237       9707 FORMAT(' FRICTION FORCE COMES ON BETWEEN HAMMER AND B.C. AT T=',
               1F15.8,' FBC(11)=',F15.8)
0238            GO TO 9706
0239       9705 FBC(11)=0.0D0
0240             IF(IFG(29).EQ.0) GO TO 9706
0241            PRINT 9709,T
0242       9709 FORMAT(' FRICTION FORCE BETWEEN HAMMER AND B.C. IS OFF AT T=',
               1F15.8)
0243             IFG(29)=0
0244       9706 FBC(12)=-CON1*XMBC*G
0245            FMG(1)=-XKMOUN*X(3) - XCMOUN*XD(3)
0246            FMG(2)=- FB(2)
0247            FMG(3)= - FB(3).
0248            FMG(4)= - FBC(7)
           C      COMPUTE FMG(5)
0249            FMG(5)=XK5*(X(L11) - X(3) )        + XC5*(XD(L11) - XD(3))
               1 - PREL
0250            IF(T.GE.TFIRE + PBORE.OR.IRND.EQ.0) GO TO 7050
0251            TNO=T - TFIRE
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      FMG(6)=XINTP(TNO,FBORX,FBORY)
      IF(IBOR.EQ.1) GO TO 7051
      IBOR=1
      PRINT 7053,T
 7053 FORMAT(' BORE FRICTION IS ON AT T=',F15.8)
      GO TO 7051
 7050 FMG(6)=0.0D0
      IF(IBOR.EQ.0) GO TO 7051
      IBOR=0
      PRINT 7052,T
 7052 FORMAT(' BORE FRICTION IS OFF AT T=',F15.8)
C      COMPUTE FMG(7)
 7051 IF(X(5).LT.0.0D0.OR.IHAM.EQ.0) GO TO 30
      FMG(7)=+XKHG*RH*DSIN(X(5)) + XCHG*RH*XD(5)*DCOS(X(5))
      IF(IFG(10).EQ.1) GO TO 31
      IFG(10)=1
      PRINT 567,T
  567 FORMAT(' HAMMER STRIKE OF THE MAIN GUN IS ON AT T=',F15.8)
      GO TO 31
   30 FMG(7)=0.0D0
      IF(IFG(10).EQ.0) GO TO 31
      IFG(10)=0
      PRINT 568,T
  568 FORMAT(' HAMMER STRIKE OF THE MAIN GUN IS OFF AT T=',F15.8)
C      COMPUTE FMG(8)      (ENTERS INTO CONSTRAINT EQN)
   31 FMG(8)=0.0D0
C      COMPUTE FMG(9)
       IF(DIFF5.LT.ALP1) GO TO 32
      FMG(9)=XK9*(-DIFF5+ ALP1) - XC9*DIFD5
      IF(FMG(9).GT.0.0D0) FMG(9)=0.0D0
      IF(IFG(11).EQ.1) GO TO 33
      IFG(11)=1
      PRINT 569,T
  569 FORMAT(' BUFFER STRIKES BACK PLATE AT T=',F15.8)
      PRINT 6666,XD(1),XD(2),XD(3)
 6666 FORMAT(/,' VELOCITIES PRIOR TO IMPACT',/,
     1' BLTD=',F15.8,' BLTCD=',F15.8,' XMGD=',F15.8,/)
      GO TO 33
   32 FMG( 9)=0.0D0
      IF(IFG(11).EQ.0) GO TO 33
      PRINT 570,T
  570 FORMAT(' BUFFER COMES OFF OF BACK PLATE AT T=',F15.8)
      PRINT 6667,XD(1),XD(2),XD(3)
 6667 FORMAT(/,' VELOCITIES AFTER IMPACT',/,
     1' BLTD=',F15.8,' BLTCD=',F15.8,' XMGD=',F15.8,/)
      IFG(11)=0
C ** SUM THE FORCES ON THE BOLT, BOLT CARRIER, AND GUN.
   33 FMG(10)=-FB(11)
      FMG(11)=-CON1*XMG*G
      FMG(12)=-FBC(10)
      FMG(13)=-FBC(11)
      FMG(14)=-(XMBUFF + XMWT)*G*CON2*DSIGN(1.D0,DIFD5)*XMUFF - XCUFF
     1*DIFD5
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0298          FMG(15)=-FR(9)
0299           FBUF(1)=-FBC(6)
0300          FBUF(2)=-FMG(9)
0301          FBUF(3)=-XK5*(X(4) - X(N11)) - XC5*(XD(4) - XD(N11)) + PREL
0302          FBUF(4)=-CON1*(XMBUFF - .0014)*G
0303          FBUF(6)=-FMG(14)
0304          IF(I4DG.EQ.1) GO TO 7042
0305          FBUFF=-XMWT*XUBUF*DSIGN(1.D0,DIFD6)*G*CON2
0306          IF(DIFF6.GT.WTI) GO TO 40
0307          IF(DIFF6.LT.-SLACK+WTI) GO TO 41
0308          IF(IFG(16).EQ.0) GO TO 42
0309          PRINT 2222,T
0310     2222 FORMAT(' FORWARD OR REARWARD INTERNAL SPRING BUFFER FORCE IS OFF A
         1T T=',F15.8)
0311          IFG(16)=0
0312          GO TO 42
0313       40 FBUFF=FBUFF - XBFORW*(DIFF6 - WTI) - XCFORW*DIFD6
0314          IF(IFG(16).EQ.1) GO TO 42
0315          IFG(16)=1
0316          PRINT 3333,T
0317     3333 FORMAT(' FORWARD INTERNAL SPRING FORCE IS ON AT T=',F15.8)
0318          GO TO 42
0319       41 FBUFF=FBUFF - XBREAR*(DIFF6 + SLACK -WTI) - XCREAR*DIFD6
0320          IF(IFG(16).EQ.1) GO TO 42
0321          IFG(16)=1
0322          PRINT 4444, T
0323     4444 FORMAT(' REAR INTERNAL BUFFER SPRING IS ON AT T=',F15.8)
0324       42 XDD(6)=(-G*XMWT*CON1+FBUFF)/XMWT
0325          GO TO 4227
0326     7042 FBUFF=0.0D0
0327          DO 8 K1=1,5
0328          XDD(5 + K1)= - XUBUF*DSIGN(1.D0,XD(5  + K1) - XD(4))*G*CON2
0329          FBUFF=XMS(K1)*XDD(5 + K1) + FBUFF
0330        8 XDD(5 + K1)=XDD(5 + K1) - CON1*G
0331          DO 8056 K1=1,4
0332          IF(X(6 + K1) - X(5 + K1).GT.0.0D0) GO TO 8057
0333          GO TO 8059
0334     8057 XDD(6 + K1)=XDD(6 + K1) - (XKS(K1+1)*(X(6 + K1) - X(5 + K1)) +
         1XCS(K1 + 1)*(XD(6 + K1) -XD(5 + K1)))/XMS(K1+1)
0335          XDD(5 + K1)=XDD(5 + K1) + (XKS(K1+1)*(X(6 + K1) - X(5 + K1)) +
         1XCS(K1 + 1)*(XD(6 + K1) -XD(5 + K1)))/XMS(K1 )
0336          IF(IFG(22 + K1).EQ.1) GO TO 8056
0337          IFG(22 + K1)=1
0338           PRINT 8060,T,(IFG(I),I=22,27)
0339     8060 FORMAT(' NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=',F15.8,
         16I3)
0340          GO TO 8056
0341     8059 IF(IFG(22 + K1).EQ.0) GO TO 8056
0342          IFG(22 + K1)=0
0343           PRINT 8060,T,(IFG(I),I=22,27)
0344     8056 CONTINUE
0345          IF(DIFF6.GT.WTI) GO TO 8070
0346          IF(IFG(22).EQ.0) GO TO 8090
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0347          IFG(22)=0
0348           PRINT 8060,T,(IFG(I),I=22,27)
0349          GO TO 8090
0350     8070 FBUFF=FBUFF - XKS(1)*(DIFF6 - WTI) - XCS(1)*DIFD6
0351          XDD(6)=XDD(6) -(XKS(1)*(DIFF6 - WTI) + XCS(1)*DIFD6)/XMS(1)
0352          IF(IFG(22).EQ.1) GO TO 8090
0353          IFG(22)=1
0354           PRINT 8060,T,(IFG(I),I=22,27)
0355     8090 IF(X(10) - X(4).LT.WTI - SLACK) GO TO 8091
0356          IFG(27).EQ.0) GO TO 4227
0357          IFG(27)=0
0358           PRINT 8060,T,(IFG(I),I=22,27)
0359          GO TO 4227
0360     8091 FBUFF=FBUFF -(X(10) - X(4) - WTI + SLACK)*XKS(6) -
             1 (XD(10) - XD(4))*XCS(6)
0361          XDD(10)=XDD(10) -((X(10) - X(4) - WTI + SLACK)*XKS(6) +
             1 (XD(10) - XD(4))*XCS(6))/XMS(5)
0362          IF(IFG(27).EQ.1) GO TO 4227
0363          IFG(27)=1
0364           PRINT 8060,T,(IFG(I),I=22,27)
0365     4227 SUM1=0.0D0
0366          SUM2=0.0D0
0367          SUM3=0.0D0
0368          SUM4=0.0D0
0369          FBUF(5)=-FBUFF
0370          XDD(L11)=( - FMG(5) - XK5*(X(L11) - X(L11 - 1))
             1- XC5*(XD(L11) - XD(L11 -1))-PREL)/XMDSP
0371          XDD(N11)=(- FBUF(3) - XK5*(X(N11) - X(N11 + 1))
             1-XC5*(XD(N11) - XD(N11 + 1))+PREL)/XMDSP
0372          DO 1952 I=N11M,L11M
0373     1952 XDD(I)=(XK5*(X(I + 1) - 2.*X(I) + X(I - 1))
             1+XC5*(XD(I + 1) - 2.*XD(I) + XD(I - 1)))/XMDSP
0374          DO 70 I=1,12
0375          SUM1=SUM1 + FB(I)
0376          SUM2=SUM2 + FBC(I)
0377       70 SUM3=SUM3 + FMG(I)
0378          DO 71 I=1,6
0379       71 SUM4=SUM4 + FBUF(I)
0380          SUM3=SUM3 + FMG(13) +FMG(14) + FMG(15)
0381          XDD(4)=SUM4/XMBUFF
0382          IF(IHAM.EQ.1.AND.IFLAG.EQ.1) GO TO 800
0383          GO TO 801
        C *** FOLLOWING COMPUTES THE ACCELERATION OF THE HAMMER WHEN
        C              THE GUN IS ABOUT TO FIRE
0384      800 XDD(5)=(TSPRIN - RH*FMG(7)*DCOS(X(5) + ALPHA))/XIH
0385          IF(IFG(12).EQ.1) GO TO 804
0386          PRINT 571,T
0387      571 FORMAT(' HAMMER IS CONTROLLED BY HAMMER SPRING AND MAIN GUN STOP A
             1T T=',F15.8)
0388          IFG(12)=1
0389          GO TO 804
0390      801 IF(ISEP.EQ.0  .OR.IFIRE.EQ.0.OR.ITEST.EQ.0.OR.DIFD3.LT.0.0D0.
             1OR.DIFF3.LT. 0.0D0) GO TO 803
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0391              IF(INIT.EQ.1) GO TO 804
0392              INIT=1
          C  ***** AFTER THE WEAPON HAS FIRED, THE HAMMER IS GIVEN AN INITIAL VELOCITY
          C       DETERMINED BY THE CONSTRAINT THE BOLT CARRIER HAS PUT ON THE HAMMER
          C  (I.E. ROTATIONAL VELOCITY)
0393              SEM=(RHH0*DSIN(ALH0) - DIFF3)/(RHH0*DC0)
0394              XD(5)=-(1./(1. + SEM*SEM))*DIFD3/(RHH0*DC0)
0395              X(5)=-ALH0 + DATAN(SEM)
0396              IFG(12)=5
0397              PRINT 572,T
0398          572 FORMAT(' HAMMER BEGINS TO COCK AT T=',F15.8)
0399              GO TO 804
          C  ***** THE ROTATIONAL VELOCITY AND ACCELERATION OF THE HAMMER ARE ZERO EXCEPT
          C       WHEN THE WEAPON IS ABOUT TO BE FIRED, AND WHEN IT IS COCKED
0400          803 IF(INIT.EQ.1.AND.IFIRE.EQ.1) GO TO 830
0401              SEM=(RHH0*DSIN(ALH0) - DIFF3)/(RHH0*DC0)
0402              IF((ITEST.EQ.1.AND.(X(5).LT.THETO.OR.(XD(5).GT.0.0D0.AND.
                 1    -ALH0 + DATAN(SEM ).LT.THETK.AND.X(5).LT.
                 1 THETK))).OR.IFG(12).EQ.2) GO TO 8509
0403              IF(INIT.EQ.0) GO TO 8509
0404              IF(X(5).LT.-ALH0 + DATAN(SEM)) GO TO 5577
0405              FBC(8)=DCOS(X(5))*(X(5)+ ALH0 - DATAN(SEM))*HK + DCOS(X(5))*HC
                 1*(  DIFD3/((1. + SEM*SEM)*RHH0*DC0) + XD(5))
0406              RHHU=DSQRT((RHH0*DS0 - DIFF3)*(RHH0*DS0 - DIFF3) + RHH0*RHH0*DC0
                 1*DC0)
0407              ALTEM=DARSIN(RHH0*DS0/RHHU)
0408              IF(IHD.EQ.1) GO TO 5555
0409              IHD=1
0410              PRINT 5578,T,FBC(8)
0411         5578 FORMAT(' HAMMER FORCE IS ON AT T=',F15.8,' FBC(8)=',F15.8)
0412              GO TO 5555
0413         5577 IHD=0
0414         5555 XDD(5)=(TSPRIN- RHHU*FBC(8)*DCOS(X(5)  +ALTEM))/XIM
0415              IF(IFG(12).EQ.1) GO TO 804
0416              PRINT 691,T
0417          691 FORMAT(' HAMMER CONSTRAINT FORCE IS OFF AT T=',F15.8 )
0418              IFG(12)=1
0419              GO TO 804
0420         8509  XDD(5)=0.0D0
0421              XD(5)=0.0D0
0422              IF(IUNLO.EQ.1.AND.X(5).GT.THETO)X(5)=THETO
0423              IF(IFG(12).EQ.2) GO TO 804
0424              PRINT 573,T
0425          573 FORMAT(' HAMMER ROTATIONAL VELOCITY AND ACCELERATION ARE ZERO STAR
                 1TING AT T=',F15.8)
0426              IFG(12)=2
0427          804 IF(INIT.EQ.1.AND.IFIRE.EQ.1) GO TO 830
0428              SOMV=DCOS(X(5))*RO*XMHAM
0429              IF(DABS(XDD(5) + XD(5)).LT.1.D-10) SOMV=0.0D0
0430              XDD(3)=(-(RO*(XDD(5)*DCOS(X(5)) - XD(5)*XD(5)*DSIN(X(5)))*
                 1XMHAM + FBC(8)+FMG(7)) + SUM3)/(XMHAM + XMG - SOMV*SOMV/XIM)
0431              IF((DIFF2.GE..52083D-2.AND.DIFF2.LE.XLD.AND.IBRC.EQ.0).AND.
                 1(IFG(13).EQ.1.OR.ITEST.EQ.1)) GO TO 879
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0432            IF(IFG(13).EQ.1.AND.ITFST.EQ.0) GO TO 879
          C ****** IF THE GUN ISN'T BEING COCKED OR LOCKED UP THEN THE ACCELFRATIONS OF
          C       THE BOLT AND BOLT CARRIER ARE COMPUTED HERE
0433            IF(IUNLO.EQ.0)XDD(3)=(XDD(3)*(XMHAM + XMG -SOMV*SOMV/XIM) + SUM1)/
               1(XMHAM + XMG + XMBV(IFLG) - SOMV*SOMV/XIM)
0434            XDD(1)=SUM1/XMBV(IFLG)
0435            XDD(2)=SUM2/XMBC
0436            IF(IUNLO.EQ.0)XDD(1)=XDD(3)
0437            IF(IFG(13).EQ.0) GO TO 875
0438            PRINT 575,T
0439        575 FORMAT(' WEAPON LOCKING FORCE IS OFF AT T=',F15.8)
0440            IFG(13)=0
0441            XMOM1=(XMG + XMBV(IFLG)+XMHAM)*XD(1) + XMBC*XD(2)
0442            XKEN=(XMG+XMBV(IFLG)+XMHAM)*XD(1)*XD(1)/2.+XMBC*XD(2)*XD(2)/2.+
               1 XIB*DERIV*DERIV*(XD(2)-XD(1))*(XD(2)-XD(1))/2.
0443            SA=XMG+XMBV(IFLG) +XMHAM
0444            SB=XMBC
0445            SC=(XMG+XMBV(IFLG)+XMHAM)/2.
0446            SD=XMBC/2.
0447            SE=0.0D0
0448            PRINT 2200,XD(1),XD(2)
0449            XD(2)=POSRT(SA,SB,SC,SD,SE,XMOM1,XKEN,1)
0450            XD(1)=(XMOM1 - SB*XD(2))/SA
0451            XD(3)=XD(1)
0452            PRINT 2300,XD(1),XD(2)
0453            GO TO 875
          C **** IF BEING LOCKED UP BUT NOT BEING COCKED THEN THE FOLLOWING EQUATIONS
          C       COMPUTE THE ACCFLERATIONS OF THE BOLT AND BOLT CARRIER
0454        879 C1= - XIB*DERIV*DERIV
0455            IF(IUNLO.EQ.0) GO TO 1701
0456            TEMP=(XMBV(IFLG) - C1)*(XMBC - C1) - C1*C1
0457            XDD(1)=(SUM1*(XMBC - C1) - SUM2*C1) /TEMP
0458            XDD(2)=(-SUM1*C1 + (XMBV(IFLG) - C1)*SUM2)/TEMP
0459            GO TO 1700
0460       1701 TEMP=(XMBV(IFLG) + XMG + XMHAM - C1)*(XMBC - C1) - C1*C1
0461            TEMP=TEMP - SOMV*SOMV*(XMBC - C1)/XIM
0462            SUM1=SUM1 + XDD(3)*(XMHAM + XMG - SOMV*SOMV/XIM)
0463            XDD(1)=(SUM1*(XMBC - C1) - SUM2*C1) /TEMP
0464            XDD(2)=(-SUM1*C1 + (XMBV(IFLG) + XMG + XMHAM - C1 -SOMV*SOMV/XIM)
               1*SUM2)/TEMP
0465            XDD(3)=XDD(1)
          C****     CONSTRAINT FORCE ACTING ON THE BOLT DURING LOCKING
0466       1700 FB(7)= C1*(XDD(1) - XDD(2))
0467            FBC(5)=-FB(7)
0468            IF(IFG(13).EQ.1) GO TO 875
0469            PRINT 574,T
0470        574 FORMAT(' WEAPON LOCKING FORCE IS ON AT T=',F15.8)
0471            IFG(13)=1
0472            XMOM1=(XMG + XMBV(IFLG)+XMHAM)*XD(1) + XMBC*XD(2)
0473            IF(IUNLO.EQ.1)XMOM1=XMBV(IFLG)*XD(1) + XMBC*XD(2)
0474            XKEN=(XMG+XMBV(IFLG)+XMHAM)*XD(1)*XD(1)/2.+XMBC*XD(2)*XD(2)/2.
0475            IF(IUNLO.EQ.1)XKEN=XMBV(IFLG)*XD(1)*XD(1)/2. + XMBC*XD(2)*XD(2)/2.
0476            SA=XMG+XMBV(IFLG) +XMHAM
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0477           IF(IUNLO.EQ.1)SA=XMBV(IFLG)
0478           SB=XMBC
0479           SC=(XMG+XMBV(IFLG)+XMHAM)/2.
0480           IF(IUNLO.EQ.1)SC=XMBV(IFLG)/2.
0481           SD=XMBC/2.
0482           SE=XIB*DERIV*DERIV/2.
0483           PRINT 2200,XD(1),XD(2)
0484      2200 FORMAT(/,' VELOCITIES OF THE BOLT AND BOLT CARRIER PRIOR TO THE DI
              1SCONTINUITY OF THE CAM',/,' BLTD=',F15.6,' BLTCD=',F15.6)
0485           XD(2)=POSRT(SA,SB,SC,SD,SE,XMOM1,XKEN,1)
0486           XD(1)=(XMOM1 - SB*XD(2))/SA
0487           IF(IUNLO.EQ.0) XD(3)=XD(1)
0488           PRINT 2300,XD(1),XD(2)
0489      2300 FORMAT(/,' VELOCITIES OF THE BOLT AND BOLT CARRIER AFTER THE DISCO
              1NTINUITY OF THE CAM',/,' BLTD=',F15.6,' BLTCD=',F15.6)
0490           GO TO 875
          C ***  FROM THIS POINT TO $$$$ , THE EQUATIONS ARE USED TO COMPUTE THE
          C      THE ACCELERATIONS OF THE BOLT AND THE BOLT CARRIER IF ONLY THE WEAPON IS
          C      BEING UNLOCKED
0491       830 RHHU=DSQRT((RHH0*DS0 - DIFF3)*(RHH0*DS0 - DIFF3) + RHH0*RHH0
              1*DC0*DC0)
0492           ALTEM=DARSIN(RHH0*DS0/RHHU)
0493           TEMP1=RHHU*DCOS(X(5) +ALTEM)
0494           SEM=(RHH0*DS0 - DIFF3)/(RHH0*DC0)
0495           THES=-1./((1. + SEM*SEM)*(RHH0*DC0))
0496           SEM1=(1. + SEM*SEM)*RHH0*DC0
0497           FDDP=-2.*SEM/(SEM1*SEM1)
0498           TEMP2=XIM*THES/TEMP1
0499           SOMV=DCOS(X(5))*RO*XMHAM
0500           IF(DABS(XDD(5) + XD(5)).LT.1.D-10) SOMV=0.0D0
0501           TT1=XMBC - TEMP2
0502           TT2=TEMP2 +SOMV/TEMP1
0503            TT3=SUM2 + TSPRIN/TEMP1 - FDDP*DIFD3*DIFD3*XIM/TEMP1
0504           TEMP3=RO*DCOS(X(5))
0505           V1=-XMHAM*TEMP3*THES + TEMP2
0506           V2=XMG + XMHAM*TEMP3*THES + XMHAM - TEMP2- SOMV*SOMV/XIM
              1 - SOMV/TEMP1
0507           V3=SUM3 + XMHAM*XD(5)*XD(5)*DSIN(X(5))*RO - TSPRIN/TEMP1 - FMG(7)
              1 + XIM*DIFD3*DIFD3*FDDP/TEMP1 - XMHAM*RO*DCOS(X(5))*FDDP*DIFD3*
              2DIFD3
0508           IF((DIFF2.GE..52083D-2.AND.DIFF2.LE.XLD.AND.IBRC.EQ.0).AND.
              1(IFG(14).EQ.1.OR.ITEST.EQ.1)) GO TO 831
0509           IF(IFG(14).EQ.1.AND.ITEST.EQ.0) GO TO 831
0510           IF(IUNLO.EQ.0) GO TO 1702
0511           GO TO 1703
0512      1702 DIVI=TT1*V2 - V1*TT2
0513           XGE   =(TT1*V3 - V1*TT3)/DIVI
0514           V2=V2 + XMBV(IFLG)
0515           V3=V3 + SUM1
0516      1703 XDD(1)=SUM1/XMBV(IFLG)
0517           XBE=XDD(1)
0518           DIVI=TT1*V2 - V1*TT2
0519           XDD(2)=(TT3 * V2 - TT2*V3)/DIVI
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0520          XDD(3)=(TT1*V3 - V1*TT3)/DIVI
0521          IF(IUNLO.EQ.0)XDD(1)=XDD(3)
0522          IF(IFG(14).EQ.0) GO TO 877
0523          IFG(14)=0
0524          PRINT 596,T
0525      596 FORMAT(' WEAPON IS UNLOCKED AT T=',F15.8)
0526          XMOM1=(XMG + XMBV(IFLG)+XMHAM)*XD(1) + XMBC*XD(2)
0527          XKEN=(XMG+XMBV(IFLG)+XMHAM)*XD(1)*XD(1)/2.+XMBC*XD(2)*XD(2)/2.+
             1 XIB*DERIV*DERIV*(XD(2)-XD(1))*(XD(2)-XD(1))/2.
0528          SA=XMG+XMBV(IFLG) +XMHAM
0529          SB=XMBC
0530          SC=(XMG+XMBV(IFLG)+XMHAM)/2.
0531          SD=XMBC/2.
0532          SE=0.0D0
0533          PRINT 2200,XD(1),XD(2)
0534          XD(2)=POSRT(SA,SB,SC,SD,SE,XMOM1,XKEN,0)
0535          XD(1)=(XMOM1 - SB*XD(2))/SA
0536          XD(3)=XD(1)
0537          PRINT 2300,XD(1),XD(2)
        C     $$$$
0538          GO TO 877
        C     THESE EQUATIONS ARE USED TO COMPUTE THE ACCELERATIONS OF THE BOLT
        C  AND BOLT CARRIER  IF THE WEAPON IS UNLOCKING (TO &&&&)
0539      831 C1=-XIB*DERIV*DERIV
0540     1705 CONX=1. - DSIGN(1.D0,FCONX)*CONB
0541          TT1=XMBC - C1 - TEMP2 -V1*(CONX - 1.)
0542          TT2=C1 + TT2  -(CONX -1.)*(V2 + XMARC)
0543          TT3=TT3 - V3*(CONX - 1.)
0544          V1=V1*CONX + C1
0545          V2=(V2 + XMARC)*CONX - XMARC + XMBV(IFLG) - C1
0546          V3=V3*CONX + SUM1
0547          DIVI=TT1*V2 - V1*TT2
0548          XDD(2)=(TT3 * V2 - TT2*V3)/DIVI
0549          XDD(3)=(TT1*V3 - V1*TT3)/DIVI
0550          XDD(1)=XDD(3)
        C      &&&&
        C      CONSTRAINT FORCE ACTING ON THE BOLT DURING UNLOCKING
0551     1704 FCONX=(-C1*(XDD(1) - XDD(2)) + (XMBV(IFLG) - XMARC)*XDD(1)
             1 - SUM1)/CONX
0552          FB(6)=FCONX*(CONX - 1.) + C1*(XDD(1)- XDD(2))
0553          FBC(3)=-FB(6)
0554          IF(IFG(14).EQ.1) GO TO 877
0555          IFG(14)=1
0556          PRINT 595,T
0557      595 FORMAT(' WEAPON BEGINS TO UNLOCK AT T=',F15.8)
0558          XMOM1=(XMG + XMBV(IFLG)+XMHAM)*XD(1) + XMBC*XD(2)
0559          XKEN=(XMG+XMBV(IFLG)+XMHAM)*XD(1)*XD(1)/2.+XMBC*XD(2)*XD(2)/2.
0560          SA=XMG+XMBV(IFLG) +XMHAM
0561          SB=XMBC
0562          SC=(XMG+XMBV(IFLG)+XMHAM)/2.
0563          SD=XMBC/2.
0564          SE=XIB*DERIV*DERIV/2.
0565          PRINT 2200,XD(1),XD(2)
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0566        XD(2)=POSRT(SA,SB,SC,SD,SE,XMOM1,XKEN,0)
0567        XD(1)=(XMOM1 - SB*XD(2))/SA
0568        XD(3)=XD(1)
0569        PRINT 2300,XD(1),XD(2)
      C ****  CONSTRAINT FORCE ACTING BETWEEN THE THE BOLT CARRIER AND GUN
      C        DURING COCKING OF THE HAMMER
0570    877 FBC(8)=(-XIM*THES*(XDD(3) - XDD(2)) + TSPRIN + XIM*2.*DIFD3*DIFD3
           1*SEM/(SEM1*SEM1) -SUMV*XDD(3))/(RHHU*DCOS(X(5) + ALTEM))
      C ***** COMPUTED ACCELERATION OF THE HAMMER . THIS PROVIDES A CHECK ON THE
      C        CORRECT SOLVING OF THE SYSTEM OF ALGEBRAIC EQUATIONS NECESSARY TO GET THE
      C        ACCELERATIONS
0571        XDD(5)=(TSPRIN - RHHU*FBC(8)*DCOS(X(5) +ALTEM))/XIM
      C        CONSTRAINT FORCE ACTING AT THE HAMMER PIVOT POINT
0572    875 XDD(5)=XDD(5) - SOMV*XDD(3)/XIM
0573        FMG(8)=-(XMHAM*RO*(XDD(5)*DCOS(X(5)) - XD(5)*XD(5)*DSIN(X(5))
           1 + XDD(3)/RO) + FBC(8) + FMG(7))
0574                          XDD(L1)=(R20*FMG(1) - R10*(XDD(3)* XMG
           1 - FMG(1)) - XKG* X(L1) - XCG*XD(L1))/XIG
0575   1450 IF(IUNLO.NE.0.OR.IFLAG.NE.0.OR.IFLG.EQ.3) GO TO 1775
0576        IF(IUNLO.EQ.0.AND.IFLAG.EQ.0.AND.(XGE-XBE)/(1./XMBV(IFLG)+1./XMG)
           1.GT.FSTIC)IUNLO=1
0577   1775 IF(IUNLO.EQ.1) GO TO 951
0578        GO TO 950
0579    951 IF(IFG(15).EQ.0) RETURN
0580        IFG(15)=0
0581        PRINT 598,T
0582    598 FORMAT(' BOLT AND MAIN GUN ARE TWO MASSES AT T=',F15.8)
0583        TBRC=T
0584        RETURN
0585    950 IF(IFG(15).EQ.1) RETURN
0586        IFG(15)=1
0587        PRINT 597,T
0588    597 FORMAT(' BOLT AND MAIN GUN ARE CONSIDERED AS ONE MASS AT T=',
           1 F15.8)
0589        RETURN
0590        ENTRY READIT
0591        NAMELIST/MAG/NC,NI
0592        NAMELIST/MAG1/      YE,YF,XKMAG,XFL,XUMAG1,XUMAG2,XUMAG3
           1 ,XPROJ,SPR,EPSOL,DIAC
      C        XPROJ MUST BE LESS THAN XPUC1
0593        NAMELIST/MANP/TBFO,XMDSP
0594        READ (5,MANP)
0595        XMDSP=XMDSP/G
0596        PRINT 9560,TBFO,XMDSP
0597   9560 FORMAT(' TBFO',5X,'DELAY BETWEEN HAMMER STRIKE AND IGNITION',
           120X,F20.8,2X,'SEC',// ,' XMDSP',4X,'MASS OF THE DRIVE SPRING',
           136X,F20.8,2X,'SLUGS',/)
0598        READ (5,MAG)
0599        READ (5,MAG1)
0600        PREL=XK5*BETA
0601        TEMP=NMASS + 1
0602        XK5=XK5*TEMP
0603        XMDSP=XMDSP/(TEMP - 1.)
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0604          PRINT 9500,NC,YE,YF,NI,XKMAG
0605     9500 FORMAT(//,10X,' INPUT PARAMETERS FOR FEED MECHANISM',//,
             1' NC=',I2,' (NUMBER OF ROUNDS WHEN MAGAZINE IS FULL)',/,
             2' YE=',F10.5,'FT',' (HT. OF MAG. SPRING WHEN EMPTY)',/,
             3' YF=',F10.5,'FT',' (HT. OF MAG. SPRING WHEN FULL)',/,
             4' NI=',I2,' (INITIAL NUMBER OF ROUNDS IN THE MAGAZINE)',/,
             5' XKMAG=',F10.5,'LBS/FT',' (MAGAZINE SPRING CONSTANT)')
0606          PRINT 9501,XFL,XUMAG1,XUMAG2,XUMAG3,XPROJ,SPR,EPSOL,DIAC
0607     9501 FORMAT(' XFL=',F10.5,'FT',' (FREE LENGTH OF MAGAZINE SPRING)',/,
             1' XUMAG1=',F10.5,' (COFF. OF FRICTION BETWEEN B.C. AND ROUNDS)',/,
             2' XUMAG2=',F10.5,' (COFF. OF FRICTION BETWEEN MAG. AND ROUNDS)',/,
             3' XUMAG3=',F10.5,' (COFF. OF FRICTION BETWEEN ROUNDS)',/,' XPROJ='
             4           ,F10.5,'FT',' (DISTANCE BOLT CARRIER MOVES WHEN FRICTION B
             5ETWEEN B.C. AND ROUNDS IS OFF)',/,
             5' SPR=',F10.5,'LBS',' (A THIRD THE SPRING MASS AND THE FOLLOWER MA
             5SS)',/,
             6' EPSOL=',F10.5,' (EFFICIENCY OF MAGAZINE SPRING SYSTEM)',/,
             7' DIAC=',F10.5,'FT',' (DIAM. OF CASE)')
0608          L1=7 + NMASS
0609          IF(I4DG.EQ.1)L1=11 + NMASS
0610          L11=L1 - 1
0611          N11=7
0612          IF(I4DG.EQ.1)N11=11
0613          L11M=L11 - 1
0614          N11M=N11 + 1
0615          TEMP=NC
0616           DFLSP=(YE - YF)/TEMP
0617          TEMP1=NI
0618          XME=TEMP1*(XMBUL + XMARC) + SPR/G
0619          XMG=TEMP1*(XMBUL + XMARC) + XMG
0620          YIHT=YE - DELSP*TEMP1
0621          SINM1=DELSP/DIAC
0622          COSM1=DSQRT(1.D0 - SINM1*SINM1)
0623          PRINT 9056,DELSP,YIHT,XME
0624     9056 FORMAT(/,10X,' COMPUTED PARAMETERS ASSOCIATED WITH THE MAG.',//,
             1' DELSP=',F10.5,'FT',' (SPACE PER ROUND)',/,
             2' YIHT=',F10.5,'FT',' (INITIAL HT. OF THE MAG. SPRING)',/,
             1' XME=',F10.5,'SLUGS',' (EFFECTIVE MASS ASSOC. WITH THE MAGAZINE)'
             1)
0625          CONF1=DCOS(ALH1)
0626          DS0=DSIN(ALH0)
0627          DC0=DCOS(ALH0)
0628          CONT=DCOS(ALPHA)
0629          CONB=2.*XMUB*(R2B*R2B + R1B*R2B + R1B*R1B)*DERIV/
             1((R1B + R2B)*3.)
0630          RETURN
0631          ENTRY READUM
0632          READ 1718,XMS,XKS,XCS
0633     1718 FORMAT(5F10.0,/,6F10.0,/,6F10.0)
0634          PRINT 1719,XMS,XKS,XCS
0635     1719 FORMAT(/,'        MASSES, SPRING CONSTANTS, AND DAMPING COEFS.',
             1/,' UNITS ARE SLUGS, LBS/FT, AND LBS-SEC/FT, RESPECTIVELY',
             1/,' THE FIRST MASS CORRESPONDS TO THE MASS NEAREST THE BOLT CARRIE
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              2R ','/',' XMS=',5F15.8,/,' XKS=',6F10.2,/,' XCS=',6F10.2)
0636           NAMELIST/ACCUR/R10,R20,XIG,XKG,XCG
0637           READ(5,ACCUR)
0638           PRINT 1720,R10,R20,XIG,XKG,XCG
0639      1720 FORMAT(//,15X,'PARAMETERS FOR ACCURACY',//,
              1' R10=',F15.8,'FT (BARREL OFFSET FROM THE C.G.)',/,
              2' R20=',F15.8,'FT (MOUNT FORCE OFFSET FROM THE C.G.)',/,
              2' XIG=',F15.8,'SLUG - FT**2 (MOMENT OF INERTIA OF THE GUN)',/,
              3' XKG=',F15.8,'FT-LBS/RAD (TORSIONAL SPRING CONSTANT)',/,
              3' XCG=',F15.8,'FT-LBS-SEC/RAD (DAMPING FOR TORSIONAL SPRING)')
0640           RETURN
0641           END
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0001            SUBROUTINE OUTP(T,N)
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            REAL*8 FBMA(12)/12*0.0D0/, FBCMA(12)/12*0.0D0/,FMGMA(15)/15*0.0D0/
               1,FBUFMA(6)/6*0.0D0/
0004            REAL*4 ITMIN,ITMAX,ITX,ITY
0005            DATA IQUIT,K/0,1/,IDUMB/0/,IAIM/0/,IDPR/0/,FMCI/0.0D0/,
               1FMCA/0.0D0/
0006            COMMON/MAS/NMASS
0007            COMMON/LAIN/ITMIN,ITMAX,ITX,ITY
0008            COMMON/DIF/DIFF1,DIFF2,DIFF3,DIFF4,DIFF5,DIFF6
0009            COMMON/RUNKA/PRMT(4),X(21),XD(21),XDD(21)
0010            COMMON/MODES/IMODE,IFG(30),IPT,IHAM,IUNLO,ISEP,IBBC,INIT,IRND
               1,NROUN,ISR,IFREQ,IRX,IPICK,I4DG
0011            COMMON/ASTHT/PBRCX     ,PBRCY(50),TFIRE,CCL,XLAMCC,XMUCC,ACC,BOREA,
               1 XKHYP,XCHYP,CF4,XMU4,XMB,XLD,PBRL,PCAL,CARTF(50),CARTX,PCAR
0012            COMMON/ASIT/XBBC,XKBBC,XCBBC,DERIV,XMBUL,XMARC ,PUC,AREBOL,
               1 TDEL,XIB,G,TPORT,XCHCMG,XMUBCG,EJCAR,FLEV            ,XKLBC,XCLBC
0013            COMMON/NUM/NBRCH,IFLAG,NCAV,IFLG,IFIRE
0014            COMMON/ASTHEE/PCAVT     ,PCAVY(50),FBORY(50),PBORE,FBORX
0015            COMMON/ER/XKBUFF,XCBUFF,XMBC,XMBV(3),XIM,RH,ALPHA,TSPRIN,
               1 RHH0,ALHC,RHH1,ALH1,HK,HC,
               1    DCOCK ,XKMOUN,XCMOUN,XUHBC,XDHA,EXTC,EXTF,FDIS,FSTIC
               1,R1B,R2B,XMUB,FCONX
0016            COMMON/S/XK9,XC9,XK5,XC5,BETA,ALP1,THETO,XKHG,XCHG,XMHAM,XMG,
               1     XMBUFF,CON1,XPUC,XPUC1,RO,CON2,XMUFF,XCUFF
0017            COMMON/BUFF/XBFORW,XCFORW,XBREAR,XCREAR,SLACK,XUBUF,XMWT,WTI
0018            COMMON/PRT/FB(12),FBC(12),FMG(15),FBUF(6)
0019            COMMON/GRAP/TGR1,TGR2
0020            COMMON/MSSS/XMS(5),XKS(6),XCS(6)
0021            REAL*4 IMT(600),IBLT(600),IBLTD(600),IBLTC(600),IBLTCD(600),
               1IBUF(600),IBUFD(600),IBFW(600),IBFWD(600),IGUN(600),IGUND(600),
               2IBFWX(600),IROT(600),IROTD(600),IDRB(600),IDRG(600)
0022            REAL*4 IBLTDD(600),ICDD(600),IGDD(600),IBDD(600),IHDD(600),
               1IWDD(600),TAXI(600),FB6(600),FB7(600),FBC8(600)
0023            COMMON/PLOP/MTX
0024            INTEGER*2 MTX(51,111)
0025            INTEGER*2 CHR,CHR1,CHS(5)/'1','2','3','4','5'/
0026            INTEGER XCORD(4)/'TIME',' (SE','C)  ','    '/,YCORD(4),NAME(4)
0027            REAL*4        BUFMI,BUFMA,GUNMI,GUNMA,GUNDMI,GUNDMA,BFWMI,BFWMA,
               1ROTMI,ROTMA,ROTDMI,ROTDMA,FMTMI,FMTMA,BLTMI,BLTMA,
               2BLTDMI,BLTDMA,BLCDMI,BLCDMA            ,BUFDMI,BUFDMA,BBWMI,BBWMA,
               2BFWDMI,BFWDMA,BLDDMI,BLDDMA,CADDMI,CADDMA,GUDDMI,GUDDMA,BUDDMI,
               3BUDDMA,HDDMI,HDDMA,WDDMI,WDDMA,FENMI,FENMA
0028            IF(IMMAX.EQ.0) GO TO 5000
0029            DO 5001 I=1,12
0030            FBMA(I)=DMAX1(FBMA(I),DABS(FB(I)))
0031       5001 FBCMA(I)=DMAX1(FBCMA(I),DABS(FBC(I)))
0032            DO 5002 I=1,15
0033       5002 FMGMA(I)=DMAX1(FMGMA(I),DABS(FMG(I)))
0034            DO 5003 I=1,6
0035       5003 FBUFMA(I)=DMAX1(FBUFMA(I),DABS(FBUF(I)))
0036            FMCI=DMIN1(FCONX,FMCI)
0037            FMCA=DMAX1(FCONX,FMCA)
0038       5000 IF(IRND.EQ.1.AND.T.GE.TFIRE + TMUZ.AND.IAIM.EQ.0)
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     1 GO TO 4995
 4996 IF(IRND.GT.1.AND.IRND.GT.IDPR) GO TO 4899
      GO TO 4897
 4899 CALL ACCUM
      IDPR=IRND
      PRINT 1000,T,X,XD,XDD
       PRINT 1001,FB,FBC,FMG,FBUF,XMBV(IFLG),XMG
 4897 IF(IPT.NE.-1) GO TO 9270
      PRINT 1000,T,X,XD,XDD
       PRINT 1001,FB,FBC,FMG,FBUF,XMBV(IFLG),XMG
      REAL*4 IFB(12),EFB(12),IFBC(12),EFBC(12),IFMG(15),
     1EFMG(15),IFBUF(6),EFBUF(6)
      COMMON/ACKY/IFB,EFB,IFBC,EFBC,IFMG,EFMG,IFBUF,EFBUF
      DO 7880 I=1,12
      IFB(I)=FB(I)/2.
      EFB(I)=FB(I)*XD(1)/2.
      IFBC(I)=FBC(I)/2.
 7880 EFBC(I)=FBC(I)*XD(2)/2.
      DO 7881 I=1,15
      IFMG(I)=FMG(I)/2.
 7881 EFMG(I)=FMG(I)*XD(3)/2.
      DO 7882 I=1,6
      IFBUF(I)=FBUF(I)/2.
 7882 EFBUF(I)=FBUF(I)*XD(4)/2.
 9270 DO 7883 I=1,12
      IFB(I)=IFB(I) + FB(I)
      EFB(I)=EFB(I) + FB(I)*XD(1)
      IFBC(I)=IFBC(I) + FBC(I)
 7883 EFBC(I) = EFBC(I) + FBC(I)*XD(2)
      DO 7884 I=1,15
      IFMG(I)=IFMG(I) + FMG(I)
 7884 EFMG(I)=EFMG(I) + FMG(I)*XD(3)
      DO 7885 I=1,6
      IFBUF(I)=IFBUF(I) + FBUF(I)
 7885 EFBUF(I)=EFBUF(I) + FBUF(I)*XD(4)
      IF(DIFF3.GT.ALP1.AND.IDUMB.EQ.0) GO TO 7886
      GO TO 9200
 7886 IDUMB=1
      CALL ACCUM
 9200 IPT=IPT + 1
      IF(IMODE.EQ.1) GO TO 1600
      IF(MOD(IPT,ISR).NE.0)GO TO 1600
      IF(K.GT.600) GO TO 1600
      IDRB(K)=FBUF(3)
      IDRG(K)=FMG(5)
      IBUF(K)=X(4) - X(3)
      IGUN(K)=X(3)
      IGUND(K)=XD(3)
      IF(I4DG.EQ.1) GO TO 2000
      IBFWX(K)=X(6) - X(3)
      IBFWD(K)=XD(6)
      GO TO 2001
 2000 IBFWX(K)=(X(6)*XMS(1) + X(7)*XMS(2) + X(8)*XMS(3) +
     1X(9)*XMS(4) + X(10)*XMS(5))/XMWT - X(3)
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0090              IBFWD(K)=(XD(6)*XMS(1) + XD(7)*XMS(2) + XD(8)*XMS(3) +
                1 XD(9)*XMS(4) + XD(10)*XMS(5))/XMWT
0091         2001 IROT(K)=X(5)
0092              IF(IWSV.NE.1) GO TO 2042
0093              FB6(K)=FB(6)
0094              FB7(K)=FB(7)
0095              FBC8(K)=FBC(8)
0096         2042 IROTD(K)=XD(5)
0097              IMT(K)=FMG(1)
0098              IBLT(K)=DIFF1
0099              IBLTD(K)=XD(1)
0100              IBLTCD(K)=XD(2)
0101              IBLTC(K)=- DIFF3
0102              IBUFD(K)=XD(4)
0103              IBFW(K)=DIFF6
0104              IF(I4DG.NE.1) GO TO 1880
0105              ITX=T
0106              ITY=DIFF6
0107              CALL DETER(ITX,ITY,ITMIN,ITMAX,BBWMI,BBWMA,I,J)
0108              MIX(J,I)=CHS(1)
0109              ITY=X(7) - X(4) - .057
0110              CALL DETER(ITX,ITY,ITMIN,ITMAX,BBWMI,BBWMA,I,J)
0111              MIX(J,I)=CHS(2)
0112              ITY=X(8) - X(4) -.114
0113              CALL DETER(ITX,ITY,ITMIN,ITMAX,BBWMI,BBWMA,I,J)
0114              MTX(J,I)=CHS(3)
0115              ITY=X(9) - X(4) -.171
0116              CALL DETER(ITX,ITY,ITMIN,ITMAX,BBWMI,BBWMA,I,J)
0117              MTX(J,I)=CHS(4)
0118              ITY=X(10) - X(4) - .253666
0119              CALL DETER(ITX,ITY,ITMIN,ITMAX,BBWMI,BBWMA,I,J)
0120              MTX(J,I)=CHS(5)
0121         1880 IF(IOPT.EQ.1) GO TO 1922
0122              TAXI(K)=T
0123              K=K + 1
0124              GO TO 1600
0125         1922 IBLTDD(K)=XDD(1)
0126              ICDD(K)=XDD(2)
0127              IGDD(K)=XDD(3)
0128              IBDD(K)=XDD(4)
0129              IHDD(K)=XDD(5)
0130              IWDD(K)=XDD(6)
0131              TAXI(K)=T
0132              K=K + 1
0133         1600 IF(IRND.GT.1.AND.IRND.GT.IRX.AND.T.GE.TFIRE + TMUZ) GO TO 50
0134              IF(IMODE.EQ.0) GO TO 99
            C      ////////////     PRINT FORCES
0135              IF(MOD(IPT,IFREQ) .NE.0) GO TO 99
0136              PRINT 1000,T,X,XD,XDD
0137               PRINT 1001,FB,FBC,FMG,FBUF,XMBV(IFLG),XMG
0138         1000 FORMAT(' T=',E12.5,'X=',11E10.3,/,1X,10E12.5,/,
                1' XD=',11E11.4,/,1X,10E11.4,/,'XDD=',11E11.4,/,1X,10E11.4)
0139         1001 FORMAT(' FB=',6D12.5,/,6D13.5,/,
                1' FBC=',6D12.5,/,6D13.5,/,
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                  2' FMG=',7D12.5,/,8D13.5,/,
                  3' FBUF=',6D12.5,/,' XMB=',E12.5,' XMG=',E12.5,/)
              C    /////////////     PRINT FORCES
0140            99 IF(T - TFIRE.GT..110D0) GO TO 100
0141               IF(IPICK.EQ.0.AND.T - TFIRE.GT..06D0) GO TO 201
0142               RETURN
0143           100 PRINT 101,T
0144           101 FORMAT(' TIME IS EQUAL TO ',F15.8,/,' /////////WEAPON HAS FAILED T
                  1O FIRE AFTER 110 MS //////////')
0145               GO TO 200
0146           201 PRINT 203
0147           203 FORMAT(' ///////// WEAPON HAS FAILED TO RECOIL FAR ENOUGH TO PICK
                  1UP ANOTHER ROUND //////////')
0148           200 IQUIT=1
0149               PRINT 1000,T,X,XD,XDD
0150                PRINT 1001,FB,FBC,FMG,FBUF,XMBV(IFLG),XMG
0151               GO TO 1500
0152            50 PRINT 1000,T,X,XD,XDD
0153                PRINT 1001,FB,FBC,FMG,FBUF,XMBV(IFLG),XMG
0154            51 FORMAT(' PROGRAM TERMINATED SINCE ROUND REQUEST IS MET')
0155          1500 PRINT 150
0156           150 FORMAT(//,' TERM',20X,'DEFINITION',15X,'UNITS',//,
                  1 ' XMG',6X,'GUN DISPLACEMENT' ,24X,'FT',//
                  2' XMGD',5X,'VELOCITY OF THE GUN',21X,'FT/SEC',//,
                  3' BLT',6X,'BOLT DISPLACEMENT',23X,'FT',//,
                  4' BLTD',5X,'BOLT VELOCITY',27X,'FT/SEC',//,
                  5' BLTC',5X,'BOLT CARRIER DISPLACEMENT',15X,'FT',//,
                  6' BLTCD',4X,'BOLT CARRIER VELOCITY',19X,'FT/SEC',/)
0157               PRINT 151
0158           151 FORMAT(' BUFF',5X,'BUFFER DISPLACEMENT',21X,'FT',//,
                  1' BUFFD' ,4X,'BUFFER VELOCITY',25X,'FT/SEC',//,
                  2 ' BUFW',5X,'INTERNAL BUFFER WT. DISPLACEMENT',8X,'FT',//
                  3 ' BUFWD',4X,'INTERNAL BUFFER WT. VELOCITY',12X,'FT/SEC',//,
                  4 ' HAMR',5X,'ROTATION OF THE HAMMER',18X,'RAD',//,
                  5' HAMRD',4X,'ROTATIONAL VELOCITY OF THE HAMMER',7X,'RAD/SEC',//)
0159               PRINT 152
0160           152 FORMAT(' NOTE:BOLT, BOLT CARRIER, AND BUFFER DISPLACEMENTS ARE WIT
                  1H RESPECT TO THE GUN AND NOT THE INERTIAL REFERENCE FRAME')
0161               IF(IMMAX.EQ.0) GO TO 5010
0162               PRINT 5011,FBMA,FBCMA,FMGMA,FBUFMA
0163          5011 FORMAT(//,' ABSOLUTE MAXIMUMS OF INDIVIDUAL FORCES',
                  1/,' MAX FB=',6E12.5,/,6E12.5,/,' MAX FBC=',6E12.5,/,6E12.5,/,
                  1' MAX FMG=',7E12.5,/,8E12.5,/,' MAX FBUF=',6E12.5)
0164               PRINT 9076,FMCI,FMCA
0165          9076 FORMAT(' FMCI=',E12.5,' FMCA=',E12.5)
0166          5010 K=K - 1
0167               IDUMB=0
0168          4995 IAIM=1
0169               TEMP=DSIN(X(L1) + PHIE)
0170               TEMP1=DCOS(X(L1) + PHIE)
0171               ZI=SQR1*TEMP
0172               ZDI=XD(L1)*SQR1*TEMP1 + DSIN(X(L1))*VPR
0173               YI=SQR1*TEMP1
0174               YDI=-XD(L1)*SQR1*TEMP + DCOS(X(L1))*VPR
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0175          TEMP=RANGE - YI
0176          ZCAP=-.5*G*TEMP*TEMP/
             1(YDI*YDI) + ZDI*TEMP/YDI + ZI
0177          PRINT 2988,ZCAP,X(L1),XD(L1)
0178     2988 FORMAT(' ZCAP=',F15.8,'FT (DEFLECTION OF PROJECTILE)',/,
             1' X(L1)=',F15.8,'RAD (GUN ROTATION)',/,
             2' XD(L1)=',F15.8,'RAD/SEC (ROTATIONAL VELOCITY OF THE GUN)')
0179          IF(IRND.EQ.1.AND.IQUIT.NE.1) GO TO 4996
0180     2985 IF(IMODE.EQ.1) CALL EXIT
0181          IF(IWSV.NE.1) GO TO 2043
0182          PRINT 2045,(TAXI(I),FB6(I),FB7(I),FBC8(I),I=1,K)
0183     2045 FORMAT(' CONSTRAINT FORCES FB(6) FB(7) FBC(8)',/,(4E10.3,2X,4E10.3
             1,/))
0184     2043 IF(IPRI.NE.1) GO TO 1920
0185          PRINT 1917,(TAXI(I),IBLTC(I),I=1,K)
0186     1917 FORMAT(//,40X,'BOLT CARRIER DISPLACEMENT',/,(10F12.5,/))
0187          PRINT 1918,(TAXI(I),IBLTCD(I),I=1,K)
0188     1918 FORMAT(//,40X,'BOLT CARRIER VELOCITY',/,(10F12.5,/))
0189     1920 IF(I4DG.NE.1) GO TO 2073
0190          CALL PLOT2(ITMIN,ITMAX,BB*MI,BBWMA,XCORD,16HDISP. (FT)           ,
             1   16HBUFW-BUFF             ,0.0,0.0,16H                    ,1,
             1CHR,CHR1)
0191     2073 CALL PLOT0(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                    ,0,CHR,K)
0192          CALL PLOT1(IMT,1H*,FMTMI,FMTMA)
0193          CALL PLOT2(ITMIN,ITMAX,FMTMI,FMTMA,XCORD,16HFORCE (LBS)         ,
             1 16HMOUNT FORCE         ,0.0,0.0,1,16H                    ,CHR,
             1CHR1)
0194          CALL PLOT0(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                    ,0,CHR,K)
         C    CALL PLOT1(IBLT,1H*,BLTMI,BLTMA)
         C    CALL PLOT1(IBLTC,1H+,BLTMI,BLTMA)
         C    CALL PLOT2(ITMIN,ITMAX,BLTMI,BLTMA,XCORD,16HDISP. (FT)           ,
         C   1 16HBLT-* BLTC-+        ,0.0,0.0,16H                    ,1,CHR,
         C   1CHR1)
0195          CALL PLOT1(IDRB,1H*,FENMI,FENMA)
0196          CALL PLOT1(IDRG,1H+,FENMI,FENMA)
0197          CALL PLOT2(ITMIN,ITMAX,FENMI,FENMA,XCORD,16HFORCE (LBS)         ,
             1  16HFRUF-* FMG-+       ,0.0,0.0,16H
             2,1,CHR,CHR1)
0198          CALL PLOT0(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                    ,0,CHR,K)
0199          CALL PLOT1(IBLTD,1H*,BLTDMI,BLTDMA)
0200          CALL PLOT1(IBLT,1H+,BLTMI,BLTMA)
0201          CALL PLOT2(ITMIN,ITMAX,BLTDMI,BLTDMA,XCORD,16HVELOC. (FT/SEC)     ,
             1  16HBLTD-* BLT-+       ,BLTMI,BLTMA,16HDISP. (FT) (+)      ,
             1 2,'*','+')
0202          CALL PLOT0(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                    ,0,CHR,K)
0203          CALL PLOT1(IBLTCD,1H*,BLCDMI,BLCDMA)
0204          CALL PLOT1(IBLTC,1H+,BLTMI,BLTMA)
0205          CALL PLOT2(ITMIN,ITMAX,BLCDMI,BLCDMA,XCORD,16HVELOC. (FT/SEC)     ,
             1  16HBLTCD-* BLTC-+           ,BLTMI,BLTMA,16HDISP. (FT) (+)    ,
             12,'*','+')
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0206          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0207          CALL PLOT1(IBUFD,1H*,BUFDMI,BUFDMA)
0208          CALL PLOT1(IBUF,1H+,BUFMI,BUFMA)
0209          CALL PLOT2(ITMIN,ITMAX,BUFDMI,BUFDMA,XCORD,16HVELOC. (FT/SEC)    ,
             1  16HBUFFD-* BUFF-+       ,BUFMI,BUFMA,16HDISP. (FT) (+)          ,2,
             2'*','+')
0210          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0211          CALL PLOT1(IBFWD,1H*,BFWDMI,BFWDMA)
0212          CALL PLOT1(IBFWX,1H+,BFWMI,BFWMA)
0213          CALL PLOT2(ITMIN,ITMAX,BFWDMI,BFWDMA,XCORD,16HVELOC. (FT/SEC)    ,
             1  16HBUFWD-* BUFW-+       ,BFWMI,BFWMA,16HDISP. (FT) (+)          ,2,
             1'*','+')
0214          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0215          CALL PLOT1(IGUN,'+',GUNMI,GUNMA)
0216          CALL PLOT1(IGUND,'*',GUNDMI,GUNDMA)
0217          CALL PLOT2(ITMIN,ITMAX,GUNDMI,GUNDMA,XCORD,16HVELOC. (FT/SEC)    ,
             116HXMGD-* XMG-+          ,GUNMI,GUNMA,16HDISP. (FT) (+)          ,
             2  2,'*','+')
0218          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0219          CALL PLOT1(IROT,'+',ROTMI,ROTMA)
0220          CALL PLOT1(IROTD,'*',ROTDMI,ROTDMA)
0221          CALL PLOT2(ITMIN,ITMAX,ROTDMI,ROTDMA,XCORD,16HHAMRD (RAD/SEC)    ,
             1  16HHAMRD-* HAMR-+       ,ROTMI,ROTMA,16HHAMR (RAD) (+)          ,
             2 2,'*','+')
0222          IF(I4DG.EQ.1) GO TO 2700
0223          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0224          CALL PLOT1(IBFW,1H*,BBWMI,BBWMA)
0225          CALL PLOT2(ITMIN,ITMAX,BBWMI,BBWMA,XCORD,16HDISP. (FT)          ,
             1   16HBUFW-BUFF           ,0.0,0.0,16H                           ,1,
             1CHR,CHR1)
0226     2700 IF(IOPT.NE.1) GO TO 1865
0227          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0228          CALL MINMAX(IBLTDD,BLDDMI,BLDDMA,K)
0229          CALL PLOT1(IBLTDD,'*',BLDDMI,BLDDMA)
0230          CALL PLOT2(ITMIN,ITMAX,BLDDMI,BLDDMA,XCORD,16HACC. (FT/SEC**2)   ,
             116HBLTDD-*          ,0.0,0.0,16H              ,1,CHR,CHR1)
0231          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0232          CALL MINMAX(ICDD   ,CADDMI,CADDMA,K)
0233          CALL PLOT1(ICDD   ,'*',CADDMI,CADDMA)
0234          CALL PLOT2(ITMIN,ITMAX,CADDMI,CADDMA,XCORD,16HACC. (FT/SEC**2)   ,
             116HBLTCDD-*         ,0.0,0.0,16H              ,1,CHR,CHR1)
0235          CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
             1IMT,0.0,0.0,16H                   ,0,CHR,K)
0236          CALL MINMAX(IGDD   ,GUDDMI,GUDDMA,K)
0237          CALL PLOT1(IGDD,'*',GUDDMI,GUDDMA)
0238          CALL PLOT2(ITMIN,ITMAX,GUDDMI,GUDDMA,XCORD,16HACC. (FT/SEC**2)   ,
             116HXMGDD-*          ,0.0,0.0,16H              ,1,CHR,CHR1)
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0239           CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
              1IMT,0.0,0.0,16H                     ,0,CHR,K)
0240           CALL MINMAX(IBDD   ,BUDDMI,BUDDMA,K)
0241           CALL PLOT1(IBDD,'*',BUDDMI,BUDDMA)
0242           CALL PLOT2(ITMIN,ITMAX,BUDDMI,BUDDMA,XCORD,16HACC. (FT/SEC**2)    ,
              116HBUFFDD-*         ,0.0,0.0,16H                  ,1,CHR,CHR1)
0243           CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
              1IMT,0.0,0.0,16H                     ,0,CHR,K)
0244           CALL MINMAX(IHDD   ,HDDMI ,HDDMA ,K)
0245           CALL PLOT1(IHDD,'*',HDDMI,HDDMA)
0246           CALL PLOT2(ITMIN,ITMAX,HDDMI ,HDDMA ,XCORD,16HACC. (FT/SEC**2)    ,
              116HHAMPDD-*         ,0.0,0.0,16H                  ,1,CHR,CHR1)
0247           CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
              1IMT,0.0,0.0,16H                     ,0,CHR,K)
0248           CALL MINMAX(IWDD   ,WDDMI ,WDDMA ,K)
0249           CALL PLOT1(IWDD,'*',WDDMI,WDDMA)
0250           CALL PLOT2(ITMIN,ITMAX,WDDMI ,WDDMA ,XCORD,16HACC. (FT/SEC**2)    ,
              116HBUFWDD-*         ,0.0,0.0,16H                  ,1,CHR,CHR1)
0251      1865 PRINT 18
0252        18 FORMAT('1')
0253           IF(IRND.LE.NROUN) GO TO 700
0254           PRINT 51
0255           CALL EXIT
0256       700 IRX=IRND
0257           K=1
0258           ITMIN=T
0259           ITMAX=ITMIN + TGR2
0260           IF(IQUIT.EQ.1) CALL EXIT
0261           IF(I4DG.NE.1) GO TO 2074
0262           CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
              1IMT,0.0,0.0,16H                     ,0,CHR,K)
0263      2074 RETURN
0264             ENTRY READIN
         C       BUFMI,BUFMA        EST. MAX. AND MIN. VALUES FOR BUFFER DISPLACEMENT
         C       GUNMI,GUNMA        EST. MAX. AND MIN. VALUES FOR GUN DISPLACEMENT
         C       GUNDMI,GUNDMA        EST. MAX. AND MIN. VALUES FOR GUN VELOCITIES
         C       BFWMI,BFWMA         EST. MAX AND MIN. VALUES FOR BUFFER WEIGHT DISPL.
         C       (WITH RESPECT TO THE GUN)
         C       ROTMI,ROTMA        MAX. AND MIN. VALUES OF ROTATION
         C       ROTDMI,ROTDMA,         MAX. AND MIN. VALUES OF ROTATIONAL VELOCITY
         C       FMTMI,FMTMA        MAX. AND MIN. VALUES OF THE MOUNT FORCE
         C       BLTMI,BLTMA        MAX. AND MIN. VALUES OF THE BOLT DISPL.
         C       BLCDMI,BLCDMA        MAX. AND MIN. VALUFS OF THF BOLT CARRIER VELOCITY
         C       BUFDMI,BUFDMA        MAX. AND MIN. VALUFS OF THF BUFFER VELOCITY
         C       BBWMI,BBWMA        MAX. AND MIN. VALUES OF THE BUFFER WT. WITH RESPECT TO
         C       THE BUFFER
         C       BFWDMI,BFWDMA        MAX. AND MIN. VALUES OF THE BUFFER WT. VELOCITY
         C       FENMI,FENMA        EST. MAX AND MIN VALUES FOR DRIVE SPRING CONSTANT
0265           NAMELIST/INPT6/BUFMI,BUFMA,GUNMI,GUNMA,GUNDMI,GUNDMA,BFWMI,BFWMA,
              1ROTMI,ROTMA,ROTDMI,ROTDMA,FMTMI,FMTMA,BLTMI,BLTMA,
              2BLTDMI,BLTDMA,BLCDMI,BLCDMA,             BUFDMI,BUFDMA,BBWMI,BBWMA,
              2BFWDMI,BFWDMA,FENMI,FENMA
0266           READ(5,INPT6)
0267           L1=7 +NMASS
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0268              IF(I4DG.EQ.1)   L1=11 +NMASS
            C      FOR PLOTTING PURPOSES MAX AND MIN ACCELERATIONS ARE COMPUTED
            C      BLDDMI,BLDDMA      BOLT MAXIMUM AND MINIMUM ACCELERATIONS
            C      CADDMI,CADDMA       BOLT CARRIER MAX AND MIN ACCELERATIONS
            C      GUDDMI,GUDDMA       GUN MAXIMUM AND MINIMUM ACCELERATIONS
            C      BUDDMI,BUDDMA       BUFFER MAXIMUM AND MINIMUM ACCELERATIONS
            C      HDDMI,HDDMA,       HAMMER MAX AND MIN ACCELERATIONS
            C      WDDMI,WDDMA       BUFFER WT. MAX AND MIN ACCELERATIONS
            C      IF IPRI IS EQUAL TO ONE THEN THE BOLT CARRIER VELOCITY AND DISPLACEMENT
            C       WIL BE PRINTED
0269              NAMELIST/INPT7/IOPT,IPRI,IWSV,IMMAX
            C      IF IWSV IS ONE THEN THE CONSTRAINT FORCES WILL BE PRINTED
            C      IF IOPT EQUAL ONE THEN GRAPH THE ACCELERATIONS
0270              READ(5,INPT7)
0271              IF(I4DG.EQ.1)BBWMI=BBWMI - .2536666
0272              RETURN
0273              ENTRY INITAL
0274              CALL PLOTO(TAXI    ,ITMIN,ITMAX,0.0,0.0,XCORD,YCORD,NAME,CHR,
                 1IMT,0.0,0.0,16H                       ,0,CHR,K)
0275              NAMELIST/ACCUR1/XLB,XALPH,RANGE,VPR,TMUZ
0276              READ(5,ACCUR1)
0277              PRINT 1781,XLB,XALPH,RANGE,VPR,TMUZ
0278         1781 FORMAT(' XLB=',F15.8,'FT (PIVOT POINT TO BARREL END)',/,
                 1' XALPH=',F15.8,'FT (BARREL OFFSET)',/,
                 2' RANGE=',F15.8,'FT (TARGET RANGE)',
                 4/,' VPR=',F15.8,'FT/SEC (MUZZLE VELOCITY)',/,
                 1' TMUZ=',F15.8,'SEC (TIME TO REACH MUZZLE)')
0279              SQR1=DSQRT(XLB*XLB + XALPH*XALPH)
0280              PHIE=DARSIN(XALPH/SQR1)
0281              RETURN
0282              END
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0001              SURROUTINE RUNGE(N,FCT,OUTP)
0002              IMPLICIT REAL*8(A-H,O-Z)
            C      SOLVES A SYSTEM OF SECOND ORDER DIFF EQS.
            C      PRMT(1)       INITIAL TIME
            C      PRMT(2)       FINAL TIME
            C      PRMT(3)       TIME INCREMENT
            C      PRMT(4)       STOPS INTEGRATING IF THIS TERM IS NEGATIVE
            C      ITEST      ITEST IS ONE THEN A NEW TIME INCREMENT IS STARTING
            C      AND IF ITEST IS ZERO THEN IN THE MIDDLE OF A TIME INCREMENT
            C      X,XD      CONTAINS INITIAL VALUES BEFORE COMING INTO THE ROUTINE
            C      XDD      SECOND DERIVATIVE TERM
            C      FCT      SUBROUTINE TO EVALUATE THE SECOND DERIVATIVES
            C      OUTP       SUBROUTINE TO PRINT
0003              COMMON/RUNKA/PRMT(4),X(21),XD(21),XDD(21)
0004              DIMENSION AK1(21),AK2(21),AK3(21),AK4(21)
0005              DIMENSION TX(21),TXD(21)
            C      TIME INCREMENT CAN BE CHANGED BY JUST CHANGING DELT IN ROUTINE
            C      FCT AND COMPUTING THE NEW VALUE FOR SUM
0006              COMMON/CNTRL/DELT,SUM
0007              FINAL=PRMT(2)
0008              DELT=PRMT(3)
0009              T=PRMT(1)
0010              PRMT(4)=1.0D0
0011              SUM=DELT*.5D0
0012         1000 CALL FCT(T,X,XD,XDD,1)
0013              CALL OUTP(T,N)
0014              IF(T.GE.FINAL.OR.PRMT(4).LT.0.0D0) RETURN
0015              DO 1 I=1,N
0016              AK1(I)=DELT*XDD(I)
0017              TX(I)=X(I)+.5D0*DELT*XD(I) + DELT*AK1(I)*.125D0
0018            1 TXD(I)=XD(I) + .5D0*AK1(I)
0019              T=T + SUM
0020              CALL FCT(T,TX,TXD,XDD,0)
0021              DO 2 I=1,N
0022              AK2(I)=DELT*XDD(I)
0023            2 TXD(I)=XD(I) + .5D0*AK2(I)
0024              CALL FCT(T,TX,TXD,XDD,0)
0025              DO 3 I=1,N
0026              AK3(I)=DELT*XDD(I)
0027              TX(I)=X(I) + DELT*XD(I) + DELT*AK3(I)*.5D0
0028            3 TXD(I)=XD(I) + AK3(I)
0029              T=T + SUM
0030              CALL FCT(T,TX,TXD,XDD,0)
0031              DO 4 I=1,N
0032              AK4(I)=DELT*XDD(I)
0033              X(I)=X(I) + DELT*(XD(I) + .1666666666666667D0*(AK1(I)
                 1 + AK2(I) + AK3(I)))
0034            4 XD(I)=XD(I) + (AK1(I) + 2.D0*(AK2(I) + AK3(I)) + AK4(I))
                 1* .1666666666666667D0
0035              GO TO 1000
0036              END
```

```
      SUBROUTINE DETER(X,Y,XMIN,XMAX,YMIN,YMAX,I,J)
      I=(X     - XMIN)*110./(XMAX - XMIN) + 1.5
      IF(I.LT.1)I=1
      IF(I.GT.111)I=111
      J=(Y          - YMIN)*50./(YMAX - YMIN) + 1.5
      IF(J.LT.1)J=1
      IF(J.GT.51)J=51
      RETURN
      END
```

```
 0001            FUNCTION XINTP(A,X,Y)
 0002            IMPLICIT REAL*8(A-H,O-Z)
 0003            DIMENSION Y(50)
          C       ASSUMES EQUALLY SPACED POINTS
          C       X-COORDINATE STARTS AT ZERO WITH STEP SIZE OF X
 0004            I=A/X
 0005            XI=I
 0006            I=I + 1
 0007            XL = XI*X
 0008            XINTP=Y(I) + (Y(I + 1) - Y(I))*(A - XL  )/X
 0009            RETURN
 0010            END
```

```
0001      FUNCTION POSRT(SA,SB,SC,SD,SE,XMOM1,XKEN,ITE)
0002      IMPLICIT REAL*8(A-H,O-Z)
0003      CA=SC*SB*SB/(SA*SA)+SE*(SB/SA + 1.)*(SB/SA+1.) +SD
0004      CB=-2.*XMOM1*(SC*SB/(SA*SA) +  SE*(SB/SA+1.)/SA)
0005      CC=-XKEN + XMOM1*XMOM1*(SC+SE)/(SA*SA)
0006      IF(ITE.EQ.0)
         1POSRT=(-CB-DSQRT(CB*CB - 4.*CA*CC))/(2.*CA)
0007      IF(ITE.EQ.1)
         1POSRT=(-CB+DSQRT(CB*CB - 4.*CA*CC))/(2.*CA)
0008      RETURN
0009      END
```

```
      SUBROUTINE DETER(X,Y,XMIN,XMAX,YMIN,YMAX,I,J)
      I=(X     - XMIN)*110./(XMAX - XMIN) + 1.5
      IF(I.LT.1)I=1
      IF(I.GT.111)I=111
      J=(Y           - YMIN)*50./(YMAX - YMIN) + 1.5
      IF(J.LT.1)J=1
      IF(J.GT.51)J=51
      RETURN
      END
```

```
      SUBROUTINE ACCUM
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*4 IFB(12),EFB(12),IFBC(12),EFBC(12),IFMG(15),
     1EFMG(15),IFBUF(6),EFBUF(6)
      COMMON/ACKY/IFB,EFB,IFBC,EFBC,IFMG,EFMG,IFBUF,EFBUF
      COMMON/RUNKA/PRMT(4),X(11),XD(11),XDD(11)
      COMMON/PRT/FB(12),FBC(12),FMG(15),FBUF(6)
      DIMENSION SUM(15)
      DO 1 I=1,12
    1 SUM(I)=(IFB(I) - FB(I)/2.)*PRMT(3)
      PRINT 2,(SUM(I),I=1,12)
    2 FORMAT(' BOLT IMPULSES',6F15.8,/,6F15.8)
       DO 3 I=1,12
    3 SUM(I)=(EFB(I) - FB(I)*XD(1)/2.)*PRMT(3)
      PRINT 4,(SUM(I),I=1,12)
    4 FORMAT(' BOLT ENERGY',6F15.8,/,6F15.8)
      DO 5 I=1,12
    5 SUM(I)=(IFBC(I) - FBC(I )/2.)*PRMT(3)
      PRINT 6,(SUM(I),I=1,12)
    6 FORMAT(' BOLT CARRIER IMPULSES',6F15.8,/,6F15.8)
      DO 7 I=1,12
    7 SUM(I)=(EFBC(I) - XD(2)*FBC(I)/2.)*PRMT(3)
      PRINT 8,(SUM(I),I=1,12)
    8  FORMAT(' BOLT CARRIER ENERGY',6F15.8,/,6F15.8)
      DO 9 I=1,15
    9 SUM(I)=(IFMG(I) - FMG(I)/2.)*PRMT(3)
      PRINT 10,(SUM(I),I=1,15)
   10 FORMAT(' GUN IMPULSE',7F14.8,/,8F15.8)
      DO 11 I=1,15
   11 SUM(I)=(EFMG(I) - FMG(I)*XD(3)/2.)*PRMT(3)
      PRINT 12,(SUM(I),I=1,15)
   12 FORMAT(' GUN ENERGY',7F14.8,/,8F15.8)
      DO 13 I=1,6
   13 SUM(I)=(IFBUF(I) - FBUF(I)/2.)*PRMT(3)
      PRINT 14,(SUM(I),I=1,6)
   14 FORMAT(' BUFFER IMPULSE',6F15.8)
      DO 15 I=1,6
   15 SUM(I)=(EFBUF(I) - XD(4)*FBUF(I)/2.)*PRMT(3)
      PRINT 16,(SUM(I),I=1,6)
   16 FORMAT(' BUFFER ENERGY',6F15.8)
      RETURN
      END
```

```
0001        SUBROUTINE PLOTO(X   ,XMIN,XMAX,YMIN,YMAX,XCORD,YCORD,
           1 NAME,CHR,ISTORE,ZMIN,ZMAX,ZCORD,ITEST,CHR1,NPTS)
0002        COMMON/PLOP/MTX
0003        INTEGER*2 MTX(51,111),CHR,B/' '/,CHR1
0004        INTEGER XCORD(4),YCORD(4),NAME(4),ZCORD(4)
0005        REAL*4 ISTORE(600),X(600)
0006        DO 1 I=1,51
0007        DO 1 J=1,111
0008      1 MTX(I,J)=B
0009        RETURN
      C ///////////////
0010        ENTRY PLOT1(ISTORE,CHR,YMIN,YMAX)
      C ///////////////
0011        DO 1772 K=1,NPTS
0012        I=(X(K) - XMIN)*110./(XMAX - XMIN) + 1.5
0013        IF(I.LT.1)I=1
0014        IF(I.GT.111)I=111
0015        J=(ISTORE(K) - YMIN)*50./(YMAX - YMIN) + 1.5
0016        IF(J.LT.1)J=1
0017        IF(J.GT.51)J=51
0018   1772 MTX(J,I)=CHR
0019        RETURN
      C     ///////////////////////////////////////
0020        ENTRY PLOT2(XMIN,XMAX,YMIN,YMAX,XCORD,YCORD,NAME,ZMIN,ZMAX,ZCORD,
           1 ITEST,CHR,CHR1)
      C     ///////////////////////////////////////
0021        DIMENSION XVAR(12)
0022        TEMP=ALOG10(YMAX-YMIN)
0023        IF(TEMP.LT.0.0)TEMP=TEMP-1.
0024        IFY=TEMP
0025        TEMP=ALOG10(XMAX-XMIN)
0026        IF(TEMP.LT.0.0)TEMP=TEMP-1.
0027        IFX=TEMP
0028        Y1=YMAX/10.**IFY
0029        DELY=(YMAX-YMIN)/10.**(IFY+1)
0030        IF(ITEST.NE.2) GO TO 35
0031        TEMP=ALOG10(ZMAX-ZMIN)
0032        IF(TEMP.LT.0.0)TEMP=TEMP-1.
0033        IFZ=TEMP
0034        DELZ=(ZMAX-ZMIN)/10.**(IFZ+1)
0035        Z1=ZMAX/10.**IFZ
0036     35 IF(ITEST.NE.2) GO TO 36
0037        PRINT 5,CHR,CHR1,(NAME(I),I=1,4)
0038      5 FORMAT('1'/' ','CHR(',A1,')','   CHR(',A1,')',40X,4A4)
0039        GO TO 38
0040     36 PRINT 47,(NAME(I),I=1,4)
0041     47 FORMAT('1'/' ',55X,4A4)
0042     38 DO 13 L=1,11
0043        NV=56-5*L
0044        N1=N0-26
0045        IF(ITEST.NE.2) GO TO 27
0046        PRINT 28,Y1,Z1
0047     28 FORMAT(' ',F6.2,' **',F6.2)
0048        Z1=Z1-DELZ
```

```
      GO TO 26
   27 PRINT 6,Y1
    6 FORMAT(' ',9X,F6.2)
   26 Y1=Y1-DELY
      PRINT 7
    7 FORMAT('+',16X,11('|---------'),'|')
      PRINT 8,(MTX(N0,J),J=1,111)
    8 FORMAT('+',16X,111A1)
      IF(L.EQ.11) GO TO 13
      DO 12 K=1,4
      PRINT 9
    9 FORMAT(' ',16X,11('|         '),'|')
      IF(ITEST.EQ.2) GO TO 30
      IF(N0-K.EQ.24) PRINT 10,(YCORD(I),I=1,4)
   10 FORMAT('+',4A4)
      IF(N0-K.EQ.23) PRINT 11,IFY
   11 FORMAT('+SCALE 10**',I3)
      GO TO 12
   30 IF(N0-K.EQ.29)PRINT 10,YCORD
      IF(N0-K.EQ.28)PRINT 11,IFY
      IF(N0-K.EQ.24)PRINT 10,ZCORD
      IF(N0-K.EQ.23)PRINT 11,IFZ
   12 PRINT 8,(MTX(N0-K,J),J=1,111)
   13 CONTINUE
      PRINT 14
   14 FORMAT(' ',16X,11('+.........'),'+')
      XVAR(1)=XMIN/10.**IFX
      DELY=(XMAX - XMIN)/(11.*10.**IFX)
      DO 19 I=2,12
   19 XVAR(I)=XVAR(I-1)+DELY
      PRINT 15,XVAR
   15 FORMAT(' ',13X,F6.2,11(4X,F6.2))
      PRINT 16,(XCORD(I),I=1,4)
   16 FORMAT('0',55X,4A4)
      PRINT 17,IFX
   17 FORMAT(' ',55X,' SCALED BY 10**',I3)
      RETURN
      ENTRY MINMAX(ISTORE,ZMIN,ZMAX,NPTS)
       ZMIN=ISTORE(1)
      ZMAX=ISTORE(1)
      DO 50 I=2,NPTS
      ZMIN=AMIN1(ZMIN,ISTORE(I))
   50 ZMAX=AMAX1(ZMAX,ISTORE(I))
      RETURN
      END
```

CONTROL VARIABLES

```
&MODER
IRND=       0,IMODE=       0,ISEP=       0,IUNLO=        0,IHAM=        1,IFLAG=       1,IFLG=       3,IBBC=
     0,INIT=        0,IFIRE=     0,IFREQ=         6,NROUN=       1,IREAD=       0,IPICK=       0,NSMPR=
   400,I4DG=        1,NMASS=     5
&END
                             BREECH PRESSURE (LB/FT**2)
                             TIME STEP=       0.00010000
        0.0        144000.00      432000.00      720000.00     1584000.00     5184000.00
  7200000.00      8352000.00     7200000.00     5040000.00     3600000.00     2736000.00
  2160000.00      1728000.00     1440000.00
                  CAVITY PRESSURE IN BOLT CARRIER (LB/FT**2)
                             TIME STEP=       0.00026400
        0.0        180000.00      360000.00      381600.00      345600.00      288000.00
   216000.00       158400.00       86400.00       57600.00       43200.00       36000.00
    28800.00        21600.00       14400.00       14400.00        7200.00        7200.00
                             CARTRIDGE CASE FORCE (LBS)
                             TIME STEP=       0.00012500
       -0.94             0.0
                             BORE FRICTION (LBS)
                             TIME STEP=       0.00010000
      210.00          325.00         295.00         265.00         240.00         210.00
      180.00          155.00         125.00         100.00          70.00          40.00
       20.00             0.0            0.0
```

NAME	DESCRIPTION	VALUE	UNITS
BOREA	BORE AREA	0.00026159	FT**2
XKHYP	SPRING THAT PREVENTS MOTION BETWEEN THE BOLT AND GUN	100000.00000000	LBS/FT
XCHYP	DAMPING FOR THE ABOVE SPRING	18.00000000	LBS-SEC
XLD	DISTANCE BOLT CARRIER TRAVELS BEFORE GUN IS UNLOCKED	0.02083330	FT
CF4	DAMPING COEFFICIENT BETWEEN BOLT AND BOLT CARRIER	0.01000000	LBS-SEC/FT
XMU4	COEFFICIENT OF FRICTION BETWEEN BOLT AND BOLT CARRIER	0.20000000	NONE
XMB	MASS OF THE BOLT	0.00404037	SLUGS
G	GRAVITATIONAL CONSTANT	32.20000000	FT/SEC**2
XBBC	DISTANCE BOLT TRAVELS IN THE BOLT CARRIER	0.02604160	FT
XKBBC	SPRING BETWEEN BOLT AND BOLT CARRIER	100000.00000000	LBS/FT
XCBBC	DAMPING COEFFICIENT FOR ABOVE SPRING	18.00000000	LBS-SEC/FT
DERIV	DERIVATIVE OF THETA WITH RESPECT TO XB-XBC	25.13000000	RAD/FT
XKLBC	SPRING BETWEEN BOLT AND BOLT CARRIER DURING LOCKING	100000.00000000	LBS/FT
XMBUL	MASS OF THE BULLET AND THE PROPELLANT	0.00037658	SLUGS
XCLBC	DAMPING CORRESPONDING TO THE SPRING XKLBC	37.00000000	LBS-SEC/FT
AREBOL	AREA OF CAVITY THE GAS FORCE ACTS ON	0.00136600	FT**2
TDEL	TIME DELAY BETWEEN PORT AND CAVITY	0.00020000	SEC

XIB	MOMENT OF INERTIA OF THE BOLT	0.00000105	SLUGS-FT**2
XMARC	MASS OF THE CARTRIDGE CASE	0.00041078	SLUGS
TPORT	TIME IT TAKES PROJECTILE TO REACH PORT	0.00110000	SEC
XKBUFF	INTERACTION SPRING BETWEEN BOLT CARRIER AND BUFFER	100000.00000000	LBS/FT
XCBUFF	DAMPING COEFFICIENT FOR ABOVE SPRING	14.06000000	LBS-SEC/FT
XCBCMG	DAMPING BETWEEN BOLT CARRIER AND MAIN GUN	0.01000000	LBS-SEC/FT
XIM	MOMENT OF INERTIA OF THE HAMMER	0.00002000	SLUGS-FT**2
RH	RADIAL DISTANCE OF THE HAMMER TO POINT OF CONTACT	0.11083000	FT
ALPHA	REFFRENCE ANGLE ON THE HAMMER	0.30530000	RAD
XKMOUN	MOUNT SPRING CONSTANT	300.00000000	LBS/FT
XCMOUN	DAMPING FOR ABOVE SPRING	9.53100000	LBS-SEC/FT
TSPRIN	CONSTANT TORQUE DUE TO THE HAMMER SPRING	0.50000000	FT-LBS
XMUBCG	COEFFICIENT OF FRICTION BETWEEN THE BOLT CARRRIER AND GUN	0.20000000	NONE
XMBC	MASS OF THE BOLT CARRIER	0.01820807	SLUGS
XK9	BACK PLATE SPRING CONSTANT	60000.00000000	LBS/FT
XC9	DAMPING FOR THE ABOVE SPRING	10.78000000	LBS-SEC/FT
XK5	DRIVE SPRING CONSTANT	19.20000000	LBS/FT
XC5	DAMPING FOR ABOVE SPRING	0.00010000	LBS-SEC/FT
BETA	PRELOAD DISTANCE FOR THE DRIVE SPRING	0.33700000	FT
ALP1	DISTANCE BUFFER TRAVELS BEFORE BACK PLATE IS HIT	0.31700000	FT
THETO	INITIAL ANGLE OF THE HAMMER	-1.22173000	RAD
XKHG	SPRING BETWEEN HAMMER AND GUN	50000.00000000	LBS/FT
XCHG	DAMPING FOR THE ABOVE SPRING	0.27800000	LBS-SEC/FT
XMHAM	MASS OF THE HAMMER	0.00212112	SLUGS
XMG	MASS OF THE MAIN GUN	0.17670807	SLUGS
XMBUFF	MASS OF THE BUFFER	0.00306460	SLUGS
EJCAR	DISTANCE BOLT TRAVELS BEFORE CARTRIDGE EJECTION	0.17400000	FT
ELEV	ANGLE OF ELEVATION OF THE GUN	-0.00110000	RAD
XPUC	DISTANCE OF BOLT WHEN ROUND IS PICKED UP	0.23441670	FT
XPUC1	DISTANCE OF BOLT WHEN PICK UP FORCE IS OFF	0.15316670	FT

RO	DISTANCE FROM THE PIVOT POINT TO THE C.G. OF THE HAMMER	0.06250000	FT
XBFORW	EFFFCTIVE INTERNAL BUFFER SPRING CONSTANT (FORWARD)	22567.00000000	LBS/FT
XCFORW	DAMPING COEF. FOR THE ABOVE SPRING	6.99000000	LBS-SEC/FT
XBREAR	EFFFCTIVE INTERNAL BUFFER SPRING CONSTANT (REARWARD)	26405.00000000	LBS/FT
XCREAR	DAMPING COEF. FOR THE ABOVE SPRING	7.56000000	LBS-SEC/FT
SLACK	DISTANCE WTS. ARE ALLOWED TO MOVE	0.01083000	FT
XUBUF	COEF. OF FRICTION BETWEEN WTS. AND BUFFER	0.20000000	NONE
XMWT	MASS OF THE WTS IN THE BUFFER	0.00695497	SLUGS
XUHBC	COEF. OF FRICTION BETWEEN HAMMER AND B.C.	0.15000000	NONE
XDHA	DISTANCE B.C. MOVES BEFORE FRICT. BETWEEN THE HAM. AND B.C.	0.11500000	FT
EXTC	EXTRACTOR DISTANCE	0.00780000	FT
EXTF	EXTRACTOR FORCE	7.50000000	LBS
FDIS	DISTANCE BOLT CARRIER IS FROM BATTFRY, HAMMER ROTATES	0.01966700	FT
XMUFF	COEF. OF FRIC. BETWEEN GUN AND BUFFER	0.20000000	NONE
XCUFF	DAMPING BETWEEN BUFFER AND GUN	0.01000000	NONE
FSTIC	STICTION FORCE	0.0	LBS
RHH0	DISTANCE FROM HAMMER PIVOT TO B.C.	0.00783000	FT
RHH1	DISTANCE FROM PIVOT TO TOP OF HAMMER	0.13200000	FT
ALH0	ANGLE CORRESPONDING TO RHH0	0.38400000	RAD
ALH1	ANGLE CORRESPONDING TO RHH1	0.26200000	RAD
HK	HAMMER IMPACT SPRING	4500.00000000	LBS/RAD
HC	ABOVE DAMPING CONSTANT	0.08600000	LBS-SEC/RAD
R1B	INNFR RADIUS CARTRIDGE CASE	0.00781250	FT
R2B	OUTER RADIUS CARTRIDGE CASE	0.01562500	FT
CCL	CARTRIDGE CASE LENGTH	0.14580000	FT
XMUB	COEFFICIENT OF FRICTION AT THE BASE	0.20000000	NONE
TBFO	DELAY BETWEEN HAMMER STRIKE AND IGNITION	0.0	SEC
XMDSP	MASS OF THE DRIVE SPRING	0.00414907	SLUGS

INPUT PARAMETERS FOR FEED MECHANISM

NC=19 (NUMBER OF ROUNDS WHEN MAGAZINE IS FULL)

```
YE=   0.37500FT (HT. OF MAG. SPRING WHEN EMPTY)
YF=   0.08850FT (HT. OF MAG. SPRING WHEN FULL)
NI=19 (INITIAL NUMBER OF ROUNDS IN THE MAGAZINE)
XKMAG=  10.80000LBS/FT (MAGAZINE SPRING CONSTANT)
XFL=   0.64700FT (FREE LENGTH OF MAGAZINE SPRING)
XUMAG1=   0.24200 (COEF. OF FRICTION BETWEEN B.C. AND ROUNDS)
XUMAG2=   0.30200 (COEF. OF FRICTION BETWEEN MAG. AND ROUNDS)
XUMAG3=   0.30200 (COEF. OF FRICTION BETWEEN ROUNDS)
XPROJ=   0.19490FT (DISTANCE BOLT CARRIER MOVES WHEN FRICTION BETWEEN B.C. AND ROUNDS IS OFF)
SPR=   0.03000LBS (A THIRD THE SPRING MASS AND THE FOLLOWER MASS)
EPSOL=   1.00000 (EFFICIENCY OF MAGAZINE SPRING SYSTEM)
DIAC=   0.03125FT (DIAM. OF CASE)

          COMPUTED PARAMETERS ASSOCIATED WITH THE MAG.

DELSP=   0.01508FT (SPACE PER ROUND)
YIHT=   0.08850FT (INITIAL HT. OF THE MAG. SPRING)
XME=   0.01589SLUGS (EFFECTIVE MASS ASSOC. WITH THE MAGAZINE)

     MASSES, SPRING CONSTANTS, AND DAMPING COEFS.
UNITS ARE SLUGS, LBS/FT, AND LBS-SEC/FT, RESPECTIVELY
THE FIRST MASS CORRESPONDS TO THE MASS NEAREST THE BOLT CARRIER
XMS=     0.00126136     0.00126136     0.00126136     0.00126136      0.00191190
XKS=  60000.00  60000.00  60000.00  60000.00  60000.00  60000.00
XCS=     12.00     12.00     12.00     12.00     12.00     12.00


              PARAMETERS FOR ACCURACY

R1O=      0.06500000FT (BARREL OFFSET FROM THE C.G.)
R2O=     -0.02830000FT (MOUNT FORCE OFFSET FROM THE C.G.)
XIG=      0.13520000SLUG - FT**2 (MOMENT OF INERTIA OF THE GUN)
XKG=     50.00000000FT-LBS/RAD (TORSIONAL SPRING CONSTANT)
XCG=      2.82800000FT-LBS-SEC/RAD (DAMPING FOR TORSIONAL SPRING)
XLB=      1.64500000FT (PIVOT POINT TO BARREL END)
XALPH=      0.06500000FT (BARREL OFFSET)
RANGE=   450.00000000FT (TARGET RANGE)
VPR=  3180.00000000FT/SEC (MUZZLE VELOCITY)
TMUZ=      0.00130000SEC (TIME TO REACH MUZZLE)
```

COMPUTED RESULTS FOR A 1 SHOT BURST

```
(FORWARD) BOLT PIN STRIKES END OF CAM PATH AT T=      0.0
FRICTIONAL FORCE BETWEEN CARTRIDGES AND B.C. IS ON AT T=      0.0        FBC(10)=    -2.67172608
HAMMER IS CONTROLLED BY HAMMER SPRING AND MAIN GUN STOP AT T=      0.0
BOLT AND MAIN GUN ARE CONSIDERED AS ONE MASS AT T=      0.0
T= 0.0        X= 0.0        0.647D-04 0.0        0.129D-03-0.122D 01 0.129D-03 0.129D-03 0.129D-03 0.129D-03 0.129D-03 0.129D-03
 0.12942D-03 0.12942D-03 0.12942D-03 0.12942D-03 0.0        0.0        0.0        0.0        0.0        0.0
XD= 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
XDD= 0.6948D 01-0.1403D 03 0.6948D 01 0.3558D 02 0.2498D 05-0.6405D 01-0.6405D 01-0.6405D 01-0.6405D 01-0.6405D 01 0.0
 0.4704D-14 0.4704D-14 0.4704D-14-0.1797D 02-0.6402D 00 0.0        0.0        0.0        0.0        0.0
FB= 0.0        0.0        0.0        0.31091D-01 0.0        0.0
 0.0        0.64714D 01  0.0        0.0        0.0        0.17100D-03
FBC=-0.31091D-01 0.0        0.0        -0.64714D 01 0.0        0.64708D 01
 0.14835D 00  0.0        0.0        -0.26717D 01  0.0        0.64493D-03
FMG= 0.0        0.0        0.0        -0.14835D 00-0.64555D 01 0.0        0.0
-0.11476D 01  0.0        0.0        0.67889D-02  0.26717D 01  0.0        -0.64526D-01  0.0
FBUF=-0.64708D 01 0.0        0.64704D 01 0.58960D-04 0.44805D-01 0.64526D-01
XMB= 0.48277D-02 XMG= 0.19167D 00

NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00015000  1  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00017500  1  1  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00020000  0  1  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00020000  0  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00020000  1  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00022500  1  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00025000  1  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00025000  1  1  1  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00025000  0  1  1  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00025000  0  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00025000  1  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00035000  0  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00035000  0  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00035000  1  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.00052500  1  1  1  1  1  0
**********  ROUND NUMBER  1  BEGINS  **********
WEAPON FIRES AT T=      0.00990000
BREECH FORCE IS ON AT T=      0.00990000
BORE FRICTION IS ON AT T=      0.00990000
HAMMER STRIKE OF THE MAIN GUN IS ON AT T=      0.00990000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01012500  0  1  1  1  1  0
HAMMER STRIKE OF THE MAIN GUN IS OFF AT T=      0.01017500
HAMMER ROTATIONAL VELOCITY AND ACCELERATION ARE ZERO STARTING AT T=      0.01017500
(FORWARD) FORCE AT END OF CAM PATH IS OFF AT T=      0.01025000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01040000  0  0  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01042500  0  0  0  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01045000  0  0  0  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01045000  0  0  0  0  0  0
(FORWARD) BOLT PIN STRIKES END OF CAM PATH AT T=      0.01050000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01090000  1  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01092500  1  1  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01095000  1  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01100000  1  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.01105000  1  1  1  1  1  0
CAVITY FORCE IS ON AT T=      0.01122500
ZCAP=    -0.04125794FT (DEFLECTION OF PROJECTILE)
X(L1)=     0.00026430RAD (GUN ROTATION)
XD(L1)=     0.40899012RAD/SEC (ROTATIONAL VELOCITY OF THE GUN)
BREECH FORCE IS OFF AT T=      0.01132500
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BORE FRICTION IS OFF AT T=      0.01132500
(FORWARD) FORCE AT END OF CAM PATH IS OFF AT T=      0.01175000
HAMMER BEGINS TO COCK AT T=      0.01175000
WEAPON BEGINS TO UNLOCK AT T=      0.01225000

VELOCITIES OF THE BOLT AND BOLT CARRIER PRIOR TO THE DISCONTINUITY OF THE CAM
BLTD=       -2.367407 BLTCD=      -18.026211

VELOCITIES OF THE BOLT AND BOLT CARRIER AFTER THE DISCONTINUITY OF THE CAM
BLTD=       -2.392839 BLTCD=      -17.749314
WEAPON IS UNLOCKED AT T=      0.01305000

VELOCITIES OF THE BOLT AND BOLT CARRIER PRIOR TO THE DISCONTINUITY OF THE CAM
BLTD=       -1.126611 BLTCD=      -22.884965

VELOCITIES OF THE BOLT AND BOLT CARRIER AFTER THE DISCONTINUITY OF THE CAM
BLTD=       -1.090576 BLTCD=      -23.277296
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.01327500
BOLT AND MAIN GUN ARE TWO MASSES AT T=      0.01327500
EXPERIMENTAL CARTRIDGE CASE EJECTION FORCE IS ON AT T=      0.01327500
HAMMER CONSTRAINT FORCE IS OFF AT T=      0.01327500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01380000  0  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01382500  0  0  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01382500  0  0  0  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01385000  0  0  0  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01385000  0  0  0  0  0  0
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.01422500
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.01447500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01467500  0  0  0  0  0  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01485000  0  0  0  0  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01515000  0  0  0  1  1  1
CAVITY FORCE IS OFF AT T=      0.01570000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01592500  0  0  0  0  1  1
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.01605000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01635000  0  0  0  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01637500  0  0  1  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.01847500  0  1  1  1  1  1
HAMMER ROTATIONAL VELOCITY AND ACCELERATION ARE ZERO STARTING AT T=      0.01855000
FRICTION FORCE COMES ON BETWEEN HAMMER AND B.C. AT T=      0.01857500 FBC(11)=      1.20600619
EXPERIMENTAL CARTRIDGE CASE EJECTION FORCE IS OFF AT T=      0.02175000
AMMO AND BOLT FRICTION IS ON AT T=      0.02340000FRIC=      2.67172608
FRICTIONAL FORCE BETWEEN CARTRIDGES AND B.C. IS OFF AT T=      0.02340000
CARTRIDGE CASE IS EJECTED AT T=      0.02342500
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.02382500
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.02435000
BOLT AND AMMO FRICTION IS OFF AT T=      0.02750000
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.02797500
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.02860000
BUFFER STRIKES BACK PLATE AT T=      0.03042500

VELOCITIES PRIOR TO IMPACT
BLTD=   -16.52286533 BLTCD=   -15.73988025 XMGD=     -0.90620382

BOLT IMPULSES      -1.23365625      0.01095406      0.0             0.00026084     -0.18698827     -0.02008823
   0.0              0.18188831      0.0             0.80843809      0.00395857      0.00000497
BOLT ENERGY         2.58875684     -0.17248485      0.0            -0.00507367      2.63359922      0.03631125
   0.0             -0.42042500      0.0            -3.61488438     -0.07243737     -0.00004847
BOLT CARRIER IMPULSES      -0.00026084      0.18698827      0.02008823     -0.18188831      0.0             0.39182319
   0.00455814       0.05452869     -0.80843809      0.00447669      0.01540618      0.00002025
BOLT CARRIER ENERGY        -0.00191320     -3.66714375     -0.40927129      0.28863145      0.0            -5.30655971
  -0.10436242      -0.96319688     13.15499375     -0.55793066     -0.25632532     -0.00022124
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   0.0          -0.42455156    0.0            -3.60525625   -0.07255833   -0.00004904
BOLT CARRIER IMPULSES    -0.00029817    0.18920697    0.02006953   -0.19589825    0.0          0.43797752
   0.00460198    0.05433560   -0.80843809    0.00461028    0.01570649    0.00002044
BOLT CARRIER ENERGY    -0.00145280   -3.69377188   -0.40796426    0.29269648    0.0          -5.73168743
  -0.10396989   -0.95755605   13.12821250   -0.55841211   -0.25689453   -0.00022189
GUN IMPULSE    0.35106385   -0.01108765    0.0          -0.00460198   -0.28658475    0.21824980    0.04980952
  -0.10524895   -0.00344598   -0.00398173    0.00021519   -0.00461028   -0.01570649   -0.00398648    0.0
GUN ENERGY   -0.51334131    0.01080588    0.0          0.00627309    0.24284976   -0.13222244    0.00801545
   0.09183917    0.00620876    0.00403501   -0.00017917    0.03158816    0.01563468    0.00461456    0.0
BUFFER IMPULSE   -0.43797752    0.00344598    0.34055798    0.00000187    0.05149081    0.00398648
BUFFER ENERGY    5.54423899   -0.08664709   -4.46477598   -0.00002028   -0.68780433   -0.08263077
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.03172500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03362500  0  0  0  0  0  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03377500  0  0  0  0  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03392500  0  0  0  1  1  1
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=     0.03395000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03405000  0  0  1  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03415000  0  1  1  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03507500  0  0  1  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03507500  0  0  0  1  1  1
BUFFER COMES OFF OF BACK PLATE AT T=     0.03512500

VELOCITIES AFTER IMPACT
BLTD=     6.71591603 BLTCD=     5.81941633 XMGD=    -3.35274859

NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03512500  0  0  0  0  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03515000  0  0  0  0  0  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03627500  0  0  0  0  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03747500  0  0  0  1  1  1
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.03920000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.03942500  0  0  1  1  1  1
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=     0.03982500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.04032500  0  1  1  1  1  1
FORCE TO PICK UP NEW ROUND IS ON AT T=     0.04045000

INELASTIC COLLISION OF THE BOLT AND ROUND

MASS OF THE GUN AND VELOCITIES OF THE BOLT AND ROUND
PRIOR TO ROUND BEING PICKED UP
GUN MASS=     0.19166786  BOLT VELOCITY=     5.40006587 ROUND VELOCITY (VELOCITY OF THE GUN)=    -2.64627849

MASS OF THE GUN AND VELOCITIES OF THE BOLT AND ROUND
AFTER ROUND IS PICKED UP
GUN MASS=     0.19088050  BOLT VELOCITY=     4.27179363 ROUND VELOCITY =     4.27179363
FRICTIONAL FORCE TO PICK UP NEW ROUND IS     -6.90975269
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.04232500
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=     0.04310000
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.04425000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.04967500  0  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.04992500  0  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.04995000  0  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.04997500  0  1  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05000000  0  0  0  0  0  0
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=     0.05065000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05070000  1  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05075000  1  1  0  0  0  0
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.05085000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05085000  1  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05100000  1  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05115000  1  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=     0.05247500  1  0  1  1  1  0
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NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05247500  1  0  0  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05247500  1  0  0  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05247500  1  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05247500  0  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05407500  0  0  0  0  0  1
FORCE TO PICK UP NEW ROUND IS OFF AT T=      0.05435000

RESPONSE TIME OF THE MAGAZINE IS       0.00893390 FMI=      5.86894737 FMA=        6.03180000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05435000  0  0  0  0  1  1
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.05470000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05487500  0  0  0  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05585000  0  0  1  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05745000  0  1  1  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05880000  0  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05885000  0  1  1  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05885000  0  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05887500  0  0  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.05887500  0  0  0  0  0  0
FRICTION FORCE BETWEEN HAMMER AND B.C. IS OFF AT T=      0.06150000
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.06217500
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.06300000
FRICTIONAL FORCE BETWEEN CARTIRIDGES AND B.C.IS ON AT T=      0.06330000 FBC(10)=      -2.59290541
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.06405000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07007500  0  0  0  0  0  1
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.07020000
SPRING EXTRACTOR FORCE IS ON AT T=      0.07062500 CONSTANT FORCE      -7.50000000
SPRING FORCE BETWEEN THE BOLT AND BOLT CARRIER IS ON AT T=      0.07082500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07097500  0  0  0  0  0  0
SPRING FORCE BETWEEN BOLT AND BOLT CARRIER IS OFF AT T=      0.07147500
WEAPON LOCKING FORCE IS ON AT T=      0.07150000

VELOCITIES OF THE BOLT AND BOLT CARRIER PRIOR TO THE DISCONTINUITY OF THE CAM
BLTD=      8.322588 BLTCD=      9.911152

VELOCITIES OF THE BOLT AND BOLT CARRIER AFTER THE DISCONTINUITY OF THE CAM
BLTD=      8.419253 BLTCD=      9.885522
HAMMER IS CONTROLLED BY HAMMER SPRING AND MAIN GUN STOP AT T=      0.07157500
SPRING BETWEEN BOLT AND BARREL IS OFF AT T=      0.07175000
BOLT AND MAIN GUN ARE CONSIDERED AS ONE MASS AT T=      0.07175000
WEAPON LOCKING FORCE IS OFF AT T=      0.07340000

VELOCITIES OF THE BOLT AND BOLT CARRIER PRIOR TO THE DISCONTINUITY OF THE CAM
BLTD=      0.856913 BLTCD=      9.685520

VELOCITIES OF THE BOLT AND BOLT CARRIER AFTER THE DISCONTINUITY OF THE CAM
BLTD=      0.842262 BLTCD=      9.844710
(FORWARD) BOLT PIN STRIKES END OF CAM PATH AT T=      0.07395000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07437500  1  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07460000  1  1  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07490000  1  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07535000  1  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07545000  1  1  1  1  1  0
(FORWARD) FORCE AT END OF CAM PATH IS OFF AT T=      0.07590000
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07660000  1  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07665000  1  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07667500  1  1  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07670000  1  0  0  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07672500  0  0  0  0  0  0
(FORWARD) BOLT PIN STRIKES END OF CAM PATH AT T=      0.07822500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07875000  0  0  0  0  0  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07917500  0  0  0  0  1  1
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NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.07972500  0  0  0  1  1  1
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08090000  0  0  1  1  1  1
**********  ROUND NUMBER  2  BEGINS  **********
WEAPON FIRES AT T=       0.08205000
BREECH FORCE IS ON AT T=       0.08205000
BORE FRICTION IS ON AT T=       0.08205000
HAMMER STRIKE OF THE MAIN GUN IS ON AT T=       0.08205000
BOLT IMPULSES    -1.23365625     0.01108765    -0.05113695     0.00030866     0.02720405    -0.02006953
    0.00526676     0.48969722    -0.09604452     0.80843809     0.00398173     0.00001337
BOLT ENERGY      2.58804180    -0.17320861    -0.29506406    -0.00900426     2.85447773     0.03640155
    0.02719227     0.03226258    -0.41878594    -3.60525625    -0.07255833    -0.00000329
BOLT CARRIER IMPULSES  -0.00030866    -0.02720405     0.02006953    -0.48969722    -0.00526676     1.30187764
   -0.00426197     0.05433560    -0.80843809    -0.01878916    -0.01766694     0.00005291
BOLT CARRIER ENERGY  -0.00170448    -3.95102500    -0.40796426    -0.84181964    -0.05151292    -5.46915919
   -0.17355852    -0.95755605    13.12821250    -0.83779028    -0.46797012    -0.00003194
GUN IMPULSE      1.52004713    -0.01108765     0.05113695     0.00426197    -0.87370159     0.21299980     0.04880393
   -0.14541239    -0.50360117    -0.00398173     0.00055583     0.01878916     0.01766694     0.00174806     0.09604452
GUN ENERGY      -2.22065129     0.01080588     0.03438353    -0.00230822     0.82701963    -0.13234693     0.00799160
    0.02269532     1.14934707     0.00403501    -0.00034499     0.03681586    -0.03627704     0.00020290    -0.14212896
BUFFER IMPULSE  -1.30187764     0.50360117     0.83480412     0.00000484    -0.02178150    -0.00174806
BUFFER ENERGY    4.69725447    -1.74261152    -1.54798262    -0.00000292    -1.22874798    -0.12940998
T= 0.82050D-01X=-0.495D-01-0.494D-01-0.508D-01-0.493D-01 0.582D-02-0.594D-01-0.604D-01-0.604D-01-0.603D-01-0.603D-01 0.538D-02
 0.48289D-01 0.59566D-01 0.23713D-01-0.27782D-01 0.87248D-02 0.0          0.0          0.0          0.0          0.0
XD= 0.1888D 01 0.2000D 01 0.1888D 01 0.3044D 01 0.2588D 03 0.2044D 00 0.4187D 01 0.4113D 01 0.3946D 01 0.3643D 01 0.1043D 02
 0.1029D 02 0.1146D 02 0.7829D 01 0.6316D 01-0.9727D-01 0.0          0.0          0.0          0.0          0.0
DD= 0.1203D 04 0.8606D 03 0.1203D 04-0.2847D 04-0.1152D 06 0.6475D 01 0.3105D 03 0.1728D 03-0.3158D 03-0.1342D 04-0.1642D 04
-0.4391D 04-0.6543D 04-0.2171D 04 0.3957D 04-0.1123D 03 0.0          0.0          0.0          0.0          0.0
FB= 0.0          0.0          0.0          0.29782D-01 0.0          0.0
 0.0          0.66191D 01  0.0          0.0          0.0          0.15766D-03
FBC=-0.29782D-01 0.0          0.0          -0.66191D 01 0.0          0.25058D 02
-0.14704D 00  0.0          0.0          -0.25929D 01  0.0          0.64493D-03
FMG=-0.27681D 01 0.0          0.0          0.14704D 00-0.51083D 01 0.21000D 03 0.40223D 02
-0.16850D 02  0.0          0.0          0.67610D-02  0.25929D 01  0.0          0.76078D-01  0.0
FBUF=-0.25058D 02 0.0          0.14062D 02 0.58960D-04 0.23469D 01-0.76078D-01
XMB= 0.44511D-02 XMG= 0.19088D 00

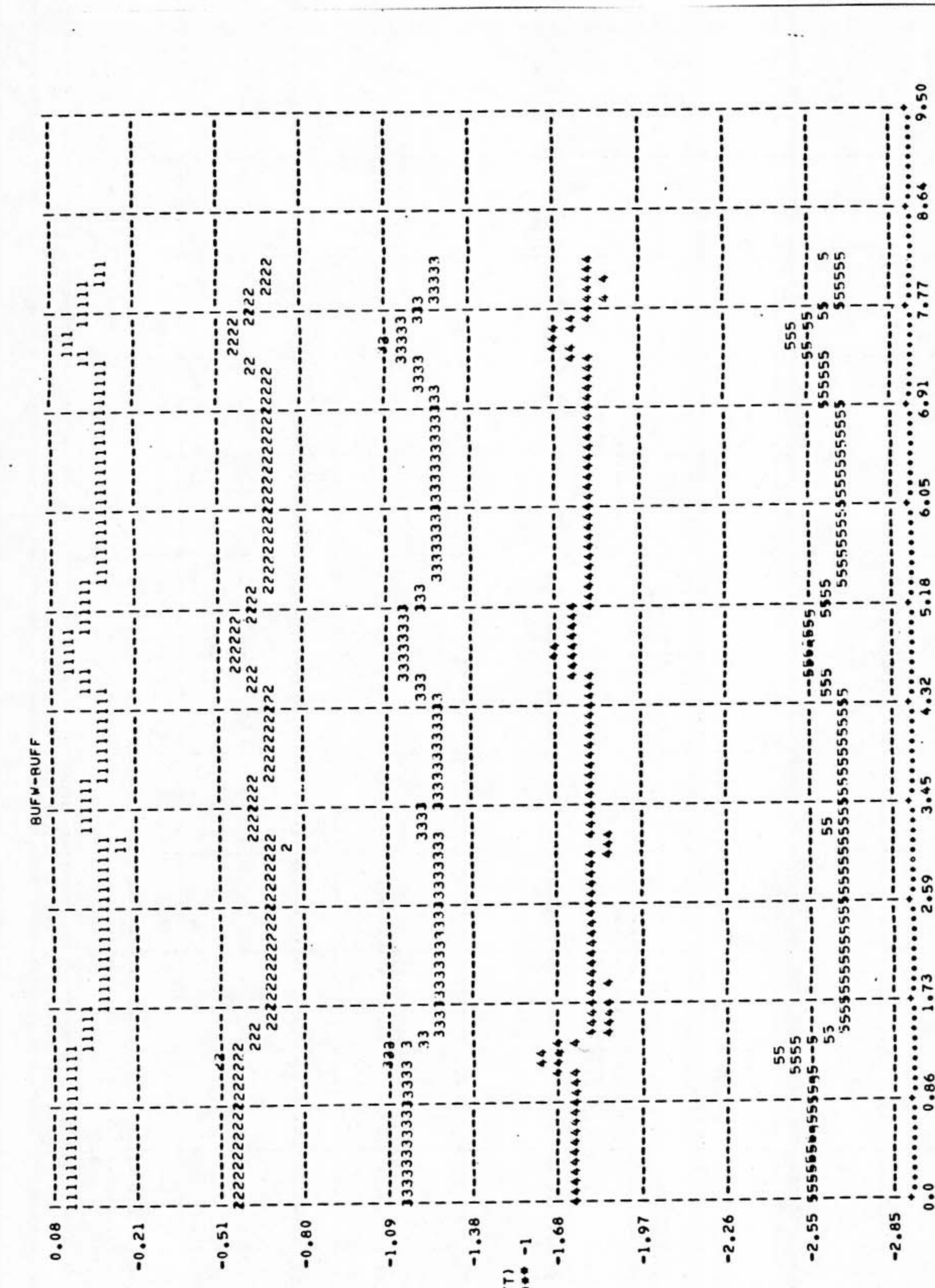
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08217500  0  0  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08225000  0  0  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08227500  0  0  0  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08227500  0  0  0  0  0  0
HAMMER STRIKE OF THE MAIN GUN IS OFF AT T=      0.08232500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08232500  0  1  0  0  0  0
HAMMER ROTATIONAL VELOCITY AND ACCELERATION ARE ZERO STARTING AT T=       0.08232500
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08235000  0  1  1  0  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08242500  0  1  1  1  0  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08260000  0  1  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08317500  0  0  1  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08320000  0  0  0  1  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08320000  0  0  0  0  1  0
NEW STATUS OF COLLISIONS OF BUFFER WTS. AT T=      0.08322500  0  0  0  0  0  0
CAVITY FORCE IS ON AT T=       0.08337500
T= 0.83400D-01X=-0.491D-01-0.478D-01-0.504D-01-0.474D-01 0.484D-01-0.558D-01-0.558D-01-0.559D-01-0.559D-01-0.559D-01 0.175D-01
 0.58258D-01 0.68930D-01 0.32449D-01-0.16010D-01 0.87907D-02 0.0          0.0          0.0          0.0          0.0
XD=-0.2362D 01-0.2032D 01-0.2362D 01-0.2048D 01 0.0          0.3450D 01 0.3324D 01 0.3213D 01 0.3043D 01 0.2946D 01 0.7125D 01
 0.4526D 01 0.2415D 01 0.5229D 01 0.1067D 02 0.2916D 00 0.0          0.0          0.0          0.0          0.0
DD=-0.9739D 03-0.8176D 04-0.9739D 03-0.7852D 04 0.0          -0.6405D 01-0.6405D 01-0.6405D 01-0.6405D 01-0.6405D 01-0.3353D 04
-0.4178D 04-0.6545D 04-0.1663D 04 0.1953D 04 0.9024D 02 0.0          0.0          0.0          0.0          0.0
FB=-0.41436D 03 0.0          0.0          0.31968D-01 0.0          0.0
 0.0          0.13880D 03  0.0          0.46568D 02  0.0          0.15766D-03
FBC=-0.31968D-01 0.0          0.0          -0.13880D 03 0.0          0.39275D 02
-0.14923D 00  0.0          -0.46568D 02 -0.25929D 01  0.0          0.64493D-03
```

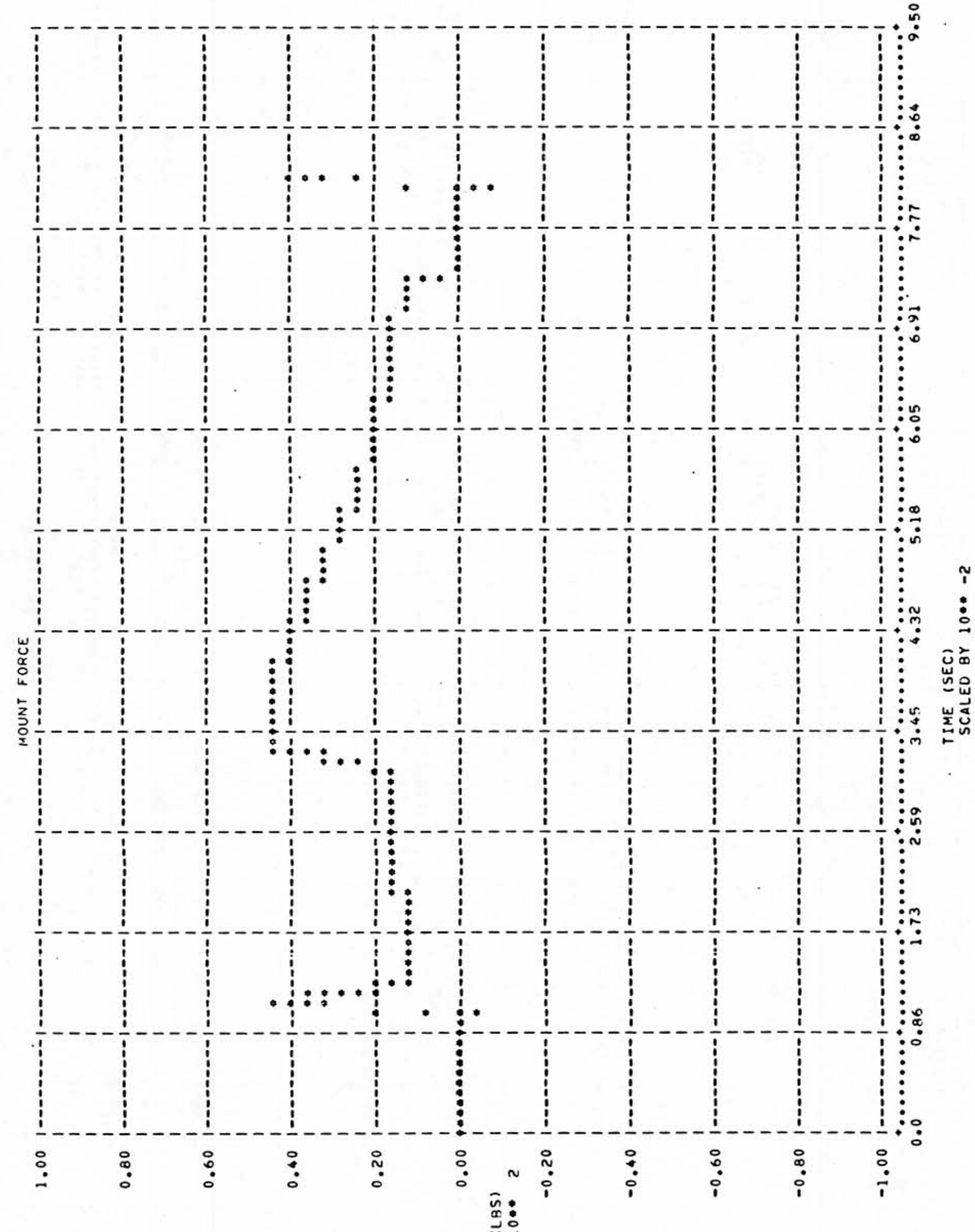
```
FMG= 0.37631D 02 0.0         0.0         0.14923D 00-0.37957D 01 0.0         0.0
 0.20658D 01  0.0         0.0         0.67610D-02  0.25929D 01  0.0         0.67672D-01  0.0
FBUF=-0.39275D 02 0.0         0.15236D 02 0.58960D-04 0.44805D-01-0.67672D-01
XMB= 0.44511D-02 XMG= 0.19088D 00
```

TERM	DEFINITION	UNITS
XMG	GUN DISPLACEMENT	FT
XMGD	VELOCITY OF THE GUN	FT/SEC
BLT	BOLT DISPLACEMENT	FT
BLTD	BOLT VELOCITY	FT/SEC
BLTC	BOLT CARRIER DISPLACEMENT	FT
BLTCD	BOLT CARRIER VELOCITY	FT/SEC
BUFF	BUFFER DISPLACEMENT	FT
BUFFD	BUFFER VELOCITY	FT/SEC
BUFW	INTERNAL BUFFER WT. DISPLACEMENT	FT
BUFWD	INTERNAL BUFFER WT. VELOCITY	FT/SEC
HAMR	ROTATION OF THE HAMMER	RAD
HAMRD	ROTATIONAL VELOCITY OF THE HAMMER	RAD/SEC

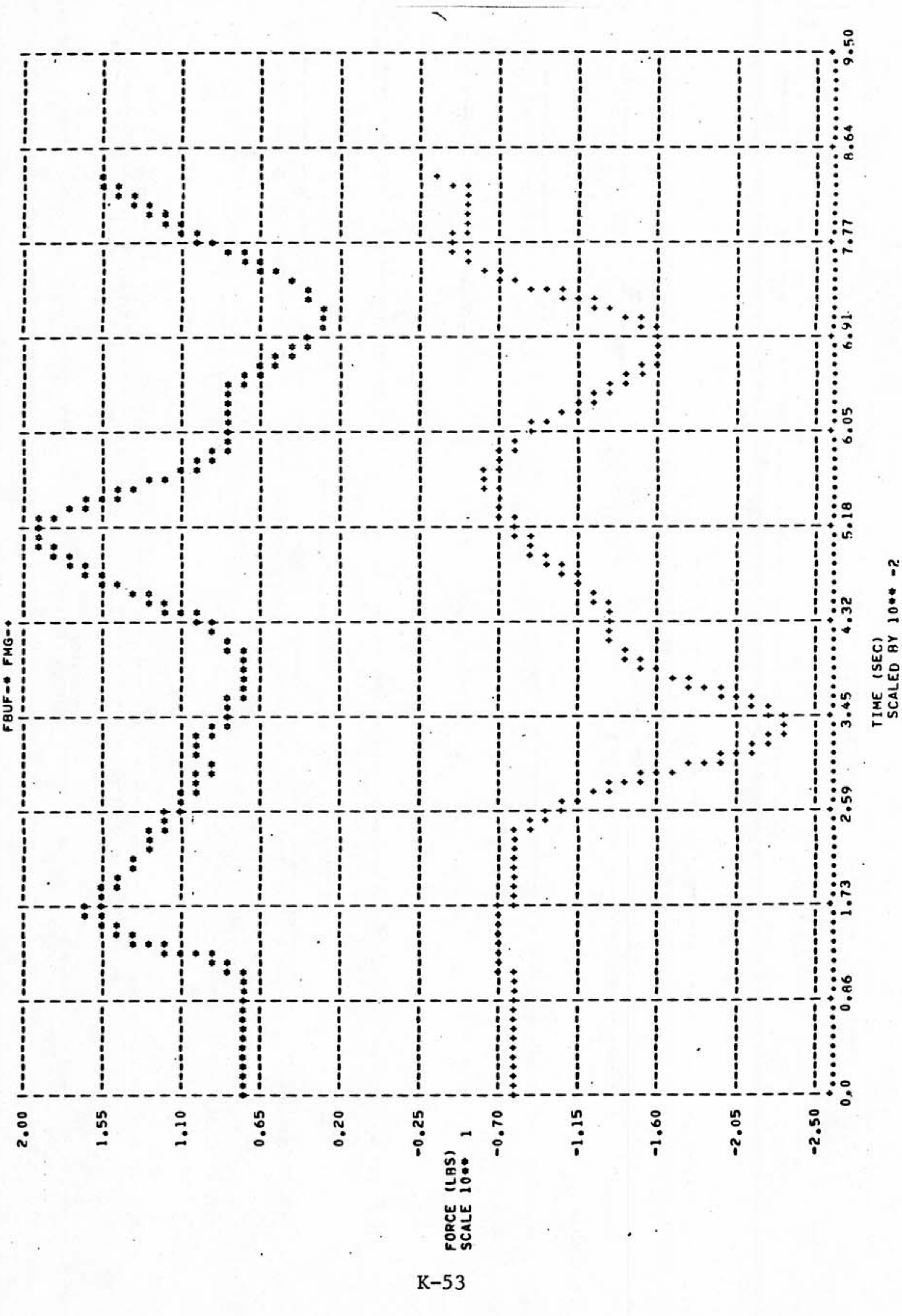
NOTE:BOLT, BOLT CARRIER, AND BUFFER DISPLACEMENTS ARE WITH RESPECT TO THE GUN AND NOT THE INERTIAL REFFRENCE FRAME

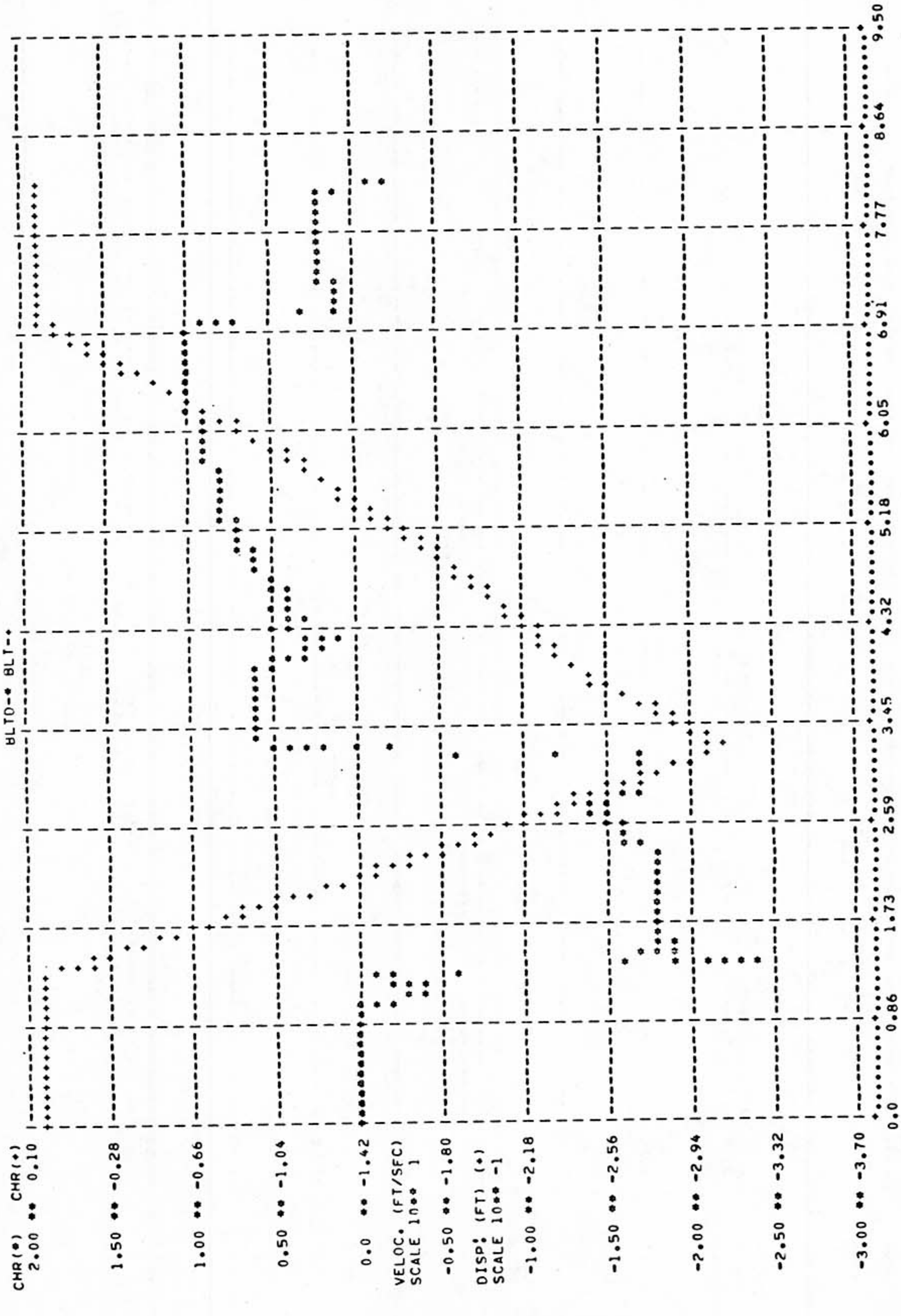
```
ABSOLUTE MAXIMUMS OF INDIVIDUAL FORCES
MAX FB= 0.21848D 04 0.26717D 01 0.19305D 03 0.25568D 00 0.43364D 03 0.34970D 02
0.22636D 02 0.34686D 03 0.69098D 01 0.51978D 03 0.93983D 00 0.17100D-03
MAX FBC= 0.25568D 00 0.43364D 03 0.34970D 02 0.34686D 03 0.22636D 02 0.31211D 03
0.37294D 00 0.51655D 02 0.51978D 03 0.26717D 01 0.12059D 01 0.64493D-03
MAX FMG= 0.43385D 02 0.26717D 01 0.19305D 03 0.37294D 00 0.23069D 02 0.32500D 03 0.26475D 03
0.87628D 02 0.32991D 03 0.93983D 00 0.67889D-02 0.26717D 01 0.12059D 01 0.32797D 00 0.69098D 01
MAX FBUF= 0.31211D 03 0.32991D 03 0.19207D 02 0.58960D-04 0.16385D 03 0.32797D 00
FMCI=-0.42921D 03 FMCA= 0.0
ZCAP=      3.76849016FT (DEFLECTION OF PROJECTILE)
X(L1)=      0.00879073RAD (GUN ROTATION)
XD(L1)=      0.29159483RAD/SEC (ROTATIONAL VELOCITY OF THE GUN)
```

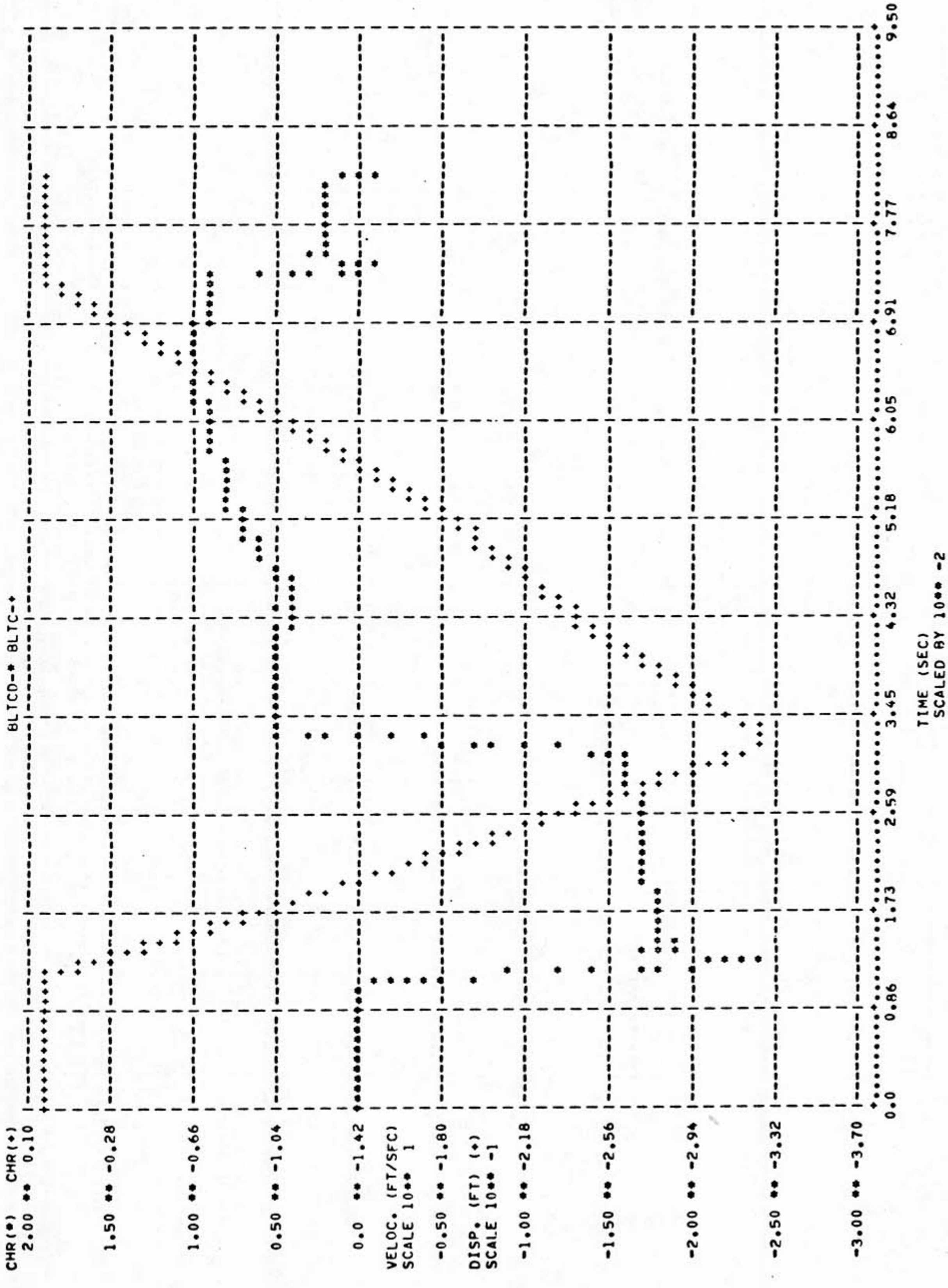

BUFW-RUFF
DISP. (FT)
SCALE 10** -1
0.08
-0.21
-0.51
-0.80
-1.09
-1.38
-1.68
-1.97
-2.26
-2.55
-2.85
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2



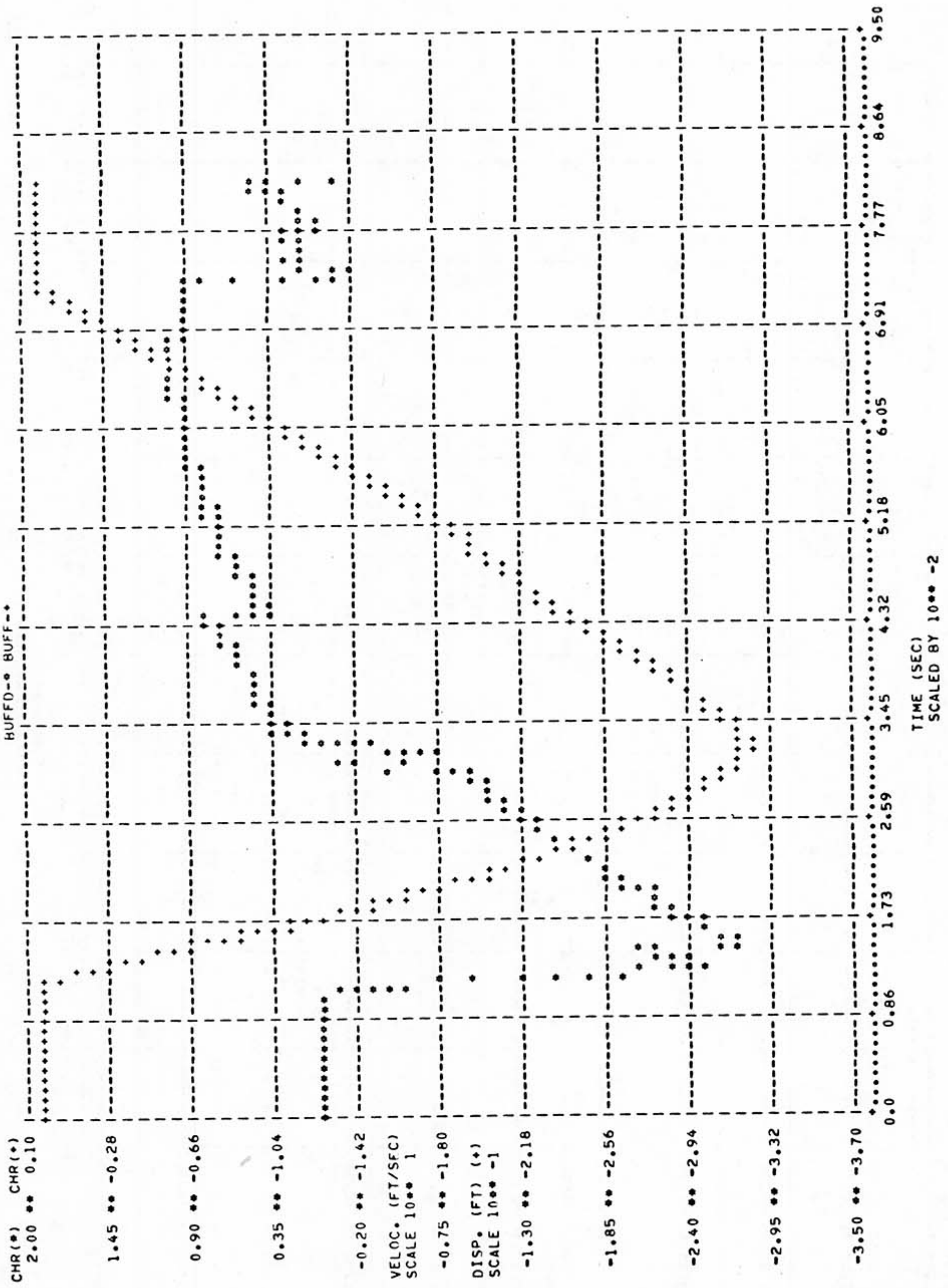
MOUNT FORCE
FORCE (LBS)
SCALE 10** 2
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2

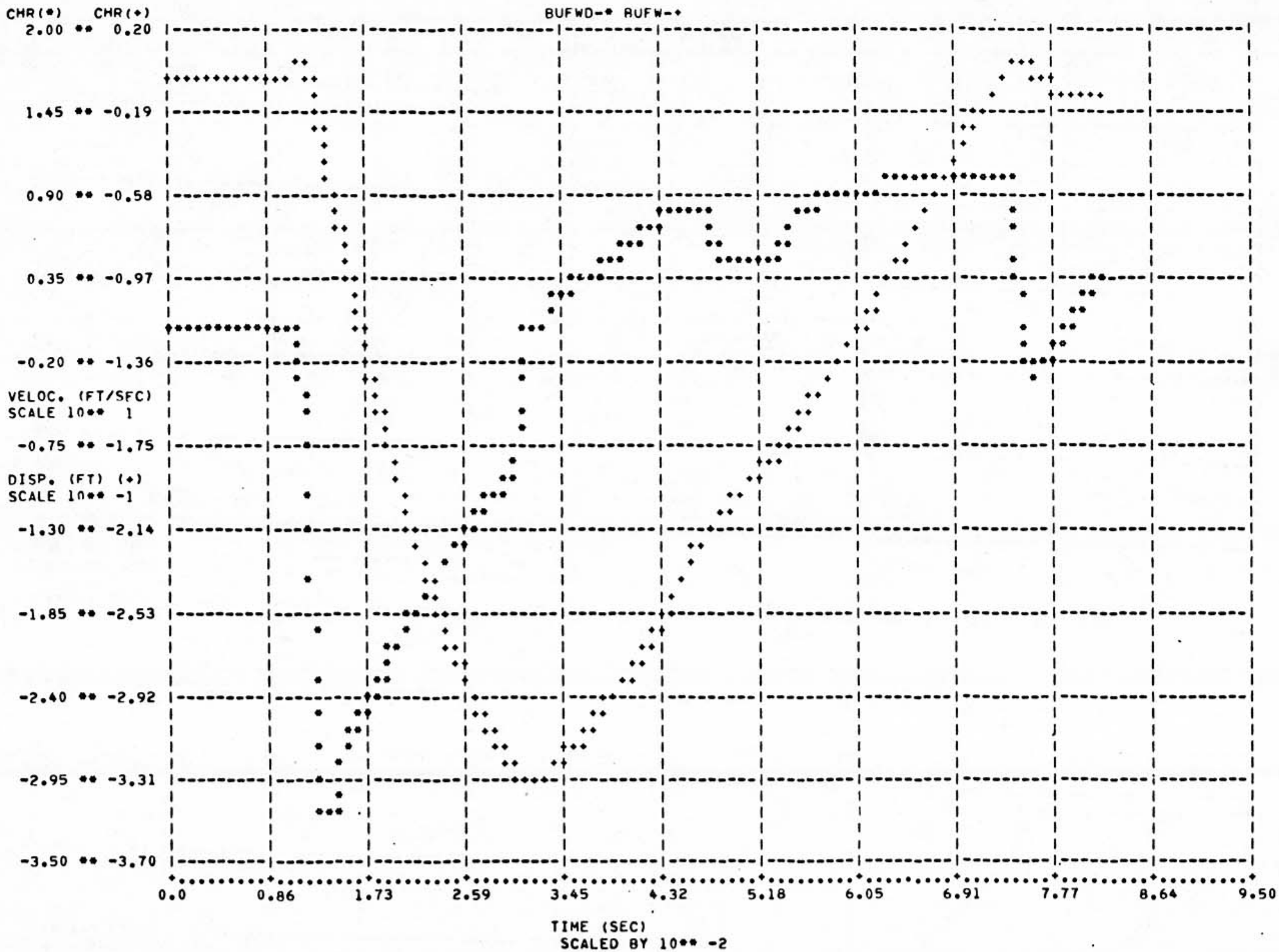

FBUF-* FMG-+
2.00
1.55
1.10
0.65
0.20
-0.25
-0.70
-1.15
-1.60
-2.05
-2.50
FORCE (LBS)
SCALE 10** 1
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2

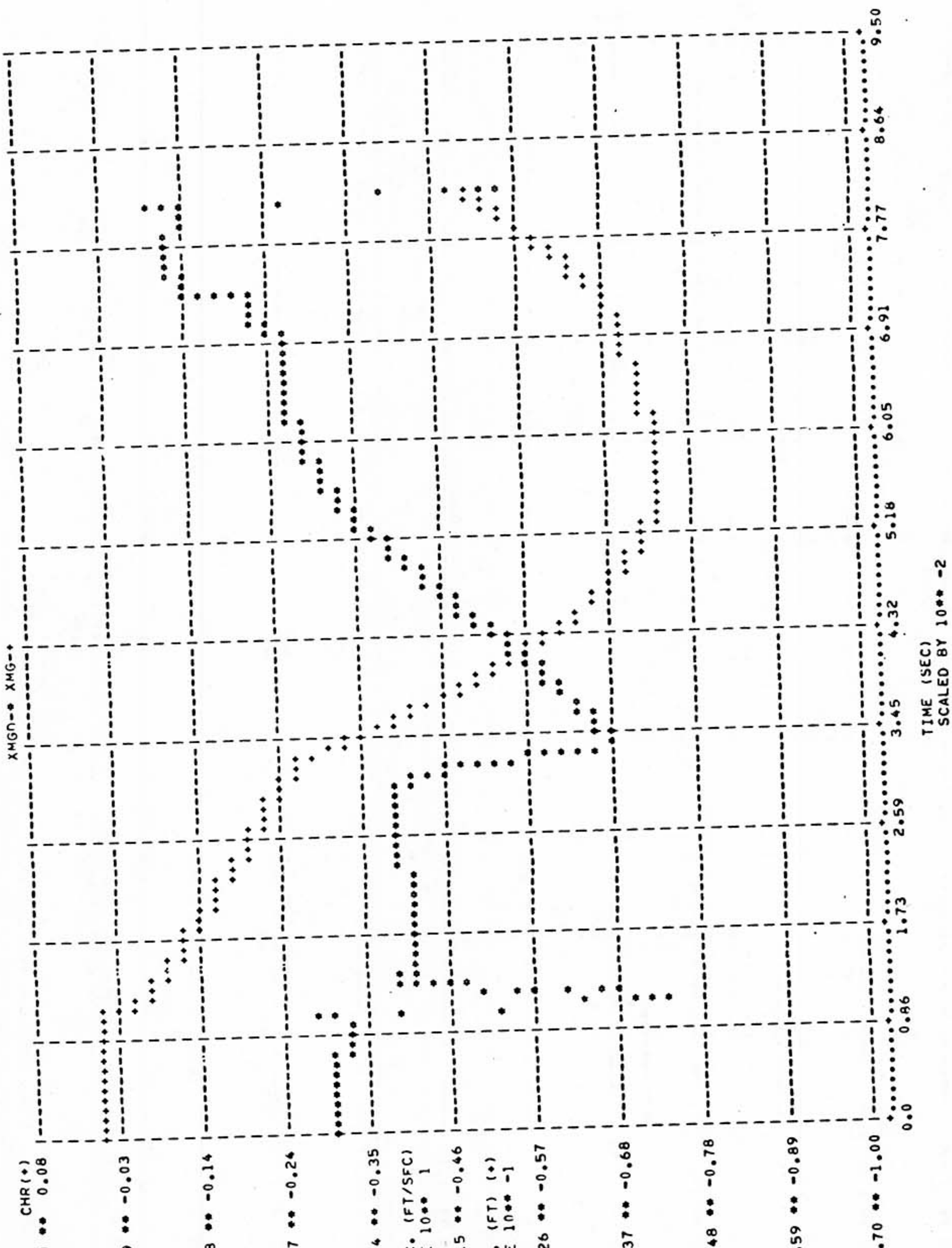

BLTD-* BLT-+
CHR(*) CHR(+)
2.00 ** 0.10
1.50 ** -0.28
1.00 ** -0.66
0.50 ** -1.04
0.0 ** -1.42
VELOC. (FT/SFC)
SCALE 10** 1
-0.50 ** -1.80
DISP. (FT) (+)
SCALE 10** -1
-1.00 ** -2.18
-1.50 ** -2.56
-2.00 ** -2.94
-2.50 ** -3.32
-3.00 ** -3.70
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2

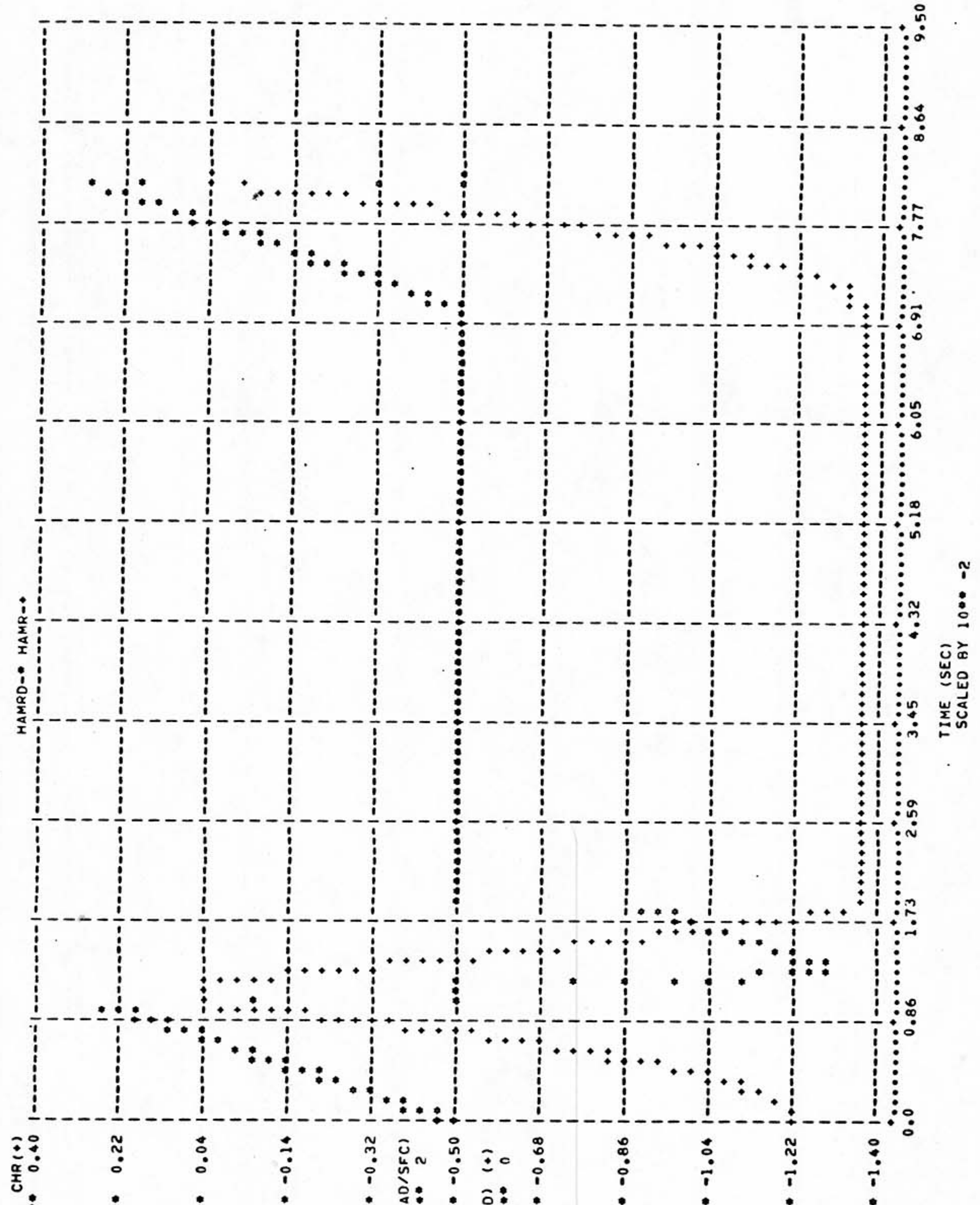


BLTCD-* BLTC-+
CHR(*) CHR(+)
2.00 ** 0.10
1.50 ** -0.28
1.00 ** -0.66
0.50 ** -1.04
0.0 ** -1.42
VELOC. (FT/SEC)
SCALE 10** 1
-0.50 ** -1.80
DISP. (FT) (+)
SCALE 10** -1
-1.00 ** -2.18
-1.50 ** -2.56
-2.00 ** -2.94
-2.50 ** -3.32
-3.00 ** -3.70
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2


BUFFD-* BUFF-+
CHR(*) CHR(+)
2.00 ** 0.10
1.45 ** -0.28
0.90 ** -0.66
0.35 ** -1.04
-0.20 ** -1.42
VELOC. (FT/SFC) SCALE 10** 1
-0.75 ** -1.80
DISP. (FT) (+) SCALE 10** -1
-1.30 ** -2.18
-1.85 ** -2.56
-2.40 ** -2.94
-2.95 ** -3.32
-3.50 ** -3.70
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2


BUFWD-* BUFW-+
CHR(*) CHR(+)
2.00 ** 0.20
1.45 ** -0.19
0.90 ** -0.58
0.35 ** -0.97
-0.20 ** -1.36
VELOC. (FT/SEC)
SCALE 10** 1
-0.75 ** -1.75
DISP. (FT) (+)
SCALE 10** -1
-1.30 ** -2.14
-1.85 ** -2.53
-2.40 ** -2.92
-2.95 ** -3.31
-3.50 ** -3.70
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2


XMGD-* XMG-+
CHR(*) CHR(+)
0.40 ** 0.08
0.29 ** -0.03
0.18 ** -0.14
0.07 ** -0.24
-0.04 ** -0.35
VELOC. (FT/SEC)
SCALE 10** 1
-0.15 ** -0.46
DISP. (FT) (+)
SCALE 10** -1
-0.26 ** -0.57
-0.37 ** -0.68
-0.48 ** -0.78
-0.59 ** -0.89
-0.70 ** -1.00
0.0
0.86
1.73
2.59
3.45
4.32
5.18
6.05
6.91
7.77
8.64
9.50
TIME (SEC)
SCALED BY 10** -2



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APPENDIX L

COMPUTER PROGRAM FOR SPRING SURGE AND SAMPLE OUTPUT

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UNIVERSITY OF IOWA....START JOB 134......7.49.26 AM  1 NOV 72................RAHE     ....M-16  ................UNIVERSITY OF IOWA
UNIVERSITY OF IOWA....START JOB 134......7.49.26 AM  1 NOV 72................RAHE     ....M-16  ................UNIVERSITY OF IOWA
UNIVERSITY OF IOWA....START JOB 134......7.49.26 AM  1 NOV 72................RAHE     ....M-16  ................UNIVERSITY OF IOWA
UNIVERSITY OF IOWA....START JOB 134......7.49.26 AM  1 NOV 72................RAHE     ....M-16  ................UNIVERSITY OF IOWA

//RAHE   JOB (--------,4),'M-16  '                                   JOB 134
C       PROGRAM WRITTEN BY ALBERT PATSCHE
      DIMENSION XBUF(7), XDBUF(7), XDDBUF(7), BUFK(9), BUFKON(4),
     1 BUFC(9), BUFCON(4), BUFDEL(9), BUFM(3)
      NRCDS=0
      REWIND 4
      READ 1, NCORD
      DIMENSION XCORD(20), YCORD(20)
      READ 2, (XCORD(I), YCORD(I), I=1,NCORD )
      DO 90 I=1,NCORD
      XCORD(I)=XCORD(I)*0.08/173.1
   90 YCORD(I)=32.0 - YCORD(I)*3.645/ 97.9
      READ 2, XMG, XMGD, XOP, XOPD, TEND,H
    1 FORMAT(I10)
    2 FORMAT(8F10.0)
      DO 10 I=1,7
      XBUF(I)=0.0
   10 XDBUF(I)=0.0
      READ 2,(BUFKON(I), BUFCON(I), I=1,4)
      DO 11 I=1,9
      BUFK(I)=0.0
   11 BUFC(I)=0.0
      READ 2,(BUFM(I),I=1,3)
      READ 2,(BUFDEL(I),I=1,9)
      T=0.0
         XOPT0=0.0
          IXOP=0
          TLAG=0.2
      DIMENSION XDS(44), XDOTDS(44)
      READ 1, NCOILS
      READ 2, DSK,DSC,DSFL,DSIL,DSM
      PRINT 70, XMG, XMGD, XOP, XOPD, TEND, H, (BUFKON(I),I=1,4),
     1   (BUFCON(I),I=1,4), (BUFM(I),I=1,3), (BUFDEL(I),I=1,9)
   70 FORMAT('1',50X,'I N P U T   D A T A'////54X,'B U F F E R '///
     1 3X,'XMG',11X,'XMGDOT',8X,'XOP',11X,'XOPDOT',8X,'TEND',10X,'DELTA
     2T'/6E14.6//3X,'K1',12X,'K2...K6',7X,'K8&K9'9X,'K7'12X,'C1',12X,
     3 'C2...C6',7X,'C8&C9',9X,'C7'/8E14.6//3X,'M1',12X,'M2...M6',7X,'M7
     4 '/3E14.6//50X,'DELTA 1 THRU DELTA 9'/9E14.6)
      PRINT 71, NCOILS, DSK, DSC, DSFL, DSIL, DSM
   71 FORMAT(////54X,'S P R I N G'//3X,'COILS'/I5//3X,'K',13X,'C',13X,
     1 'FREE LENGTH',3X,'INITL LENGTH',3X,'M'/5E14.6//)
      READ 1, NTHTRN
      NTIME=NTHTRN
      READ 1, NTHPLT
      NOWPLT=NTHPLT
      PRINT 73, NTHTRN, NTHPLT
   73 FORMAT(20X,I5,'=NTHTRN',20X,I5,'=NTHPLT'//)
      Z=NTHTRN
      Z=Z*H
      CALL SPRNGU(NCOILS,DSFL,DSIL,XBUF(1),XMG,DSK,DSC,DSM,Z,XDS,XDOTDS,
     1 FNOT,XDBUF(1))
```

```
      FBUF=FNOT
      CALL BUFF(BUFKON,BUFCON,BUFM)
      PRINT 72, DSK, DSC, DSM
   72 FORMAT(44X,'A F T E R   A D J U S T M E N T'//3X,'K',13X,'C',13X,
     1 'M'/3F14.6)
      INTEGER*2 IPLOT(101,51)
      CALL PLOT0(IPLOT)
      REAL AA1(15), AA2(15), AA3(15), AA4(15), AA5(15), AA6(15), AA7(15)
      REAL AA8(15)
      READ 2,(AA1(I),I=1,6)
      DO 12 I=1,6
      AA2(I)=AA1(I)
      AA3(I)=AA1(I)
      AA4(I)=AA1(I)
      AA5(I)=AA1(I)
      AA6(I)=AA1(I)
      AA7(I)=AA1(I)
   12 AA8(I)=AA1(I)
      CALL PLOT1(AA1)
      CALL PLOT1(AA2)
      CALL PLOT1(AA3)
      CALL PLOT1(AA4)
      CALL PLOT1(AA5)
      CALL PLOT1(AA6)
      CALL PLOT1(AA7)
      CALL PLOT1(AA8)
      REAL TITLE(5), VLAB(4), HLAB(4)
      READ 9,TITLE
      READ 9,VLAB
      READ 9,HLAB
    9 FORMAT(5A4)
      INTEGER*2 CH1/'*'/,CH2/'2'/,CH3/'3'/,CH4/'4'/,CH5/'5'/,CH6/'6'/,
     * CH7/'7'/,CH8/'*'/
      INTEGER*2 JPLOT(101,51)
      CALL PLOT0(JPLOT)
      REAL BBB(44,15),A(15)
      READ 2, (BBB(1,I),I=1,6)
      DIMENSION IDSPLT(44)
      READ 3,(IDSPLT(I),I=1,44)
    3 FORMAT(44I1)
      INTEGER*2 DSCHAR(44)/'A','B','C','D','E','F','G','H','I','J','K',
     * 'L','M','N','O','P','Q','R','S','T','U','V','W','X','Y','Z','1',
     * '2','3','4','5','6','7','8','9','0','*','+','$','@','=','%','#',
     * '&'/
      REAL TITL2(5), VLAB2(4), HLAB2(4)
      READ 9,TITL2
      READ 9,VLAB2
      READ 9,HLAB2
      DO 7 I=7,15
    7 BBB(1,I) = 0.0
      DO 5 I=1,44
      IF ( IDSPLT(I) .EQ. 0 ) GO TO 5
      DO 6 J=1,15
    6 BBB(I,J) = BBB(1,J)
    5 CONTINUE
      DO 15 I=1,44
      IF ( IDSPLT(I) .EQ. 0 ) GO TO 15
      DO 16 J=1,6
   16 A(J) = BBB(I,J)
      CALL PLOT1(A)
      BBB(I,7) = A(7)
```

```
      BBB(I,8) = A(8)
      BBB(I,15) = A(15)
   15 CONTINUE
      INTEGER*2 KPLOT(101,51)
      CALL PLOTO(KPLOT)
      REAL AB2(15), AB3(15), AB4(15), AB5(15), AB6(15), AB7(15)
      READ 2,(AB2(I), I=1,6)
      DO 61 I=1,6
      AB3(I)=AB2(I)
      AB4(I)=AB2(I)
      AB5(I)=AB2(I)
      AB6(I)=AB2(I)
      AB7(I)=AB2(I)
   61 CONTINUE
      CALL PLOT1(AB2)
      CALL PLOT1(AB3)
      CALL PLOT1(AB4)
      CALL PLOT1(AB5)
      CALL PLOT1(AB6)
      CALL PLOT1(AB7)
      REAL TITL3(5), VLAB3(4), HLAB3(4)
      READ 9, TITL3
      READ 9, VLAB3
      READ 9, HLAB3
      COILS=NCOILS
  100 CONTINUE
      NTIME=NTIME+1
      IF ( NTIME .LT. NTHTRN ) GO TO 150
      NTIME=0
         CALL XNOW(T,XOP,XOPD,XCORD,YCORD)
      CALL SPRING(XDS,XDOTDS,XMG,XBUF(1),XMGD,XDBUF(1),FBUF)
  150 CONTINUE
      CALL BUFFER(H,XBUF,XDBUF,BUFDEL,FBUF,XMG,XOP,XMGD,XOPD,FNOT)
      T=T+H
      NOWPLT=NOWPLT+1
      IF ( NOWPLT .LT. NTHPLT ) GO TO 60
      NOWPLT=0
      PX1=XBUF(1)+8.0
      PX2=XBUF(2)+8.0-0.3775
      PX3=XBUF(3)+8.0-0.3775-(0.684)*1.0
      PX4=XBUF(4)+8.0-0.3775-(0.684)*2.0
      PX5=XBUF(5)+8.0-0.3775-(0.684)*3.0
      PX6=XBUF(6)+8.0-0.3775-(0.684)*4.0
      PX7=XBUF(7)+8.0-0.3775-(0.684)*4.0-0.9565
      PX8=PX1-4.85
      CALL PLOT2(IPLOT,CH1,AA1,T,PX1,0)
      CALL PLOT2(IPLOT,CH2,AA2,T,PX2,0)
      CALL PLOT2(IPLOT,CH3,AA3,T,PX3,0)
      CALL PLOT2(IPLOT,CH4,AA4,T,PX4,0)
      CALL PLOT2(IPLOT,CH5,AA5,T,PX5,0)
      CALL PLOT2(IPLOT,CH6,AA6,T,PX6,0)
      CALL PLOT2(IPLOT,CH7,AA7,T,PX7,0)
      CALL PLOT2(IPLOT,CH8,AA8,T,PX8,0)
      DO 25 I=1,44
      IF ( IGSPLT(I) .EQ. 0 ) GO TO 25
      DO 26 J=9,15
   26 A(J) = BBB(I,J)
      FLOATI=I
      XCOIL = XDS(I) + (COILS + 1.0 - FLOATI)*DSIL/COILS
      CALL PLOT2(JPLOT,DSCHAR(I),A,T,XCOIL,0)
      DO 27 J=9,15
```

```
      BBB(I,J) = A(J)
   27 CONTINUE
   25 CONTINUE
      PY2=PX1-PX2
      PY3=PX1-PX3
      PY4=PX1-PX4
      PY5=PX1-PX5
      PY6=PX1-PX6
      PY7=PX1-PX7
      CALL PLOT2(KPLOT,CH2,AB2,T,PY2,0)
      CALL PLOT2(KPLOT,CH3,AB3,T,PY3,0)
      CALL PLOT2(KPLOT,CH4,AB4,T,PY4,0)
      CALL PLOT2(KPLOT,CH5,AB5,T,PY5,0)
      CALL PLOT2(KPLOT,CH6,AB6,T,PY6,0)
      CALL PLOT2(KPLOT,CH7,AB7,T,PY7,0)
      WRITE (4) T,PY2,PY3,PY4,PY5,PY6,PY7
      NRCDS=NRCDS+1
   60 CONTINUE
      IF ( T .LT. TEND ) GO TO 100
      CALL PLOT3(IPLOT,AA1,TITLE,VLAB,HLAB,1)
      CALL PLOT3(JPLOT,A,TITL2,VLAB2,HLAB2,1)
      CALL PLOT3(KPLOT,AB2,TITL3,VLAB3,HLAB3,1)
      CALL EXIT
      END
      SUBROUTINE SPRNGO(N,FL,IL,XOP,XMG,K,C,M,INCR,X,XDOT,F1,XOPDOT)
      REAL K,M,INCR,IL
      DIMENSION X(44),XDOT(44)
      DO 10 I=1,N
      X(I)=0.0
   10 XDOT(I)=0.0
      REALN=N
      CAPDEL=FL/(REALN+1.0)
      DEL=IL/(REALN+1.0)
      DEL1=XOP+(CAPDEL-DEL)
      DEL2=XMG-(CAPDEL-DEL)
      K=K*(REALN+1.0)
      C=C*(REALN+1.)
      M=M/REALN
      ALPHA=2.*C
      BETA=2.*K
      POWER=-ALPHA/(2.0*M)
      ETERM=EXP(POWER*INCR)
      IF((BETA/M)-(ALPHA/(2.*M))**2)61,62,63
   63 IFLAG=1
      ROOT=SQRT((BETA/M)-(ALPHA/(2.0*M))**2)
      COSTM=COS(ROOT*INCR)
      SINTM=SIN(ROOT*INCR)
      PRINT 903
  903 FORMAT(20X,'SPRING IS UNDFR DAMPED')
      GO TO 60
   62 IFLAG=0
      PRINT 902
  902 FORMAT(20X,'SPRING IS CRITICALLY DAMPED')
      GO TO 60
   61 IFLAG=-1
      ROOT=SQRT((ALPHA/(2.*M))**2-(BETA/M))
      ETERMP=EXP(ROOT*INCR)
      ETERMN=EXP(-ROOT*INCR)
      PRINT 901
  901 FORMAT(20X,'SPRING IS OVER DAMPED')
   60 CONTINUE
```

```
      F1=-K*(XOP-X(1)-DEL1)-C*(XOPDOT-XDOT(1))
      N1=N-1
      RETURN
      ENTRY SPRING(X,XDOT,XMG,XOP,XMGDOT,XOPDOT,F1)
      IF(IFLAG)601,602,603
  603 CONTINUE
      H=K*(X(2)+XOP+CAPDEL-DEL-DEL1)+C*(XDOT(2)+XOPDOT)
      COEF=X(1)-H/BETA
      X(1)=ETERM*(COEF*COSTM+(XDOT(1)+ALPHA*COEF/(2.*M))*SINTM/ROOT)
     1 +H/BETA
      XDOT(1)=ETERM*(XDOT(1)*COSTM-SINTM*(ALPHA*XDOT(1)+2.*BETA*COEF)/
     1 (2.*M*ROOT))
      IF (N .EQ. 2) GO TO 200
      DO 100 I=2,N1
      H=K*(X(I-1)+X(I+1))+C*(XDOT(I-1)+XDOT(I+1))
      COEF=X(I)-H/BETA
      X(I)=ETERM*(COEF*COSTM+(XDOT(I)+ALPHA*COEF/(2.*M))*SINTM/ROOT)
     1 +H/BETA
  100 XDOT(I)=ETERM*(XDOT(I)*COSTM-SINTM*(ALPHA*XDOT(I)+2.*BETA*COEF)/
     1 (2.*M*ROOT))
  200 CONTINUE
      H=K*(X(N-1)+XMG-CAPDEL+DEL-DEL2)+C*(XDOT(N-1)+XMGDOT)
      COEF=X(N)-H/BETA
      X(N)=ETERM*(COEF*COSTM+(XDOT(N)+ALPHA*COEF/(2.*M))*SINTM/ROOT)
     1 +H/BETA
      XDOT(N)=ETERM*(XDOT(N)*COSTM-SINTM*(ALPHA*XDOT(N)+2.*BETA*COEF)/
     1 (2.*M*ROOT))
      GO TO 9999
  602 CONTINUE
      H=K*(X(2)+XOP+CAPDEL-DEL-DEL1)+C*(XDOT(2)+XOPDOT)
      COEF=X(1)-H/BETA
      X(1)=ETERM*(COEF-(POWER*COEF-XDOT(1))*INCR)+H/BETA
      XDOT(1)=ETERM*((XDOT(1)-POWER*COEF)*(1.+POWER*INCR)+POWER*COEF)
      IF (N .EQ. 2) GO TO 202
      DO 102 I=2,N1
      H=K*(X(I-1)+X(I+1))+C*(XDOT(I-1)+XDOT(I+1))
      COEF=X(I)-H/BETA
      X(I)=ETERM*(COEF-(POWER*COEF-XDOT(I))*INCR)+H/BETA
  102 XDOT(I)=ETERM*((XDOT(I)-POWER*COEF)*(1.+POWER*INCR)+POWER*COEF)
  202 CONTINUE
      H=K*(X(N-1)+XMG-CAPDEL+DEL-DEL2)+C*(XDOT(N-1)+XMGDOT)
      COEF=X(N)-H/BETA
      X(N)=ETERM*(COEF-(POWER*COEF-XDOT(N))*INCR)+H/BETA
      XDOT(N)=ETERM*((XDOT(N)-POWER*COEF)*(1.+POWER*INCR)+POWER*COEF)
      GO TO 9999
  601 CONTINUE
      H=K*(X(2)+XOP+CAPDEL-DEL-DEL1)+C*(XDOT(2)+XOPDOT)
      COEF1=(XDOT(1)+(ROOT-POWER)*(X(1)-H/BETA))/(2.*ROOT)
      COEF2=(X(1)-H/BETA)-COEF1
      X(1)=ETERM*(COEF1*ETERMP+COEF2*ETERMN)+H/BETA
      XDOT(1)=ETERM*(COEF1*ETERMP*(ROOT+POWER)-COEF2*ETERMN*(ROOT-POWER)
     1 )
      IF (N .EQ. 2) GO TO 201
      DO 101 I=2,N1
      H=K*(X(I-1)+X(I+1))+C*(XDOT(I-1)+XDOT(I+1))
      COEF1=(XDOT(I)+(ROOT-POWER)*(X(I)-H/BETA))/(2.*ROOT)
      COEF2=(X(I)-H/BETA)-COEF1
      X(I)=ETERM*(COEF1*ETERMP+COEF2*ETERMN)+H/BETA
  101 XDOT(I)=ETERM*(COEF1*ETERMP*(ROOT+POWER)-COEF2*ETERMN*(ROOT-POWER)
     1 )
  201 CONTINUE
```

```
      H=K*(X(N-1)+XMG-CAPDEL+DEL-DEL2)+C*(XDOT(N-1)+XMGDOT)
      COEF1=(XDOT(N)+(ROOT-POWER)*(X(N)-H/BETA))/(2.*ROOT)
      COEF2=(X(N)-H/BETA)-COEF1
      X(N)=ETERM*(COEF1*ETERMP+COEF2*ETERMN)+H/BETA
      XDOT(N)=ETERM*(COEF1*ETERMP*(ROOT+POWER)-COEF2*ETERMN*(ROOT-POWER)
     1 )
9999  F1=-K*(XOP-X(1)-DEL1)-C*(XOPDOT-XDOT(1))
      F2=-K*(XMG-X(N)-DEL2)-C*(XMGDOT-XDOT(N))
      RETURN
      END
      SUBROUTINE XNOW(T,XOP,XOPD,X,Y)
      DIMENSION X(20), Y(20)
      I=1
      CALL LINEAR(T,X,Y,XOP,I)
      XOPD=(Y(I+1)-Y(I))/(X(I+1)-X(I))
      RETURN
      END
      SUBROUTINE LINEAR (A,X,Y,VV,I)
      DIMENSION X(20),Y(20)
    2 IF(A-X(I)) 3,1,1
    1 I=I+ 1
      GO TO 2
    3 I=I-1
      VV=Y(I)*(A-X(I+1))/(X(I)-X(I+1))+Y(I+1)*(A-X(I))/(X(I+1)-X(I))
      RETURN
      END
      SUBROUTINE BUFF(BK,BC,BM)
      DIMENSION BK(4), BC(4), BM(3)
      REAL K(9), KON(9), C(9), CON(9), M(7)
      M(1)=BM(1)
      M(7)=BM(3)
      KON(1)=BK(1)
      CON(1)=BC(1)
      KON(7)=BK(4)
      CON(7)=BC(4)
      KON(8)=BK(3)
      KON(9)=BK(3)
      CON(9)=BC(3)
      CON(8)=BC(3)
      DO 1 I=2,6
      M(I)=BM(2)
      KON(I)=BK(2)
    1 CON(I)=BC(2)
      RETURN
      ENTRY BUFFER(INCR,X,XDOT,DEL,FBUF,XMG,XOP,XMGD,XOPD,FNOT)
      REAL INCR
      DIMENSION X(7), XDOT(7), DEL(9)
      FOFT=FBUF-FNOT
      CALL ZERO(K,C)
      IF(  XOP-X(1)  .LE. DEL(1) ) K(1)=KON(1)
      IF(  XOP-X(1)  .LE. DEL(1) ) C(1)=CON(1)
      IF( X(1)-X(2)  .LE. DEL(2) ) K(2)=KON(2)
      IF( X(1)-X(2)  .LE. DEL(2) ) C(2)=CON(2)
      IF( X(7)+DEL(8)  .LE. X(1) ) K(8)=KON(8)
      IF( X(7)+DEL(8)  .LE. X(1) ) C(8)=CON(8)
      IF(  XMG-X(1)  .GE. DEL(9) ) K(9)=KON(9)
      IF(  XMG-X(1)  .GE. DEL(9) ) C(9)=CON(9)
      ALPHA=C(1) + C(2) + C(8) + C(9)
      BETA =K(1) + K(2) + K(8) + K(9)
      POWER=-ALPHA/(2.*M(1))
      ETERM=EXP(POWER*INCR)
```

```
      IF ( BETA/M(1) - (ALPHA/(2.*M(1)))**2) 11,12,13
   11 ROOT=SQRT((ALPHA/(2.*M(1)))**2 - BETA/M(1))
      ETERMP=EXP(ROOT*INCR)
      ETERMN=EXP(-ROOT*INCR)
      H=FOFT-K(1)*(DEL(1)-XOP )+C(1)* XOPD   +K(2)*(DEL(2)+X(2))+C(2)*
     1 XDOT(2)+K(9)*( XMG-DEL(9))+C(9)* XMGD   +K(8)*(X(7)+DEL(8))+C(8)*
     2 XDOT(7)
      COEF1=(XDOT(1)+(ROOT-POWER)*(X(1)-H/BETA))/(2.*ROOT)
      COEF2=X(1)-H/BETA-COEF1
      X(1)=ETERM*(COEF1*ETERMP+COEF2*ETERMN)+H/BETA
      XDOT(1)=ETERM*(COEF1*ETERMP*(ROOT+POWER)-COEF2*ETERMN*(ROOT-POWER)
     1 )
      GO TO 20
   12 BETA=BETA+10.**(-5)
   13 ROOT=SQRT(BETA/M(1) - (ALPHA/(2.*M(1)))**2)
      COSTM=COS(ROOT*INCR)
      SINTM=SIN(ROOT*INCR)
      H=FOFT-K(1)*(DEL(1)-XOP )+C(1)* XOPD   +K(2)*(DEL(2)+X(2))+C(2)*
     1 XDOT(2)+K(9)*( XMG-DEL(9))+C(9)* XMGD   +K(8)*(X(7)+DEL(8))+C(8)*
     2 XDOT(7)
      COEF=X(1)-H/BETA
      X(1)=ETERM*(COEF*COSTM+(XDOT(1)+ALPHA*COEF/(2.*M(1)))*SINTM/ROOT)
     1 +H/BETA
      XDOT(1)=ETERM*(XDOT(1)*COSTM-SINTM*(ALPHA*XDOT(1)+2.*BETA*COEF)
     1 /(2.*M(1)*ROOT))
   20 CONTINUE
      DO 30 I=2,6
      CALL ZERO(K,C)
      IF( X(I-1)-X(I)  .LE.  DEL(I)  ) K(I)=KON(I)
      IF( X(I-1)-X(I)  .LE.  DEL(I)  ) C(I)=CON(I)
      IF( X(I)-X(I+1)  .LE.  DEL(I+1)  ) K(I+1)=KON(I+1)
      IF( X(I)-X(I+1)  .LE.  DEL(I+1)  ) C(I+1)=CON(I+1)
      ALPHA=C(I)+C(I+1)
      BETA=K(I)+K(I+1)
      POWER=-ALPHA/(2.*M(I))
      ETERM=EXP(POWER*INCR)
      IF ( BETA/M(I) - (ALPHA/(2.*M(I)))**2) 21,22,23
   21 ROOT=SQRT((ALPHA/(2.*M(I)))**2 - BETA/M(I))
      ETERMP=EXP(ROOT*INCR)
      ETERMN=EXP(-ROOT*INCR)
      H=K(I)*(X(I-1)-DEL(I))+C(I)*XDOT(I-1)+K(I+1)*(X(I+1)+DEL(I+1))
     1 +C(I+1)*XDOT(I+1)
      COEF1=(XDOT(I)+(ROOT-POWER)*(X(I)-H/BETA))/(2.*ROOT)
      COEF2=X(I)-H/BETA-COEF1
      X(I)=ETERM*(COEF1*ETERMP+COEF2*ETERMN)+H/BETA
      XDOT(I)=ETERM*(COEF1*ETERMP*(ROOT+POWER)-COEF2*ETERMN*(ROOT-POWER)
     1 )
      GO TO 30
   22 BETA=BETA+10.**(-5)
   23 ROOT=SQRT(BETA/M(I) - (ALPHA/(2.*M(I)))**2)
      COSTM=COS(ROOT*INCR)
      SINTM=SIN(ROOT*INCR)
      H=K(I)*(X(I-1)-DEL(I))+C(I)*XDOT(I-1)+K(I+1)*(X(I+1)+DEL(I+1))
     1 +C(I+1)*XDOT(I+1)
      COEF=X(I)-H/BETA
      X(I)=ETERM*(COEF*COSTM+(XDOT(I)+ALPHA*COEF/(2.*M(I)))*SINTM/ROOT)
     1 +H/BETA
      XDOT(I)=ETERM*(XDOT(I)*COSTM-SINTM*(ALPHA*XDOT(I)+2.*BETA*COEF)
     1 /(2.*M(I)*ROOT))
   30 CONTINUE
      CALL ZERO(K,C)
```

```
      IF( X(6)-X(7) .LE. DEL(7) ) K(7)=KON(7)
      IF( X(6)-X(7) .LE. DEL(7) ) C(7)=CON(7)
      IF( X(7)+DEL(8) .LE. X(1) ) K(8)=KON(8)
      IF( X(7)+DEL(8) .LE. X(1) ) C(8)=CON(8)
      ALPHA=C(7)+C(8)
      BETA=K(7)+K(8)
      POWER=-ALPHA/(2.*M(7))
      ETERM=EXP(POWER*INCR)
      IF ( BETA/M(7) - (ALPHA/(2.*M(7)))**2) 31,32,33
   31 ROOT=SQRT((ALPHA/(2.*M(7)))**2 - BETA/M(7))
      ETERMP=EXP(ROOT*INCR)
      ETERMN=EXP(-ROOT*INCR)
      H=K(7)*(X(6)-DEL(7))+C(7)*XDOT(6)-K(8)*(DEL(8)-X(1))+C(8)*XDOT(1)
      COEF1=(XDOT(7)+(ROOT-POWER)*(X(7)-H/BETA))/(2.*ROOT)
      COEF2=X(7)-H/BETA-COEF1
      X(7)=ETERM*(COEF1*ETERMP+COEF2*ETERMN)+H/BETA
      XDOT(7)=ETERM*(COEF1*ETERMP*(ROOT+POWER)-COEF2*ETERMN*(ROOT-POWER)
     1 )
      GO TO 40
   32 BETA=BETA+10.**(-5)
   33 ROOT=SQRT(BETA/M(7) - (ALPHA/(2.*M(7)))**2)
      COSTM=COS(ROOT*INCR)
      SINTM=SIN(ROOT*INCR)
      H=K(7)*(X(6)-DEL(7))+C(7)*XDOT(6)-K(8)*(DEL(8)-X(1))+C(8)*XDOT(1)
      COEF=X(7)-H/BETA
      X(7)=ETERM*(COEF*COSTM+(XDOT(7)+ALPHA*COEF/(2.*M(7)))*SINTM/ROOT)
     1 +H/BETA
      XDOT(7)=ETERM*(XDOT(7)*COSTM-SINTM*(ALPHA*XDOT(7)+2.*BETA*COEF1
     1 /(2.*M(7)*ROOT))
   40 CONTINUE
      RETURN
      END
      SUBROUTINE ZERO(X,Y)
      DIMENSION X(9), Y(9)
      DO 1 I=1,9
      X(I)=0.
    1 Y(I)=0.
      RETURN
      END
/*
//GO.SYSIN DD *
         20
0.0       0.0       9.7       18.6      11.6      21.2      13.1      23.0
16.2      26.7      28.1      45.0      38.1      60.0      45.6      72.0
52.1      80.5      69.1      97.9      71.1      97.3      73.1      96.9
74.1      96.6      87.1      87.0      103.1     73.5      117.6     60.0
129.1     49.0      138.1     39.0      173.1     0.0       99990000. 0.0
10.       0.0       32.0      -123.14   0.20      .00001
50.       0.        50.       0.        50.       0.        50.       0.
.0002570  .0001057  .00005449
32.0      0.0       0.0       0.0       0.0       0.0       0.0       0.13
13.8125
         44
1.4       0.0       12.2      8.16      0.0003364
         10
          1
0.0       0.2       -2.       8.0       0.0       0.0
BUFFER MOTION
DISPLACEMENT
TIME
0.0       0.2       -0.5      9.50      0.0       0.0
```

```
1        1        1        1        1        1        1
SPRING MOTION
DISPLACEMENT
     TIME
0.0       0.2       0.0       5.0       0.0       0.0
RELATIVE MOTION
DISPLACEMENT
TIME
/*
//FT04F001 DD DSN=&TAPE4,UNIT=2314,SPACE=(324,(1000,100)),
//         DCB=(RECFM=VBS,LRECL=32,BLKSIZE=324),DISP=(NEW,DELETE)
/*
```

```
                                   I N P U T   D A T A

                                       B U F F E R

XMG             XMGDOT          XOP             XOPDOT          TEND            DELTA T
0.100000E 02    0.0             0.320000E 02   -0.123140E 03    0.200000E 00    0.100000E-04

K1              K2...K6         K8&K9           K7              C1              C2...C6         C8&C9           C7
0.500000E 02    0.500000E 02    0.500000E 02    0.500000E 02    0.0             0.0             0.0             0.0

M1              M2...M6         M7
0.257000E-03    0.105700E-03    0.544900E-04

                                                        DELTA 1 THRU DELTA 9
0.320000E 02    0.0             0.0             0.0             0.0             0.0             0.0             0.130000E 00    0.138125E 02

                                       S P R I N G

COILS
44

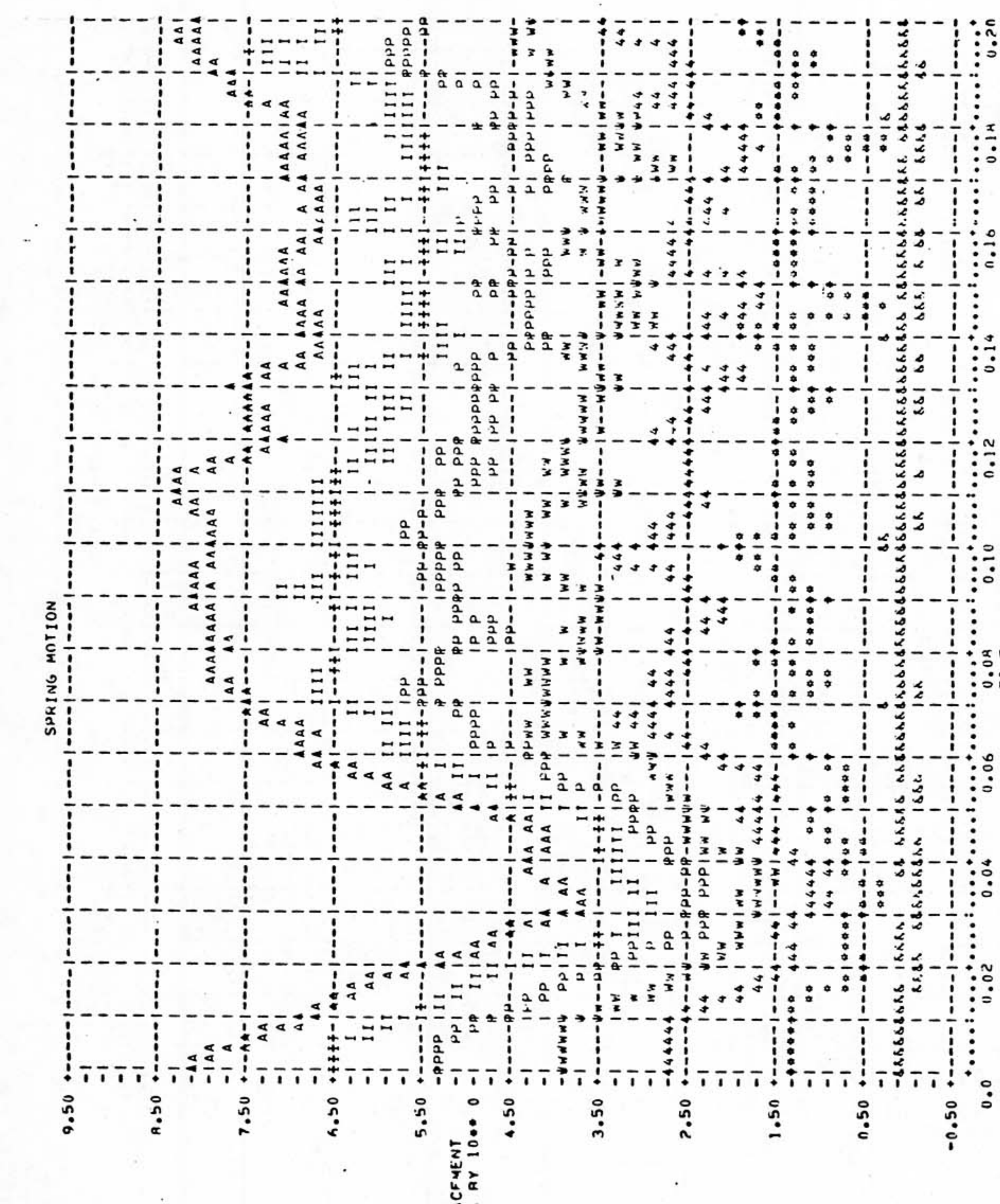
K               C                       FREE LENGTH     INITL LENGTH    M
0.140000E 01    0.0                     0.122000E 02    0.816000E 01    0.336400E-03

                10=NTHTHN                                       1=NTHPLT

            SPRING IS UNDER DAMPED     A F T E R   A D J U S T M E N T

K               C                       M
0.630000E 02    0.0                     0.764545E-05
```


BUFFER MOTION
DISPLACEMENT
MULT. BY 10** 0
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
-0.00
-1.00
-2.00
TIME
MULT. BY 10** 0
0.0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20


SPRING MOTION
DISPLACEMENT
MULT. BY 10** 0
9.50
8.50
7.50
6.50
5.50
4.50
3.50
2.50
1.50
0.50
-0.50
0.0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
TIME
MULT. BY 10** 0

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Technical Report # R-TR-75-010

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Research Directorate, General Thomas J. Rodman Laboratory
Rock Island Arsenal, Rock Island, Illinois 61201

Theory and Application of Mathematical Modeling of Shoulder-Fired Weapons
Part I of II, Final Report

Prepared By: Paul E. Ehle and Albert E. Rahe
Security Class. (of this report): Unclassified
Technical Report R-TR-75-010

243 Pages, Incl Figures

A mathematical model of the M16A1 Rifle was constructed by personnel of the Research Directorate to serve as (1) a vehicle to extend the state-of-the-art in weapon modeling, (2) a basis with which to compare performance characteristics of other weapons such as the XM19 Rifle, (3) an aid to troubleshoot the weapon (4) a means to increase present knowledge of the M16A1, and (5) a way to explore and demonstrate the capabilities and limitations of mathematical models of weapons.

UNCLASSIFIED

1. M16 Rifle 2. Dynamics
3. Simulation 4. Man Weapon
5. Kinematics 6. Mathematical Model

I. Paul E. Ehle and Albert E. Rahe
II. Rock Island Arsenal
III. Research Directorate
General Thomas J. Rodman Laboratory
Rock Island Arsenal

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For each component mass, equations of motion are written. These allow a total of eleven degrees of freedom plus a nearly unlimited number of degrees of freedom for the drive spring. To provide accuracy data, one rotational degree of freedom is allowed for weapon pitch motion. Expressions are derived for the many diverse forces acting on the masses. The resulting equations are solved on an IBM 360/65 digital computer. A sensitivity analysis is conducted, and the results are compared with those from a similar model of the XM19 Rifle. The greatest differences between the M16A1 and the XM19 sensitivities were found to be the effect of the ignition delay and the drive spring on cycle time and the mounting conditions on accuracy.

The methods developed during model construction are applicable to the modeling of many other weapons. Part II of this two-report series is an application of these techniques to the XM19 Rifle.

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